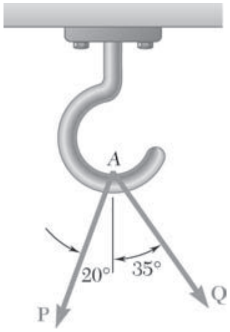
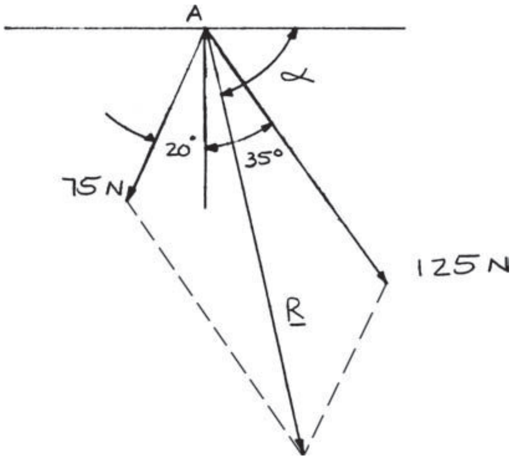
PROBLEM 2.1



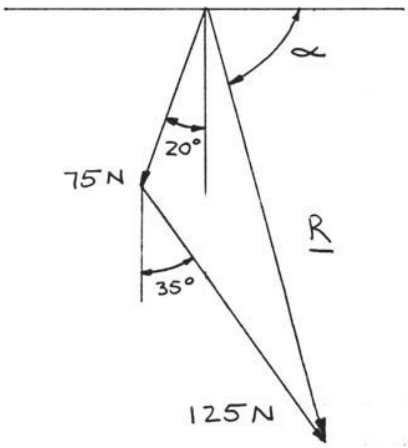
Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 75$ N and $Q = 125$ N, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

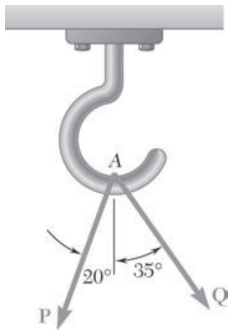
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 179$ N, $\alpha = 75.1°$ **R** $= 179$ N ⦪ $75.1°$ ◀

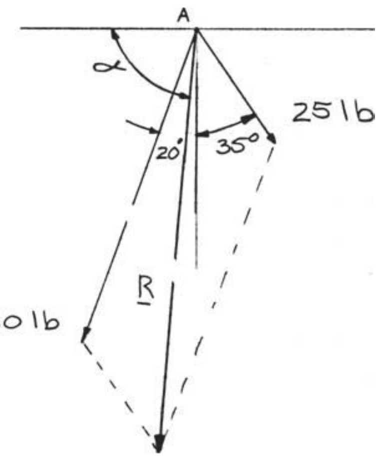


PROBLEM 2.2

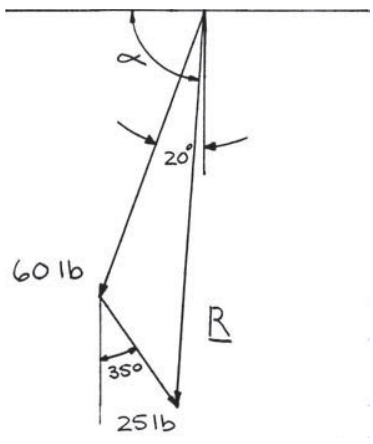
Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 60$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

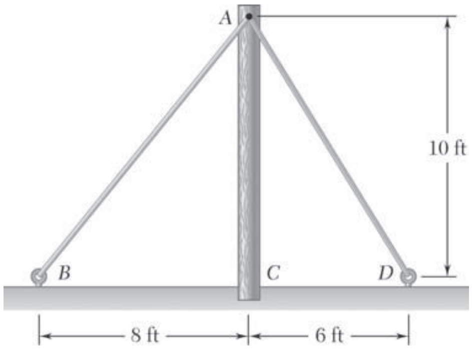
(a) Parallelogram law:



(b) Triangle rule:



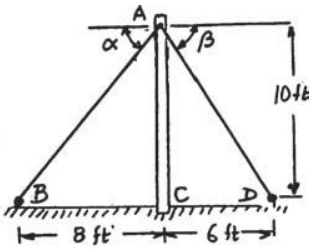
We measure: $R = 77.1$ lb, $\alpha = 85.4°$ **R** $= 77.1$ lb ⦣ $85.4°$ ◀



PROBLEM 2.3

The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 120 lb in *AB* and 40 lb in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (*a*) the parallelogram law, (*b*) the triangle rule.

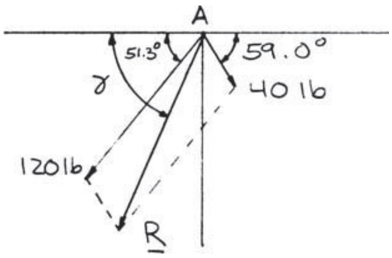
SOLUTION



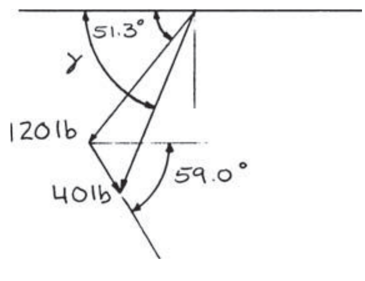
We measure: $\alpha = 51.3°$

$\beta = 59.0°$

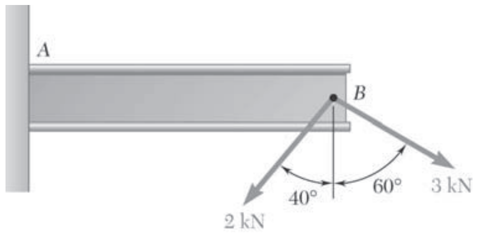
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 139.1$ lb, $\gamma = 67.0°$ $\qquad R = 139.1 \text{ lb} \measuredangle\ 67.0°$ ◀

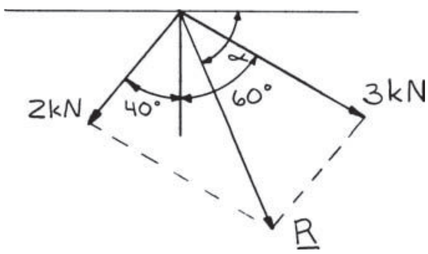


PROBLEM 2.4

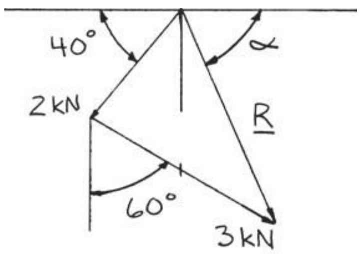
Two forces are applied at Point *B* of beam *AB*. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

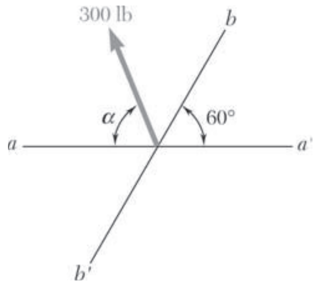
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 3.30$ kN, $\alpha = 66.6°$ **R** $= 3.30$ kN ◄ 66.6° ◄

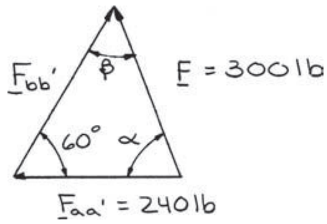


PROBLEM 2.5

The 300-lb force is to be resolved into components along lines a-a' and b-b'. (a) Determine the angle α by trigonometry knowing that the component along line a-a' is to be 240 lb. (b) What is the corresponding value of the component along b-b'?

SOLUTION

(a) Using the triangle rule and law of sines:



$$\frac{\sin\beta}{240\text{ lb}} = \frac{\sin 60^\circ}{300\text{ lb}}$$

$$\sin\beta = 0.69282$$

$$\beta = 43.854^\circ$$

$$\alpha + \beta + 60^\circ = 180^\circ$$

$$\alpha = 180^\circ - 60^\circ - 43.854^\circ$$

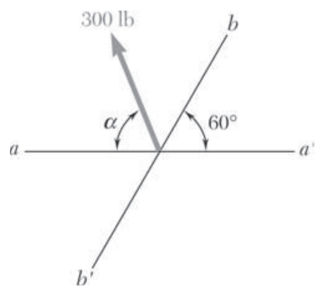
$$= 76.146^\circ$$

$\alpha = 76.1^\circ$ ◀

(b) Law of sines:

$$\frac{F_{bb'}}{\sin 76.146^\circ} = \frac{300\text{ lb}}{\sin 60^\circ}$$

$F_{bb'} = 336$ lb ◀

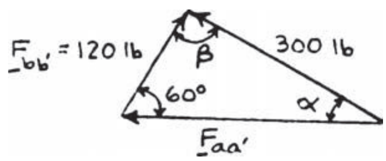


PROBLEM 2.6

The 300-lb force is to be resolved into components along lines a-a' and b-b'. (a) Determine the angle α by trigonometry knowing that the component along line b-b' is to be 120 lb. (b) What is the corresponding value of the component along a-a'?

SOLUTION

Using the triangle rule and law of sines:



(a)
$$\frac{\sin\alpha}{120 \text{ lb}} = \frac{\sin 60^\circ}{300 \text{ lb}}$$

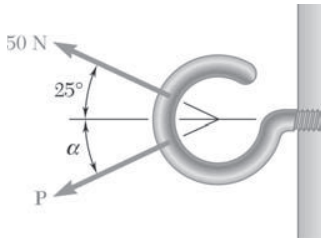
$$\sin\alpha = 0.34641$$
$$\alpha = 20.268^\circ$$

$\alpha = 20.3^\circ$ ◀

(b)
$$\alpha + \beta + 60^\circ = 180^\circ$$
$$\beta = 180^\circ - 60^\circ - 20.268^\circ$$
$$= 99.732^\circ$$

$$\frac{F_{aa'}}{\sin 99.732^\circ} = \frac{300 \text{ lb}}{\sin 60^\circ}$$

$F_{aa'} = 341$ lb ◀

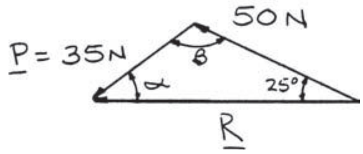


PROBLEM 2.7

Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the triangle rule and law of sines:



(*a*)

$$\frac{\sin\alpha}{50\text{ N}} = \frac{\sin 25^\circ}{35\text{ N}}$$

$$\sin\alpha = 0.60374$$

$$\alpha = 37.138^\circ$$

$\alpha = 37.1^\circ$ ◀

(*b*)

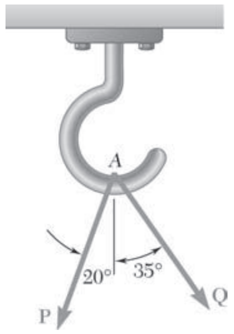
$$\alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

$$= 117.86^\circ$$

$$\frac{R}{\sin 117.86} = \frac{35\text{ N}}{\sin 25^\circ}$$

$R = 73.2$ N ◀

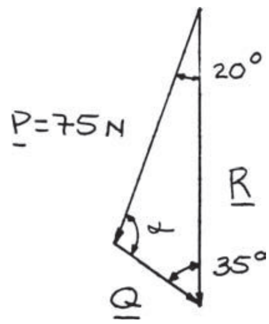


PROBLEM 2.8

For the hook support of Problem 2.1, knowing that the magnitude of **P** is 75 N, determine by trigonometry (*a*) the required magnitude of the force **Q** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

PROBLEM 2.1 Two forces **P** and **Q** are applied as shown at Point *A* of a hook support. Knowing that $P = 75$ N and $Q = 125$ N, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

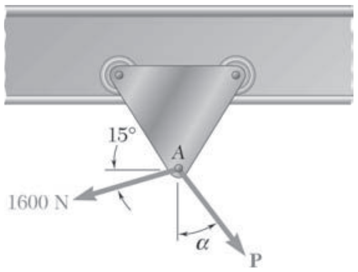
SOLUTION



Using the triangle rule and law of sines:

(*a*) $$\frac{Q}{\sin 20°} = \frac{75\text{ N}}{\sin 35°}$$ $Q = 44.7$ N ◀

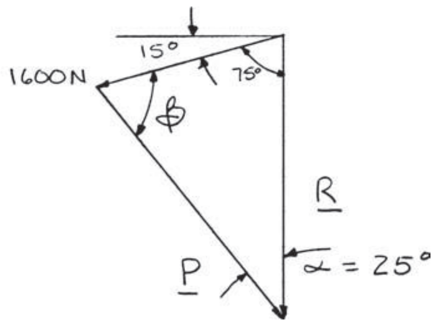
(*b*) $$\alpha + 20° + 35° = 180°$$
$$\alpha = 180° - 20° - 35°$$
$$= 125°$$
$$\frac{R}{\sin 125°} = \frac{75\text{ N}}{\sin 35°}$$ $R = 107.1$ N ◀



PROBLEM 2.9

A trolley that moves along a horizontal beam is acted upon by two forces as shown. (*a*) Knowing that $\alpha = 25°$, determine by trigonometry the magnitude of the force **P** so that the resultant force exerted on the trolley is vertical. (*b*) What is the corresponding magnitude of the resultant?

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$\frac{1600 \text{ N}}{\sin 25°} = \frac{P}{\sin 75°}$$

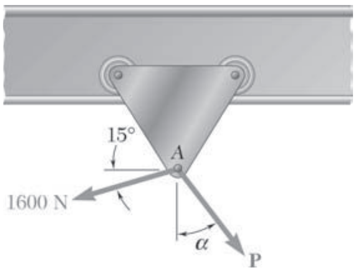
$P = 3660$ N ◀

(*b*)

$$25° + \beta + 75° = 180°$$
$$\beta = 180° - 25° - 75°$$
$$= 80°$$

$$\frac{1600 \text{ N}}{\sin 25°} = \frac{R}{\sin 80°}$$

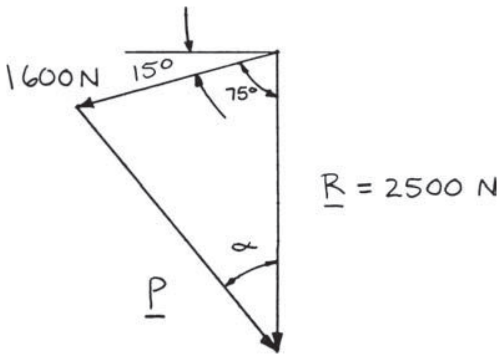
$R = 3730$ N ◀



PROBLEM 2.10

A trolley that moves along a horizontal beam is acted upon by two forces as shown. Determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 2500 N.

SOLUTION



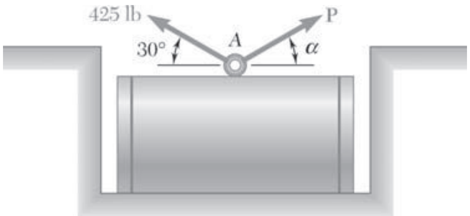
Using the law of cosines: $P^2 = (1600 \text{ N})^2 + (2500 \text{ N})^2 - 2(1600 \text{ N})(2500 \text{ N})\cos 75°$

$P = 2596 \text{ N}$

Using the law of sines: $\dfrac{\sin\alpha}{1600 \text{ N}} = \dfrac{\sin 75°}{2596 \text{ N}}$

$\alpha = 36.5°$

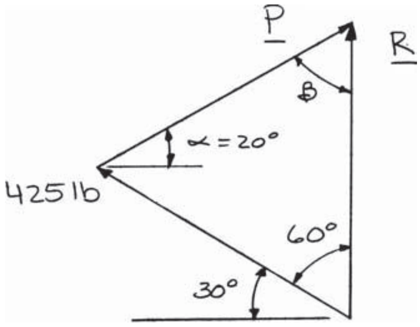
P is directed $90° - 36.5°$ or $53.5°$ below the horizontal. **P** = 2600 N ◥ 53.5° ◀



PROBLEM 2.11

A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20°$, determine by trigonometry (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$\beta + 50° + 60° = 180°$$

$$\beta = 180° - 50° - 60°$$

$$= 70°$$

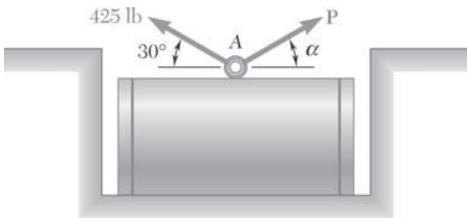
$$\frac{425 \text{ lb}}{\sin 70°} = \frac{P}{\sin 60°}$$

$P = 392$ lb ◀

(*b*)

$$\frac{425 \text{ lb}}{\sin 70°} = \frac{R}{\sin 50°}$$

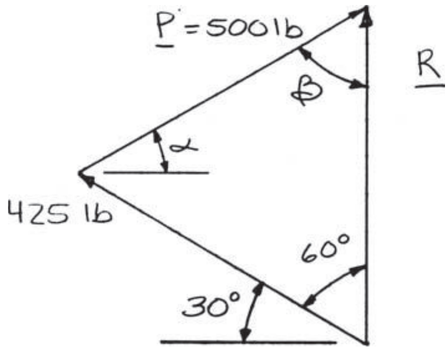
$R = 346$ lb ◀



PROBLEM 2.12

A steel tank is to be positioned in an excavation. Knowing that the magnitude of **P** is 500 lb, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$(\alpha + 30°) + 60° + \beta = 180°$$

$$\beta = 180° - (\alpha + 30°) - 60°$$

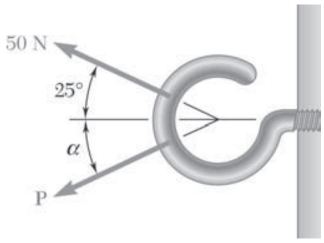
$$\beta = 90° - \alpha$$

$$\frac{\sin(90° - \alpha)}{425 \text{ lb}} = \frac{\sin 60°}{500 \text{ lb}}$$

$$90° - \alpha = 47.40° \qquad \alpha = 42.6° \blacktriangleleft$$

(*b*)

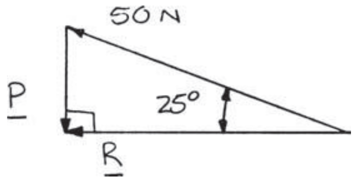
$$\frac{R}{\sin(42.6° + 30°)} = \frac{500 \text{ lb}}{\sin 60°} \qquad R = 551 \text{ lb} \blacktriangleleft$$



PROBLEM 2.13

For the hook support of Problem 2.7, determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied to the support is horizontal, (*b*) the corresponding magnitude of **R**.

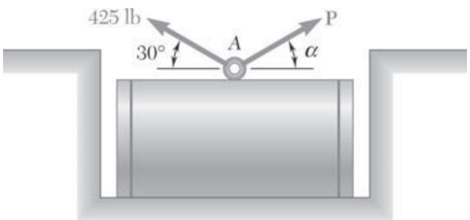
SOLUTION



The smallest force P will be perpendicular to R.

(*a*) $P = (50\text{ N})\sin 25°$ **P** $= 21.1\text{ N}\downarrow$ ◀

(*b*) $R = (50\text{ N})\cos 25°$ $R = 45.3\text{ N}$ ◀

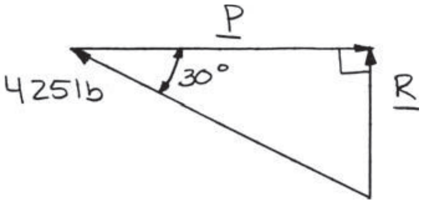


PROBLEM 2.14

For the steel tank of Problem 2.11, determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied at *A* is vertical, (*b*) the corresponding magnitude of **R**.

PROBLEM 2.11 A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20°$, determine by trigonometry (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

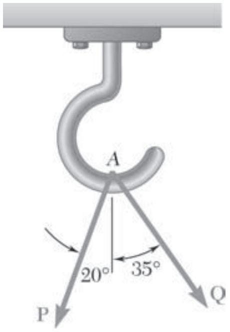
SOLUTION



The smallest force *P* will be perpendicular to *R*.

(*a*) $P = (425 \text{ lb})\cos 30°$ **P** $= 368$ lb $\longrightarrow$ ◀

(*b*) $R = (425 \text{ lb})\sin 30°$ $R = 213$ lb ◀



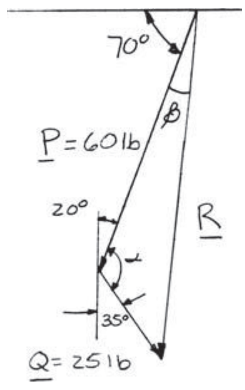
PROBLEM 2.15

Solve Problem 2.2 by trigonometry.

PROBLEM 2.2 Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 60$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

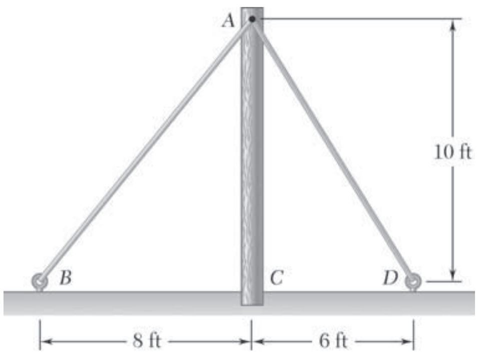
Using the triangle rule and the law of cosines:



$$20° + 35° + \alpha = 180°$$
$$\alpha = 125°$$
$$R^2 = P^2 + Q^2 - 2PQ\cos\alpha$$
$$R^2 = (60\text{ lb})^2 + (25\text{ lb})^2 - 2(60\text{ lb})(25\text{ lb})\cos 125°$$
$$R^2 = 3600 + 625 + 3000(0.5736)$$
$$R = 77.108\text{ lb}$$

Using the law of sines:
$$\frac{\sin\beta}{25\text{ lb}} = \frac{\sin 125°}{77.108\text{ lb}}$$
$$\beta = 15.402°$$
$$70° + \beta = 85.402°$$

R = 77.1 lb ⦪ 85.4° ◀

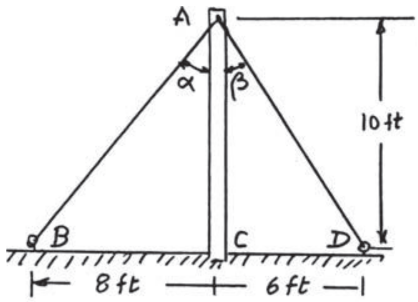


PROBLEM 2.16

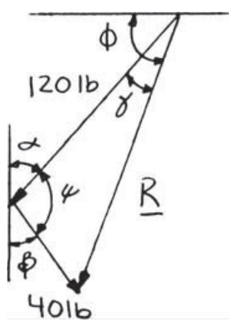
Solve Problem 2.3 by trigonometry.

PROBLEM 2.3 The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 120 lb in *AB* and 40 lb in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION



$$\tan\alpha = \frac{8}{10}$$
$$\alpha = 38.66^\circ$$
$$\tan\beta = \frac{6}{10}$$
$$\beta = 30.96^\circ$$



Using the triangle rule:
$$\alpha + \beta + \psi = 180^\circ$$
$$38.66^\circ + 30.96^\circ + \psi = 180^\circ$$
$$\psi = 110.38^\circ$$

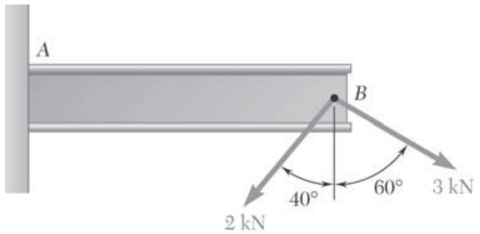
Using the law of cosines:
$$R^2 = (120\text{ lb})^2 + (40\text{ lb})^2 - 2(120\text{ lb})(40\text{ lb})\cos 110.38^\circ$$
$$R = 139.08\text{ lb}$$

Using the law of sines:
$$\frac{\sin\gamma}{40\text{ lb}} = \frac{\sin 110.38^\circ}{139.08\text{ lb}}$$

$$\gamma = 15.64^\circ$$
$$\phi = (90^\circ - \alpha) + \gamma$$
$$\phi = (90^\circ - 38.66^\circ) + 15.64^\circ$$
$$\phi = 66.98^\circ$$

R = 139.1 lb ⦪ 67.0° ◀

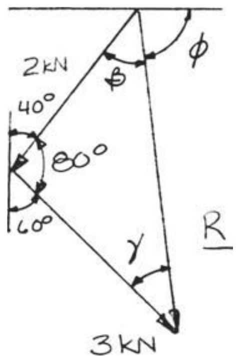
PROBLEM 2.17



Solve Problem 2.4 by trigonometry.

PROBLEM 2.4 Two forces are applied at Point *B* of beam *AB*. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION



Using the law of cosines:

$$R^2 = (2\text{ kN})^2 + (3\text{ kN})^2 - 2(2\text{ kN})(3\text{ kN})\cos 80^\circ$$

$$R = 3.304\text{ kN}$$

Using the law of sines:

$$\frac{\sin\gamma}{2\text{ kN}} = \frac{\sin 80^\circ}{3.304\text{ kN}}$$

$$\gamma = 36.59^\circ$$

$$\beta + \gamma + 80^\circ = 180^\circ$$

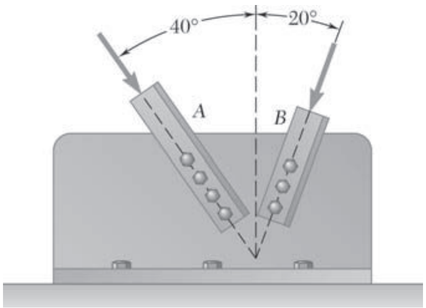
$$\gamma = 180^\circ - 80^\circ - 36.59^\circ$$

$$\gamma = 63.41^\circ$$

$$\phi = 180^\circ - \beta + 50^\circ$$

$$\phi = 66.59^\circ$$

R = 3.30 kN ⦪ 66.6° ◀

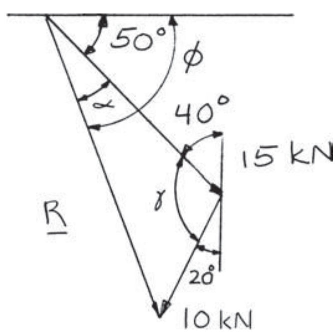


PROBLEM 2.18

Two structural members A and B are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member A and 10 kN in member B, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members A and B.

SOLUTION

Using the force triangle and the laws of cosines and sines:



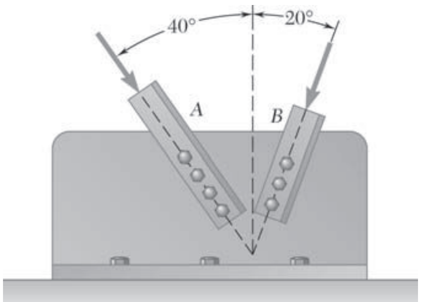
We have
$$\gamma = 180° - (40° + 20°)$$
$$= 120°$$

Then
$$R^2 = (15 \text{ kN})^2 + (10 \text{ kN})^2 - 2(15 \text{ kN})(10 \text{ kN})\cos 120°$$
$$= 475 \text{ kN}^2$$
$$R = 21.794 \text{ kN}$$

and
$$\frac{10 \text{ kN}}{\sin\alpha} = \frac{21.794 \text{ kN}}{\sin 120°}$$
$$\sin\alpha = \left(\frac{10 \text{ kN}}{21.794 \text{ kN}}\right)\sin 120°$$
$$= 0.39737$$
$$\alpha = 23.414$$

Hence:
$$\phi = \alpha + 50° = 73.414$$

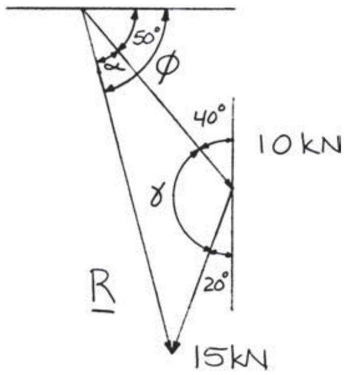
$\mathbf{R} = 21.8 \text{ kN} \measuredangle\ 73.4°$ ◀



PROBLEM 2.19

Two structural members A and B are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 10 kN in member A and 15 kN in member B, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members A and B.

SOLUTION



Using the force triangle and the laws of cosines and sines

We have
$$\gamma = 180° - (40° + 20°)$$
$$= 120°$$

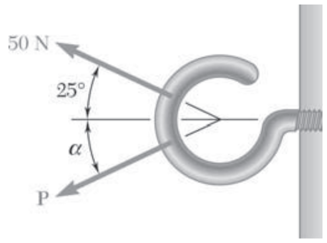
Then
$$R^2 = (10 \text{ kN})^2 + (15 \text{ kN})^2 - 2(10 \text{ kN})(15 \text{ kN})\cos 120°$$
$$= 475 \text{ kN}^2$$
$$R = 21.794 \text{ kN}$$

and
$$\frac{15 \text{ kN}}{\sin\alpha} = \frac{21.794 \text{ kN}}{\sin 120°}$$
$$\sin\alpha = \left(\frac{15 \text{ kN}}{21.794 \text{ kN}}\right)\sin 120°$$
$$= 0.59605$$
$$\alpha = 36.588°$$

Hence:
$$\phi = \alpha + 50° = 86.588°$$

R = 21.8 kN ⦪ 86.6° ◀

PROBLEM 2.20

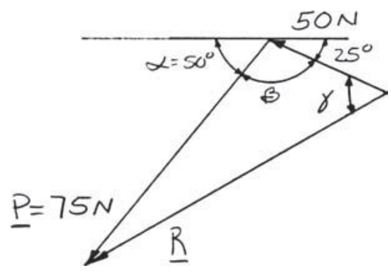


For the hook support of Problem 2.7, knowing that $P = 75\text{ N}$ and $\alpha = 50°$, determine by trigonometry the magnitude and direction of the resultant of the two forces applied to the support.

PROBLEM 2.7 Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the force triangle and the laws of cosines and sines:



We have $\beta = 180° - (50° + 25°)$

$$= 105°$$

Then $R^2 = (75\text{ N})^2 + (50\text{ N})^2$

$$-2(75\text{ N})(50\text{ N})\cos 105°$$

$$R^2 = 10066.1\text{ N}^2$$

$$R = 100.330\text{ N}$$

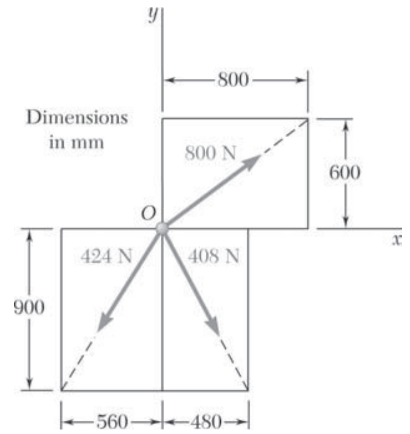
and $\dfrac{\sin\gamma}{75\text{ N}} = \dfrac{\sin 105°}{100.330\text{ N}}$

$$\sin\gamma = 0.72206$$

$$\gamma = 46.225°$$

Hence: $\gamma - 25° = 46.225° - 25° = 21.225°$

$\mathbf{R} = 100.3\text{ N} \measuredangle\ 21.2°$ ◀

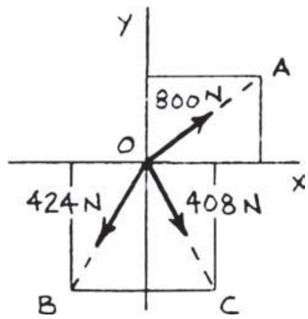


PROBLEM 2.21

Determine the x and y components of each of the forces shown.

SOLUTION

Compute the following distances:



$$OA = \sqrt{(600)^2 + (800)^2}$$
$$= 1000 \text{ mm}$$
$$OB = \sqrt{(560)^2 + (900)^2}$$
$$= 1060 \text{ mm}$$
$$OC = \sqrt{(480)^2 + (900)^2}$$
$$= 1020 \text{ mm}$$

800-N Force: $F_x = +(800 \text{ N})\dfrac{800}{1000}$ $\quad F_x = +640 \text{ N}$ ◀

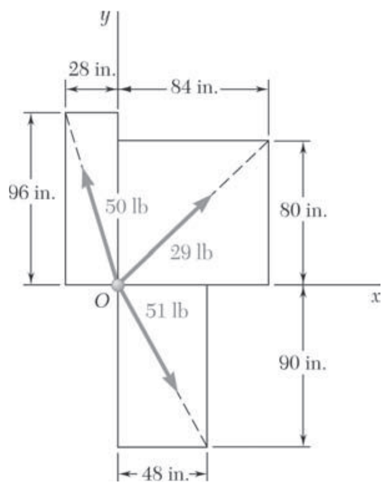
$F_y = +(800 \text{ N})\dfrac{600}{1000}$ $\quad F_y = +480 \text{ N}$ ◀

424-N Force: $F_x = -(424 \text{ N})\dfrac{560}{1060}$ $\quad F_x = -224 \text{ N}$ ◀

$F_y = -(424 \text{ N})\dfrac{900}{1060}$ $\quad F_y = -360 \text{ N}$ ◀

408-N Force: $F_x = +(408 \text{ N})\dfrac{480}{1020}$ $\quad F_x = +192.0 \text{ N}$ ◀

$F_y = -(408 \text{ N})\dfrac{900}{1020}$ $\quad F_y = -360 \text{ N}$ ◀

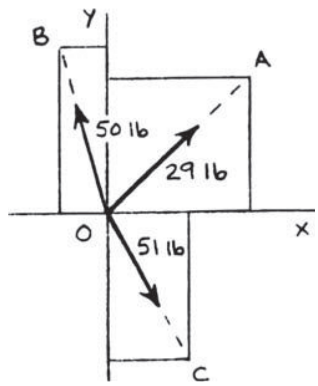


PROBLEM 2.22

Determine the x and y components of each of the forces shown.

SOLUTION

Compute the following distances:



$$OA = \sqrt{(84)^2 + (80)^2}$$
$$= 116 \text{ in.}$$
$$OB = \sqrt{(28)^2 + (96)^2}$$
$$= 100 \text{ in.}$$
$$OC = \sqrt{(48)^2 + (90)^2}$$
$$= 102 \text{ in.}$$

29-lb Force:

$$F_x = +(29 \text{ lb})\frac{84}{116} \qquad F_x = +21.0 \text{ lb} \blacktriangleleft$$

$$F_y = +(29 \text{ lb})\frac{80}{116} \qquad F_y = +20.0 \text{ lb} \blacktriangleleft$$

50-lb Force:

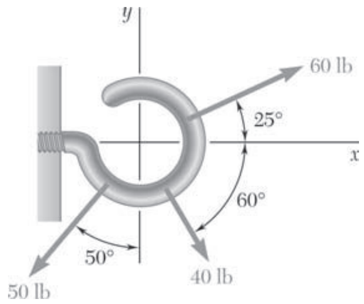
$$F_x = -(50 \text{ lb})\frac{28}{100} \qquad F_x = -14.00 \text{ lb} \blacktriangleleft$$

$$F_y = +(50 \text{ lb})\frac{96}{100} \qquad F_y = +48.0 \text{ lb} \blacktriangleleft$$

51-lb Force:

$$F_x = +(51 \text{ lb})\frac{48}{102} \qquad F_x = +24.0 \text{ lb} \blacktriangleleft$$

$$F_y = -(51 \text{ lb})\frac{90}{102} \qquad F_y = -45.0 \text{ lb} \blacktriangleleft$$



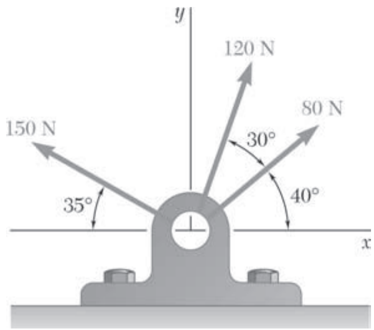
PROBLEM 2.23

Determine the x and y components of each of the forces shown.

SOLUTION

40-lb Force:	$F_x = +(40 \text{ lb})\cos 60°$	$F_x = 20.0 \text{ lb}$ ◀
	$F_y = -(40 \text{ lb})\sin 60°$	$F_y = -34.6 \text{ lb}$ ◀
50-lb Force:	$F_x = -(50 \text{ lb})\sin 50°$	$F_x = -38.3 \text{ lb}$ ◀
	$F_y = -(50 \text{ lb})\cos 50°$	$F_y = -32.1 \text{ lb}$ ◀
60-lb Force:	$F_x = +(60 \text{ lb})\cos 25°$	$F_x = 54.4 \text{ lb}$ ◀
	$F_y = +(60 \text{ lb})\sin 25°$	$F_y = 25.4 \text{ lb}$ ◀

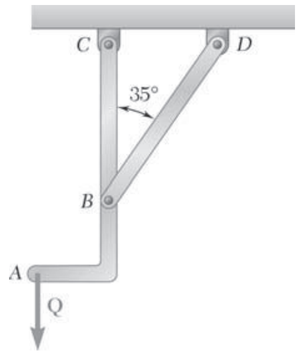
PROBLEM 2.24



Determine the x and y components of each of the forces shown.

SOLUTION

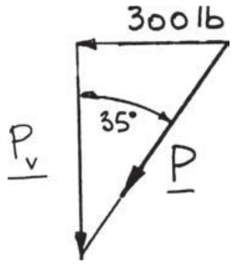
80-N Force:	$F_x = +(80\text{ N})\cos 40°$	$F_x = 61.3\text{ N}$ ◀
	$F_y = +(80\text{ N})\sin 40°$	$F_y = 51.4\text{ N}$ ◀
120-N Force:	$F_x = +(120\text{ N})\cos 70°$	$F_x = 41.0\text{ N}$ ◀
	$F_y = +(120\text{ N})\sin 70°$	$F_y = 112.8\text{ N}$ ◀
150-N Force:	$F_x = -(150\text{ N})\cos 35°$	$F_x = -122.9\text{ N}$ ◀
	$F_y = +(150\text{ N})\sin 35°$	$F_y = 86.0\text{ N}$ ◀



PROBLEM 2.25

Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 300-lb horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION



(*a*) $P \sin 35° = 300 \text{ lb}$

$$P = \frac{300 \text{ lb}}{\sin 35°}$$

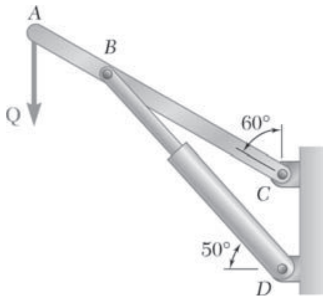
$P = 523 \text{ lb}$ ◀

(*b*) Vertical component $P_v = P \cos 35°$

$$= (523 \text{ lb}) \cos 35°$$

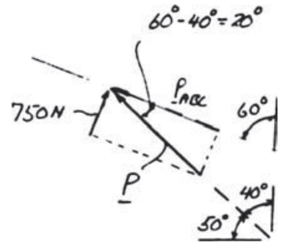
$P_v = 428 \text{ lb}$ ◀

PROBLEM 2.26



The hydraulic cylinder *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 750-N component perpendicular to member *ABC*, determine (*a*) the magnitude of the force **P**, (*b*) its component parallel to *ABC*.

SOLUTION

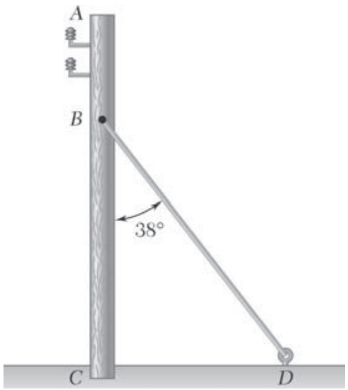


(*a*) $750\text{ N} = P\sin 20°$

$P = 2193\text{ N}$ $\qquad$ $P = 2190\text{ N}$ ◀

(*b*) $P_{ABC} = P\cos 20°$

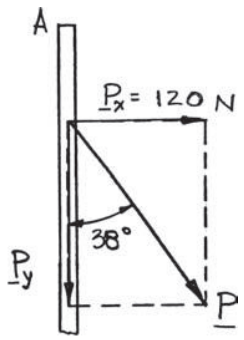
$= (2193\text{ N})\cos 20°$ $\qquad$ $P_{ABC} = 2060\text{ N}$ ◀



PROBLEM 2.27

The guy wire BD exerts on the telephone pole AC a force **P** directed along BD. Knowing that **P** must have a 120-N component perpendicular to the pole AC, determine (*a*) the magnitude of the force **P**, (*b*) its component along line AC.

SOLUTION



(*a*)

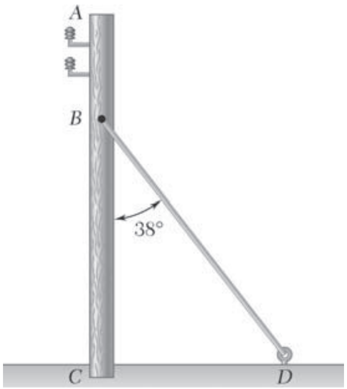
$$P = \frac{P_x}{\sin 38°}$$
$$= \frac{120 \text{ N}}{\sin 38°}$$
$$= 194.91 \text{ N}$$

or $P = 194.9 \text{ N}$ ◀

(*b*)

$$P_y = \frac{P_x}{\tan 38°}$$
$$= \frac{120 \text{ N}}{\tan 38°}$$
$$= 153.59 \text{ N}$$

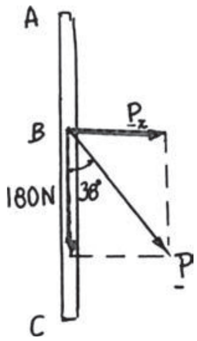
or $P_y = 153.6 \text{ N}$ ◀



PROBLEM 2.28

The guy wire BD exerts on the telephone pole AC a force **P** directed along BD. Knowing that **P** has a 180-N component along line AC, determine (*a*) the magnitude of the force **P**, (*b*) its component in a direction perpendicular to AC.

SOLUTION



(*a*)

$$P = \frac{P_y}{\cos 38°}$$

$$= \frac{180 \text{ N}}{\cos 38°}$$

$$= 228.4 \text{ N}$$

$P = 228 \text{ N}$ ◀

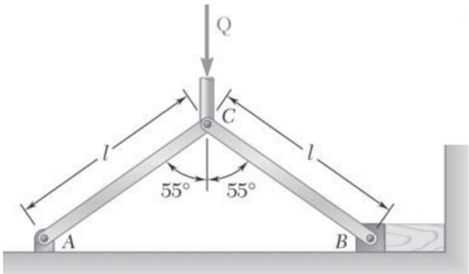
(*b*)

$$P_x = P_y \tan 38°$$

$$= (180 \text{ N}) \tan 38°$$

$$= 140.63 \text{ N}$$

$P_x = 140.6 \text{ N}$ ◀



PROBLEM 2.29

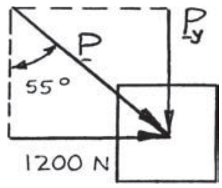
Member CB of the vise shown exerts on block B a force $\mathbf{P}$ directed along line CB. Knowing that $\mathbf{P}$ must have a 1200-N horizontal component, determine (*a*) the magnitude of the force $\mathbf{P}$, (*b*) its vertical component.

SOLUTION

We note:

CB exerts force $\mathbf{P}$ on B along CB, and the horizontal component of $\mathbf{P}$ is $P_x = 1200$ N:

Then



(*a*)

$$P_x = P \sin 55°$$

$$P = \frac{P_x}{\sin 55°}$$

$$= \frac{1200 \text{ N}}{\sin 55°}$$

$$= 1464.9 \text{ N}$$

$P = 1465$ N ◀

(*b*)

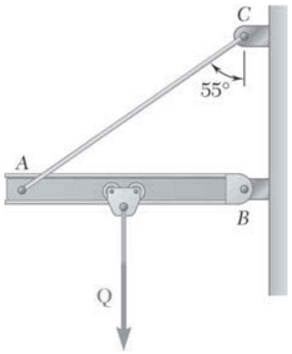
$$P_x = P_y \tan 55°$$

$$P_y = \frac{P_x}{\tan 55°}$$

$$= \frac{1200 \text{ N}}{\tan 55°}$$

$$= 840.2 \text{ N}$$

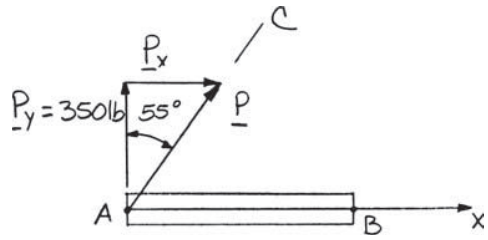
$\mathbf{P}_y = 840 \text{ N}\downarrow$ ◀



PROBLEM 2.30

Cable AC exerts on beam AB a force **P** directed along line AC. Knowing that **P** must have a 350-lb vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION

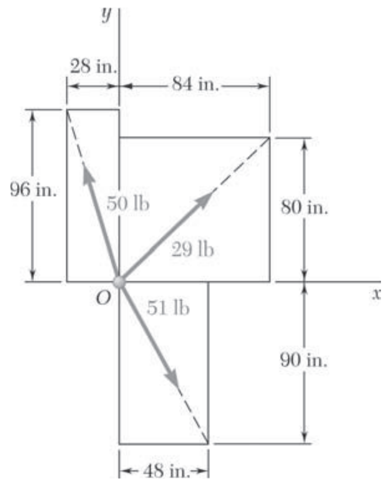


(*a*)
$$P = \frac{P_y}{\cos 55°}$$
$$= \frac{350 \text{ lb}}{\cos 55°}$$
$$= 610.2 \text{ lb}$$

$P = 610 \text{ lb}$ ◀

(*b*)
$$P_x = P \sin 55°$$
$$= (610.2 \text{ lb})\sin 55°$$
$$= 499.8 \text{ lb}$$

$P_x = 500 \text{ lb}$ ◀



PROBLEM 2.31

Determine the resultant of the three forces of Problem 2.22.

PROBLEM 2.22 Determine the *x* and *y* components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.22:

Force	*x* Comp. (lb)	*y* Comp. (lb)
29 lb	+21.0	+20.0
50 lb	–14.00	+48.0
51 lb	+24.0	–45.0
	$R_x = +31.0$	$R_y = +23.0$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (31.0\text{ lb})\mathbf{i} + (23.0\text{ lb})\mathbf{j}$$

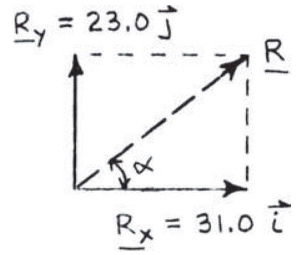
$$\tan\alpha = \frac{R_y}{R_x}$$

$$= \frac{23.0}{31.0}$$

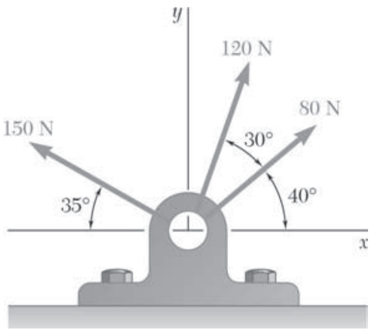
$$\alpha = 36.573°$$

$$R = \frac{23.0\text{ lb}}{\sin(36.573°)}$$

$$= 38.601\text{ lb}$$



$\mathbf{R} = 38.6\text{ lb} \measuredangle 36.6°$ ◀



PROBLEM 2.32

Determine the resultant of the three forces of Problem 2.24.

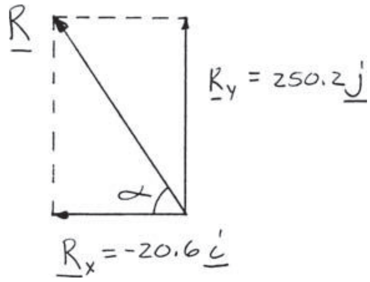
PROBLEM 2.24 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.24:

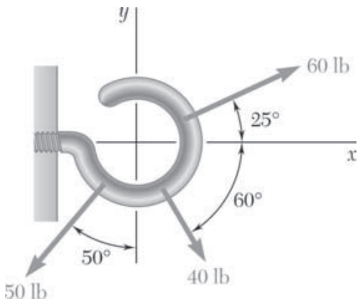
Force	x Comp. (N)	y Comp. (N)
80 N	+61.3	+51.4
120 N	+41.0	+112.8
150 N	–122.9	+86.0
	$R_x = -20.6$	$R_y = +250.2$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (-20.6 \text{ N})\mathbf{i} + (250.2 \text{ N})\mathbf{j}$$



$$\tan\alpha = \frac{R_y}{R_x}$$
$$\tan\alpha = \frac{250.2 \text{ N}}{20.6 \text{ N}}$$
$$\tan\alpha = 12.1456$$
$$\alpha = 85.293°$$
$$R = \frac{250.2 \text{ N}}{\sin 85.293°}$$

R = 251 N ⦪ 85.3° ◀



PROBLEM 2.33

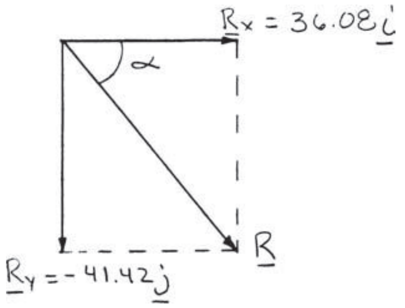
Determine the resultant of the three forces of Problem 2.23.

PROBLEM 2.23 Determine the x and y components of each of the forces shown.

SOLUTION

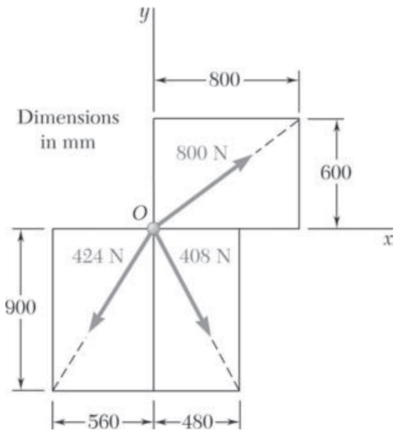
Force	x Comp. (lb)	y Comp. (lb)
40 lb	+20.00	–34.64
50 lb	–38.30	–32.14
60 lb	+54.38	+25.36
	$R_x = +36.08$	$R_y = -41.42$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (+36.08 \text{ lb})\mathbf{i} + (-41.42 \text{ lb})\mathbf{j}$$



$$\tan\alpha = \frac{R_y}{R_x}$$
$$\tan\alpha = \frac{41.42 \text{ lb}}{36.08 \text{ lb}}$$
$$\tan\alpha = 1.14800$$
$$\alpha = 48.942°$$
$$R = \frac{41.42 \text{ lb}}{\sin 48.942°}$$

R = 54.9 lb ⦪ 48.9° ◀



PROBLEM 2.34

Determine the resultant of the three forces of Problem 2.21.

PROBLEM 2.21 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.21:

Force	x Comp. (N)	y Comp. (N)
800 lb	+640	+480
424 lb	–224	–360
408 lb	+192	–360
	$R_x = +608$	$R_y = -240$

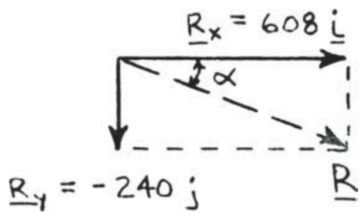
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (608 \text{ lb})\mathbf{i} + (-240 \text{ lb})\mathbf{j}$$

$$\tan\alpha = \frac{R_y}{R_x}$$

$$= \frac{240}{608}$$

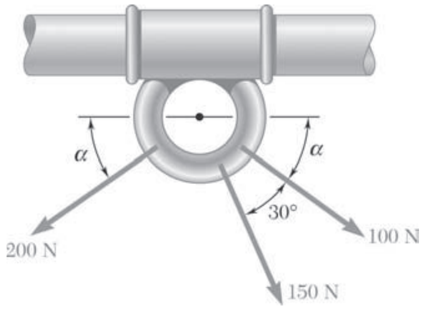
$$\alpha = 21.541°$$



$$R = \frac{240 \text{ N}}{\sin(21.541°)}$$

$$= 653.65 \text{ N}$$

R = 654 N ⦪ 21.5° ◀



PROBLEM 2.35

Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.

SOLUTION

100-N Force: $F_x = +(100\text{ N})\cos 35° = +81.915\text{ N}$

$F_y = -(100\text{ N})\sin 35° = -57.358\text{ N}$

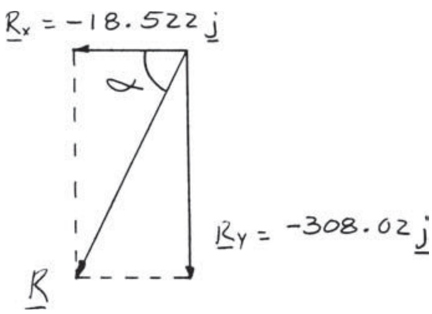
150-N Force: $F_x = +(150\text{ N})\cos 65° = +63.393\text{ N}$

$F_y = -(150\text{ N})\sin 65° = -135.946\text{ N}$

200-N Force: $F_x = -(200\text{ N})\cos 35° = -163.830\text{ N}$

$F_y = -(200\text{ N})\sin 35° = -114.715\text{ N}$

Force	x Comp. (N)	y Comp. (N)
100 N	+81.915	−57.358
150 N	+63.393	−135.946
200 N	−163.830	−114.715
	$R_x = -18.522$	$R_y = -308.02$



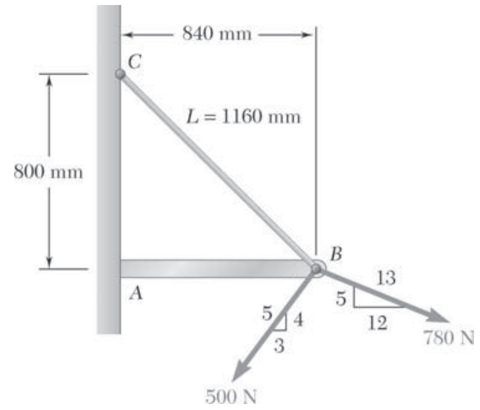
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (-18.522\text{ N})\mathbf{i} + (-308.02\text{ N})\mathbf{j}$$

$$\tan\alpha = \frac{R_y}{R_x}$$
$$= \frac{308.02}{18.522}$$
$$\alpha = 86.559°$$

$$R = \frac{308.02\text{ N}}{\sin 86.559}$$

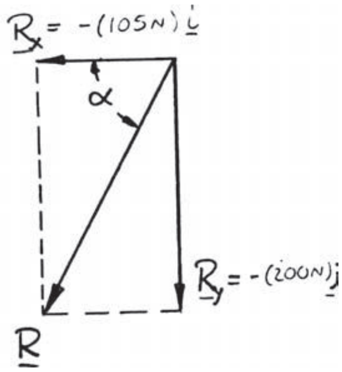
$\mathbf{R} = 309\text{ N}$ ∠ $86.6°$ ◀

PROBLEM 2.36



Knowing that the tension in cable BC is 725 N, determine the resultant of the three forces exerted at Point B of beam AB.

SOLUTION



Cable BC Force:
$$F_x = -(725\text{ N})\frac{840}{1160} = -525\text{ N}$$
$$F_y = (725\text{ N})\frac{840}{1160} = 500\text{ N}$$

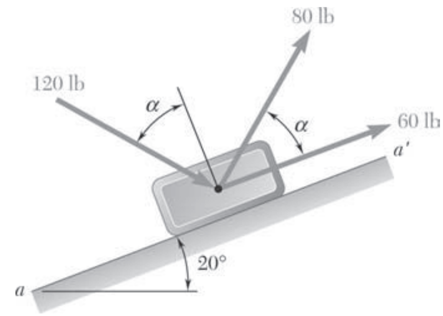
500-N Force:
$$F_x = -(500\text{ N})\frac{3}{5} = -300\text{ N}$$
$$F_y = -(500\text{ N})\frac{4}{5} = -400\text{ N}$$

780-N Force:
$$F_x = (780\text{ N})\frac{12}{13} = 720\text{ N}$$
$$F_y = -(780\text{ N})\frac{5}{13} = -300\text{ N}$$

and
$$R_x = \Sigma F_x = -105\text{ N}$$
$$R_y = \Sigma F_y = -200\text{ N}$$
$$R = \sqrt{(-105\text{ N})^2 + (-200\text{ N})^2}$$
$$= 225.89\text{ N}$$

Further:
$$\tan\alpha = \frac{200}{105}$$
$$\alpha = \tan^{-1}\frac{200}{105}$$
$$= 62.3°$$

Thus: $\mathbf{R} = 226\text{ N} \measuredangle\ 62.3°$ ◀



PROBLEM 2.37

Knowing that $\alpha = 40°$, determine the resultant of the three forces shown.

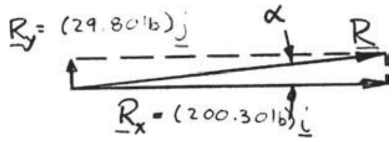
SOLUTION

60-lb Force:

$$F_x = (60 \text{ lb})\cos 20° = 56.38 \text{ lb}$$
$$F_y = (60 \text{ lb})\sin 20° = 20.52 \text{ lb}$$

80-lb Force:

$$F_x = (80 \text{ lb})\cos 60° = 40.00 \text{ lb}$$
$$F_y = (80 \text{ lb})\sin 60° = 69.28 \text{ lb}$$



120-lb Force:

$$F_x = (120 \text{ lb})\cos 30° = 103.92 \text{ lb}$$
$$F_y = -(120 \text{ lb})\sin 30° = -60.00 \text{ lb}$$

and

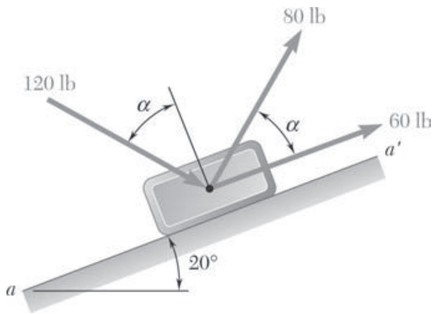
$$R_x = \Sigma F_x = 200.30 \text{ lb}$$
$$R_y = \Sigma F_y = 29.80 \text{ lb}$$
$$R = \sqrt{(200.30 \text{ lb})^2 + (29.80 \text{ lb})^2}$$
$$= 202.50 \text{ lb}$$

Further:

$$\tan\alpha = \frac{29.80}{200.30}$$

$$\alpha = \tan^{-1}\frac{29.80}{200.30}$$
$$= 8.46°$$

R = 203 lb ∠ 8.46° ◀



PROBLEM 2.38

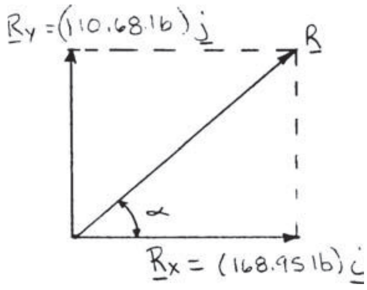
Knowing that $\alpha = 75°$, determine the resultant of the three forces shown.

SOLUTION

60-lb Force:
$$F_x = (60 \text{ lb}) \cos 20° = 56.38 \text{ lb}$$
$$F_y = (60 \text{ lb}) \sin 20° = 20.52 \text{ lb}$$

80-lb Force:
$$F_x = (80 \text{ lb}) \cos 95° = -6.97 \text{ lb}$$
$$F_y = (80 \text{ lb}) \sin 95° = 79.70 \text{ lb}$$

120-lb Force:
$$F_x = (120 \text{ lb}) \cos 5° = 119.54 \text{ lb}$$
$$F_y = (120 \text{ lb}) \sin 5° = 10.46 \text{ lb}$$

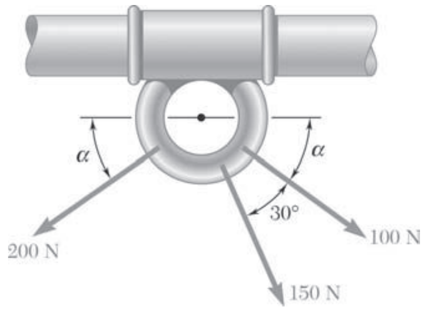


Then
$$R_x = \Sigma F_x = 168.95 \text{ lb}$$
$$R_y = \Sigma F_y = 110.68 \text{ lb}$$

and
$$R = \sqrt{(168.95 \text{ lb})^2 + (110.68 \text{ lb})^2}$$
$$= 201.98 \text{ lb}$$

$$\tan\alpha = \frac{110.68}{168.95}$$
$$\tan\alpha = 0.655$$
$$\alpha = 33.23°$$

$\mathbf{R} = 202 \text{ lb} \measuredangle 33.2°$ ◀



PROBLEM 2.39

For the collar of Problem 2.35, determine (*a*) the required value of α if the resultant of the three forces shown is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \Sigma F_x$$
$$= (100\text{ N})\cos\alpha + (150\text{ N})\cos(\alpha + 30°) - (200\text{ N})\cos\alpha$$
$$R_x = -(100\text{ N})\cos\alpha + (150\text{ N})\cos(\alpha + 30°) \qquad (1)$$

$$R_y = \Sigma F_y$$
$$= -(100\text{ N})\sin\alpha - (150\text{ N})\sin(\alpha + 30°) - (200\text{ N})\sin\alpha$$
$$R_y = -(300\text{ N})\sin\alpha - (150\text{ N})\sin(\alpha + 30°) \qquad (2)$$

(*a*) For **R** to be vertical, we must have $R_x = 0$. We make $R_x = 0$ in Eq. (1):

$$-100\cos\alpha + 150\cos(\alpha + 30°) = 0$$
$$-100\cos\alpha + 150(\cos\alpha\cos 30° - \sin\alpha\sin 30°) = 0$$
$$29.904\cos\alpha = 75\sin\alpha$$

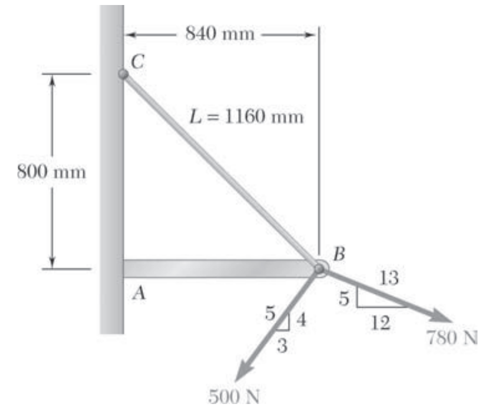
$$\tan\alpha = \frac{29.904}{75}$$
$$= 0.3988$$
$$\alpha = 21.74°$$

$\alpha = 21.7°$ ◀

(*b*) Substituting for α in Eq. (2):

$$R_y = -300\sin 21.74° - 150\sin 51.74°$$
$$= -228.9\text{ N}$$
$$R = |R_y| = 228.9\text{ N}$$

$R = 229$ N ◀



PROBLEM 2.40

For the beam of Problem 2.36, determine (*a*) the required tension in cable *BC* if the resultant of the three forces exerted at Point *B* is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \Sigma F_x = -\frac{840}{1160}T_{BC} + \frac{12}{13}(780\text{ N}) - \frac{3}{5}(500\text{ N})$$

$$R_x = -\frac{21}{29}T_{BC} + 420\text{ N} \qquad (1)$$

$$R_y = \Sigma F_y = \frac{800}{1160}T_{BC} - \frac{5}{13}(780\text{ N}) - \frac{4}{5}(500\text{ N})$$

$$R_y = \frac{20}{29}T_{BC} - 700\text{ N} \qquad (2)$$

(*a*) For **R** to be vertical, we must have $R_x = 0$

Set $R_x = 0$ in Eq. (1) $\quad -\frac{21}{29}T_{BC} + 420\text{ N} = 0 \quad$ $T_{BC} = 580\text{ N}$ ◀

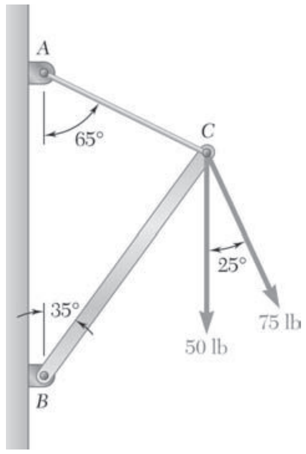
(*b*) Substituting for T_{BC} in Eq. (2):

$$R_y = \frac{20}{29}(580\text{ N}) - 700\text{ N}$$

$$R_y = -300\text{ N}$$

$$R = |R_y| = 300\text{ N}$$

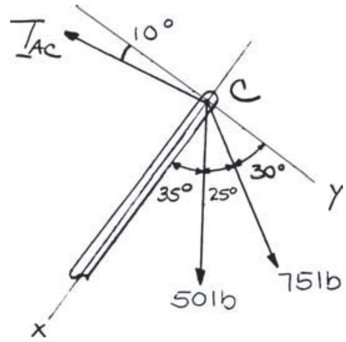
$R = 300\text{ N}$ ◀



PROBLEM 2.41

Determine (*a*) the required tension in cable *AC*, knowing that the resultant of the three forces exerted at Point *C* of boom *BC* must be directed along *BC*, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Using the *x* and *y* axes shown:

$$R_x = \Sigma F_x = T_{AC}\sin 10° + (50\text{ lb})\cos 35° + (75\text{ lb})\cos 60°$$

$$= T_{AC}\sin 10° + 78.46\text{ lb} \qquad (1)$$

$$R_y = \Sigma F_y = (50\text{ lb})\sin 35° + (75\text{ lb})\sin 60° - T_{AC}\cos 10°$$

$$R_y = 93.63\text{ lb} - T_{AC}\cos 10° \qquad (2)$$

(*a*) Set $R_y = 0$ in Eq. (2):

$$93.63\text{ lb} - T_{AC}\cos 10° = 0$$

$$T_{AC} = 95.07\text{ lb}$$

$T_{AC} = 95.1\text{ lb}$ ◀

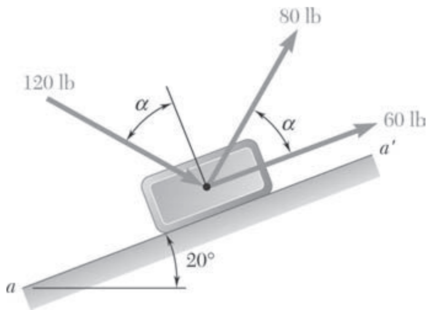
(*b*) Substituting for T_{AC} in Eq. (1):

$$R_x = (95.07\text{ lb})\sin 10° + 78.46\text{ lb}$$

$$= 94.97\text{ lb}$$

$$R = R_x$$

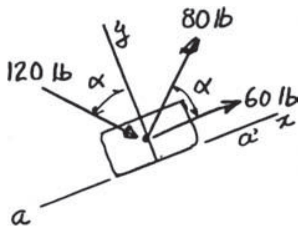
$R = 95.0\text{ lb}$ ◀



PROBLEM 2.42

For the block of Problems 2.37 and 2.38, determine (*a*) the required value of α if the resultant of the three forces shown is to be parallel to the incline, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Select the x axis to be along $a\,a'$.

Then

$$R_x = \Sigma F_x = (60 \text{ lb}) + (80 \text{ lb})\cos\alpha + (120 \text{ lb})\sin\alpha \qquad (1)$$

and

$$R_y = \Sigma F_y = (80 \text{ lb})\sin\alpha - (120 \text{ lb})\cos\alpha \qquad (2)$$

(*a*) Set $R_y = 0$ in Eq. (2).

$$(80 \text{ lb})\sin\alpha - (120 \text{ lb})\cos\alpha = 0$$

Dividing each term by $\cos\alpha$ gives:

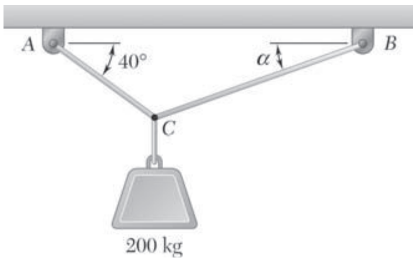
$$(80 \text{ lb})\tan\alpha = 120 \text{ lb}$$

$$\tan\alpha = \frac{120 \text{ lb}}{80 \text{ lb}}$$

$$\alpha = 56.310° \qquad \alpha = 56.3° \blacktriangleleft$$

(*b*) Substituting for α in Eq. (1) gives:

$$R_x = 60 \text{ lb} + (80 \text{ lb})\cos 56.31° + (120 \text{lb})\sin 56.31° = 204.22 \text{ lb} \qquad R_x = 204 \text{ lb} \blacktriangleleft$$

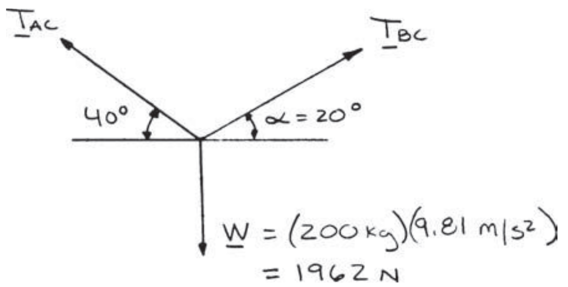


PROBLEM 2.43

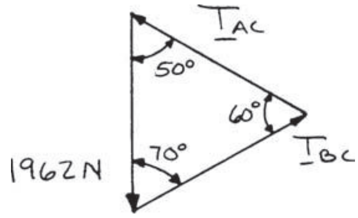
Two cables are tied together at C and are loaded as shown. Knowing that $\alpha = 20°$, determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

Free-Body Diagram



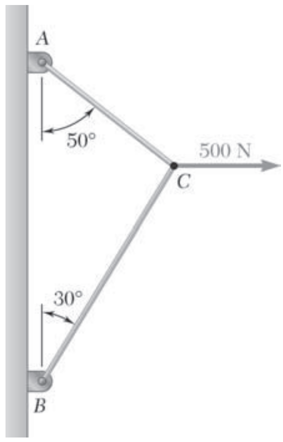
Force Triangle



Law of sines:
$$\frac{T_{AC}}{\sin 70°} = \frac{T_{BC}}{\sin 50°} = \frac{1962 \text{ N}}{\sin 60°}$$

(*a*)
$$T_{AC} = \frac{1962 \text{ N}}{\sin 60°} \sin 70° = 2128.9 \text{ N}$$
$T_{AC} = 2.13 \text{ kN}$ ◀

(*b*)
$$T_{BC} = \frac{1962 \text{ N}}{\sin 60°} \sin 50° = 1735.49 \text{ N}$$
$T_{BC} = 1.735 \text{ kN}$ ◀

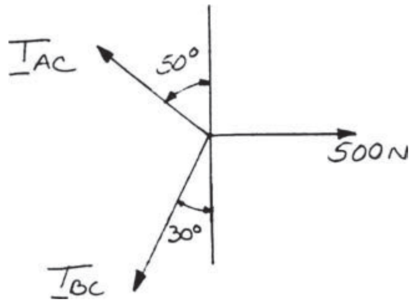


PROBLEM 2.44

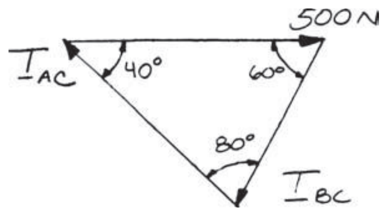
Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram



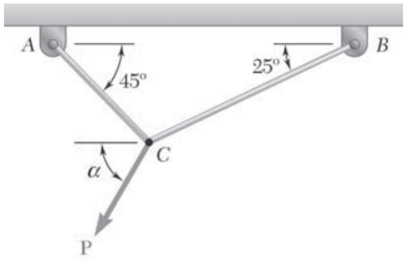
Force Triangle



Law of sines: $$\frac{T_{AC}}{\sin 60°} = \frac{T_{BC}}{\sin 40°} = \frac{500 \text{ N}}{\sin 80°}$$

(a) $$T_{AC} = \frac{500 \text{ N}}{\sin 80°} \sin 60° = 439.69 \text{ N}$$ $T_{AC} = 440$ N ◀

(b) $$T_{BC} = \frac{500 \text{ N}}{\sin 80°} \sin 40° = 326.35 \text{ N}$$ $T_{BC} = 326$ N ◀

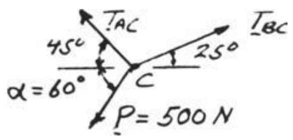


PROBLEM 2.45

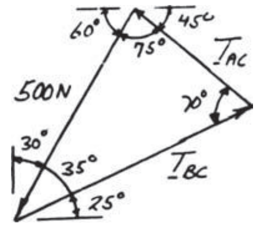
Two cables are tied together at C and are loaded as shown. Knowing that $\mathbf{P}$ = 500 N and α = 60°, determine the tension in (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle

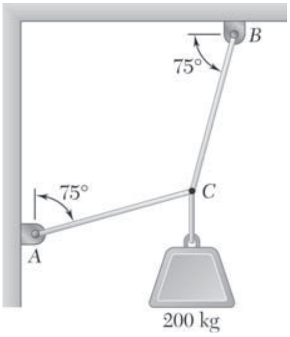


Law of sines:
$$\frac{T_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 75°} = \frac{500\text{ N}}{\sin 70°}$$

(a)
$$T_{AC} = \frac{500\text{ N}}{\sin 70°}\sin 35°$$
$T_{AC} = 305$ N ◀

(b)
$$T_{BC} = \frac{500\text{ N}}{\sin 70°}\sin 75°$$
$T_{BC} = 514$ N ◀

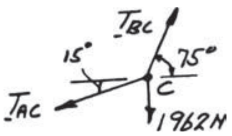
PROBLEM 2.46



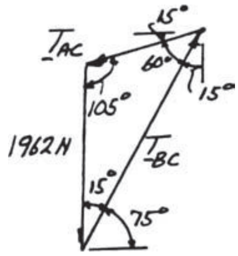
Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle

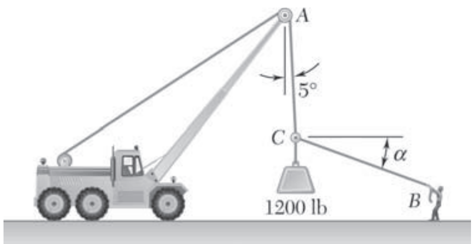


$$W = mg$$
$$= (200\text{ kg})(9.81\text{ m/s}^2)$$
$$= 1962\text{ N}$$

Law of sines: $$\frac{T_{AC}}{\sin 15°} = \frac{T_{BC}}{\sin 105°} = \frac{1962\text{ N}}{\sin 60°}$$

(a) $$T_{AC} = \frac{(1962\text{ N})\ \sin 15°}{\sin 60°}$$ $T_{AC} = 586\text{ N}$ ◀

(b) $$T_{BC} = \frac{(1962\text{ N})\sin 105°}{\sin 60°}$$ $T_{BC} = 2190\text{ N}$ ◀

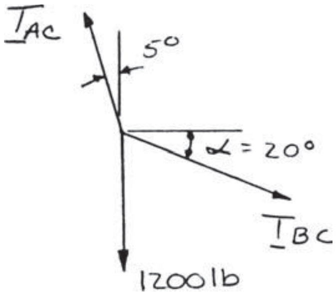


PROBLEM 2.47

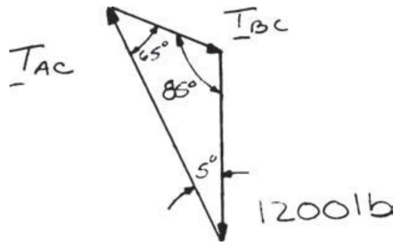
Knowing that $\alpha = 20°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

SOLUTION

Free-Body Diagram



Force Triangle

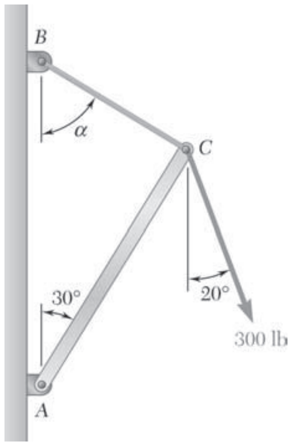


Law of sines: $$\frac{T_{AC}}{\sin 110°} = \frac{T_{BC}}{\sin 5°} = \frac{1200 \text{ lb}}{\sin 65°}$$

(*a*) $$T_{AC} = \frac{1200 \text{ lb}}{\sin 65°} \sin 110°$$ $T_{AC} = 1244 \text{ lb}$ ◀

(*b*) $$T_{BC} = \frac{1200 \text{ lb}}{\sin 65°} \sin 5°$$ $T_{BC} = 115.4 \text{ lb}$ ◀

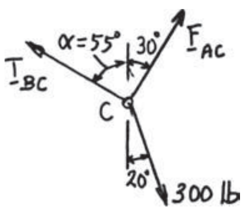
PROBLEM 2.48



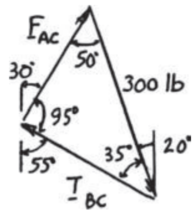
Knowing that $\alpha = 55°$ and that boom AC exerts on pin C a force directed along line AC, determine (a) the magnitude of that force, (b) the tension in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle



Law of sines:

$$\frac{F_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 50°} = \frac{300 \text{ lb}}{\sin 95°}$$

(a)

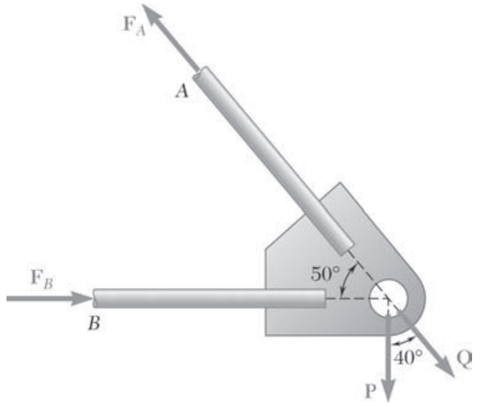
$$F_{AC} = \frac{300 \text{ lb}}{\sin 95°} \sin 35°$$

$F_{AC} = 172.7$ lb ◀

(b)

$$T_{BC} = \frac{300 \text{ lb}}{\sin 95°} \sin 50°$$

$T_{BC} = 231$ lb ◀

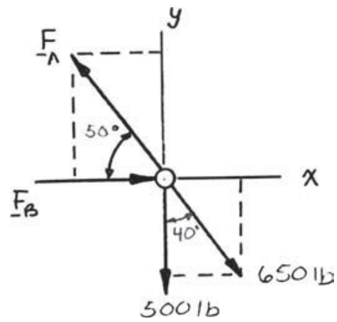


PROBLEM 2.49

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that $P = 500$ lb and $Q = 650$ lb, determine the magnitudes of the forces exerted on the rods A and B.

SOLUTION

Free-Body Diagram



Resolving the forces into x- and y-directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\mathbf{R} = -(500 \text{ lb})\mathbf{j} + [(650 \text{ lb})\cos 50°]\mathbf{i} - [(650 \text{ lb})\sin 50°]\mathbf{j} + F_B\mathbf{i} - (F_A \cos 50°)\mathbf{i} + (F_A \sin 50°)\mathbf{j} = 0$$

In the y-direction (one unknown force)

$$-500 \text{ lb} - (650 \text{ lb})\sin 50° + F_A \sin 50° = 0$$

Thus,

$$F_A = \frac{500 \text{ lb} + (650 \text{ lb})\sin 50°}{\sin 50°}$$

$$= 1302.70 \text{ lb}$$

$F_A = 1303$ lb ◀

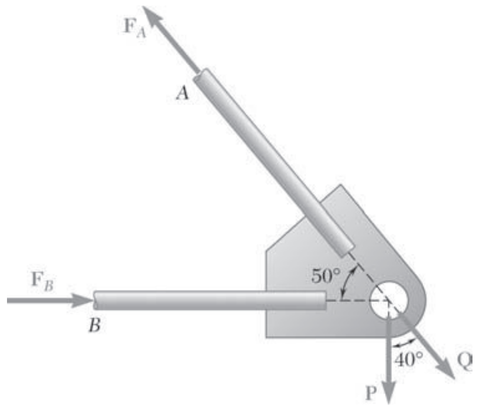
In the x-direction:

$$(650 \text{ lb})\cos 50° + F_B - F_A \cos 50° = 0$$

Thus,

$$F_B = F_A \cos 50° - (650 \text{ lb})\cos 50°$$
$$= (1302.70 \text{ lb})\cos 50° - (650 \text{ lb})\cos 50°$$
$$= 419.55 \text{ lb}$$

$F_B = 420$ lb ◀

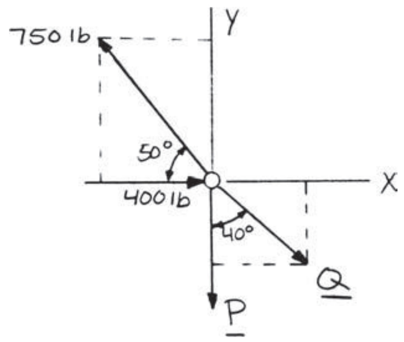


PROBLEM 2.50

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that the magnitudes of the forces exerted on rods A and B are $F_A = 750$ lb and $F_B = 400$ lb, determine the magnitudes of **P** and **Q**.

SOLUTION

Free-Body Diagram



Resolving the forces into x- and y-directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components: $\mathbf{R} = -P\mathbf{j} + Q\cos 50°\mathbf{i} - Q\sin 50°\mathbf{j}$

$$-[(750\text{ lb})\cos 50°]\mathbf{i}$$

$$+[(750\text{ lb})\sin 50°]\mathbf{j} + (400\text{ lb})\mathbf{i}$$

In the x-direction (one unknown force)

$$Q\cos 50° - [(750\text{ lb})\cos 50°] + 400\text{ lb} = 0$$

$$Q = \frac{(750\text{ lb})\cos 50° - 400\text{ lb}}{\cos 50°}$$

$$= 127.710\text{ lb}$$

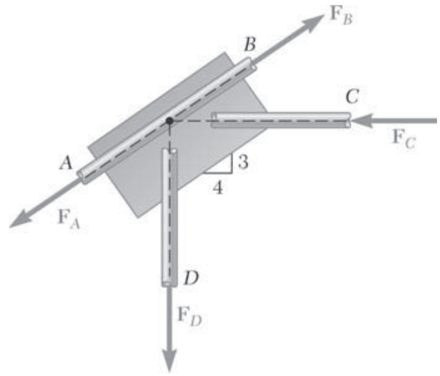
In the y-direction: $-P - Q\sin 50° + (750\text{ lb})\sin 50° = 0$

$$P = -Q\sin 50° + (750\text{ lb})\sin 50°$$

$$= -(127.710\text{ lb})\sin 50° + (750\text{ lb})\sin 50°$$

$$= 476.70\text{ lb}$$

$P = 477$ lb; $Q = 127.7$ lb ◀

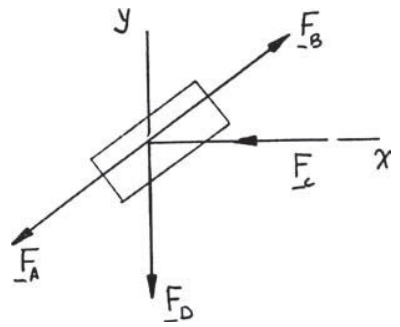


PROBLEM 2.51

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 8$ kN and $F_B = 16$ kN, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_x = 0: \quad \frac{3}{5}F_B - F_C - \frac{3}{5}F_A = 0$$

With $\quad F_A = 8$ kN

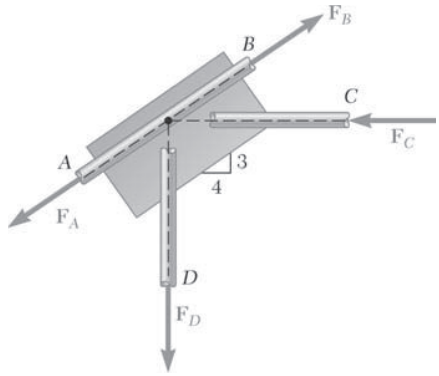
$F_B = 16$ kN

$$F_C = \frac{4}{5}(16\text{ kN}) - \frac{4}{5}(8\text{ kN}) \qquad F_C = 6.40\text{ kN} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad -F_D + \frac{3}{5}F_B - \frac{3}{5}F_A = 0$$

With F_A and F_B as above:

$$F_D = \frac{3}{5}(16\text{ kN}) - \frac{3}{5}(8\text{ kN}) \qquad F_D = 4.80\text{ kN} \blacktriangleleft$$

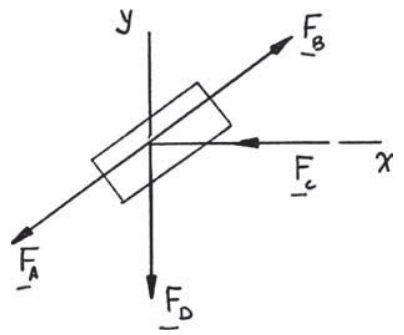


PROBLEM 2.52

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 5$ kN and $F_D = 6$ kN, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_y = 0: \quad -F_D - \frac{3}{5}F_A + \frac{3}{5}F_B = 0$$

or $$F_B = F_D + \frac{3}{5}F_A$$

With $$F_A = 5 \text{ kN}, \quad F_D = 8 \text{ kN}$$

$$F_B = \frac{5}{3}\left[6 \text{ kN} + \frac{3}{5}(5 \text{ kN})\right]$$

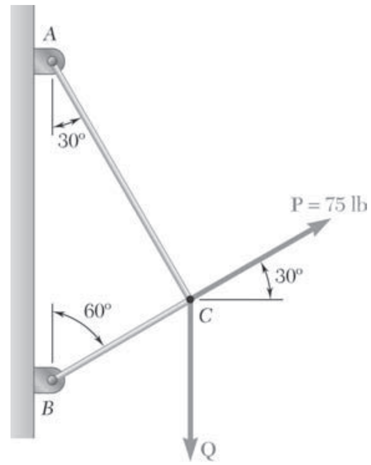
$F_B = 15.00$ kN ◀

$$\Sigma F_x = 0: \quad -F_C + \frac{4}{5}F_B - \frac{4}{5}F_A = 0$$

$$F_C = \frac{4}{5}(F_B - F_A)$$

$$= \frac{4}{5}(15 \text{ kN} - 5 \text{ kN})$$

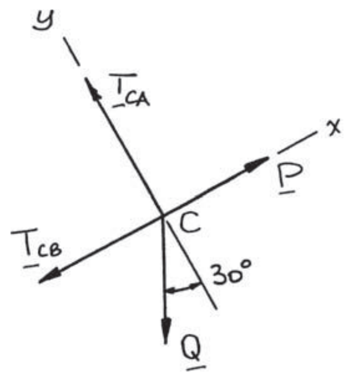
$F_C = 8.00$ kN ◀



PROBLEM 2.53

Two cables tied together at C are loaded as shown. Knowing that $Q = 60$ lb, determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION



$$\Sigma F_y = 0: \quad T_{CA} - Q\cos 30° = 0$$

With $\quad Q = 60$ lb

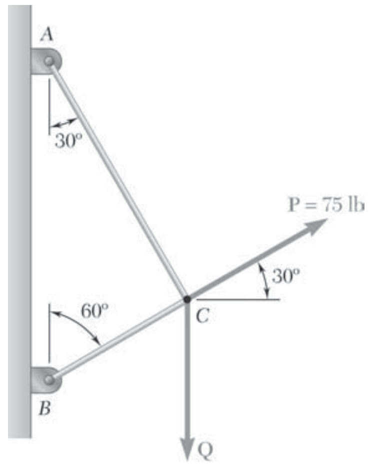
(*a*) $\quad T_{CA} = (60 \text{ lb})(0.866) \qquad T_{CA} = 52.0 \text{ lb}$ ◀

(*b*) $\quad \Sigma F_x = 0: \quad P - T_{CB} - Q\sin 30° = 0$

With $\quad P = 75$ lb

$T_{CB} = 75 \text{ lb} - (60 \text{ lb})(0.50) \qquad$ or $T_{CB} = 45.0 \text{ lb}$ ◀

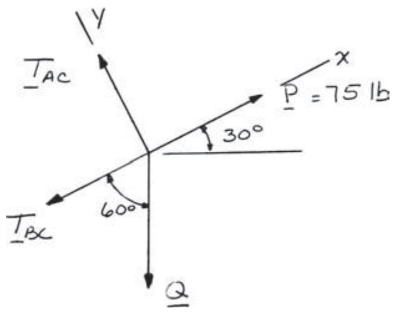
PROBLEM 2.54



Two cables tied together at C are loaded as shown. Determine the range of values of Q for which the tension will not exceed 60 lb in either cable.

SOLUTION

Free-Body Diagram



$$\Sigma F_x = 0: \quad -T_{BC} - Q\cos 60° + 75\text{ lb} = 0$$

$$T_{BC} = 75\text{ lb} - Q\cos 60° \qquad (1)$$

$$\Sigma F_y = 0: \quad T_{AC} - Q\sin 60° = 0$$

$$T_{AC} = Q\sin 60° \qquad (2)$$

Requirement $\quad T_{AC} \le 60\text{ lb}:$

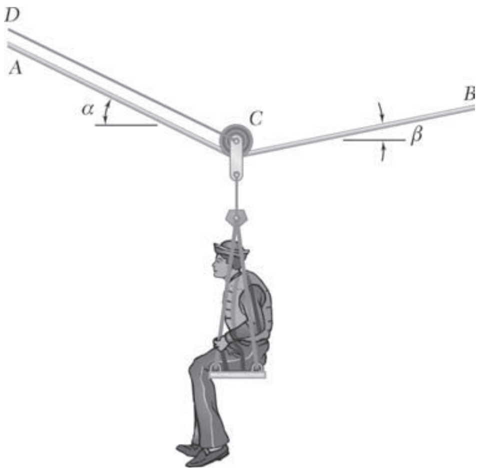
From Eq. (2): $\quad Q\sin 60° \le 60\text{ lb}$

$$Q \le 69.3\text{ lb}$$

Requirement $\quad T_{BC} \le 60\text{ lb}:$

From Eq. (1): $\quad 75\text{ lb} - Q\sin 60° \le 60\text{ lb}$

$$Q \ge 30.0\text{ lb} \qquad 30.0\text{ lb} \le Q \le 69.3\text{ lb} \blacktriangleleft$$

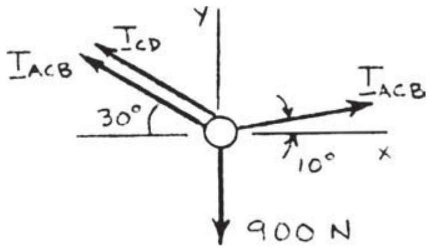


PROBLEM 2.55

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD*. Knowing that $\alpha = 30°$ and $\beta = 10°$ and that the combined weight of the boatswain's chair and the sailor is 900 N, determine the tension (*a*) in the support cable *ACB*, (*b*) in the traction cable *CD*.

SOLUTION

Free-Body Diagram



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}\cos 10° - T_{ACB}\cos 30° - T_{CD}\cos 30° = 0$$

$$T_{CD} = 0.137158 T_{ACB} \qquad (1)$$

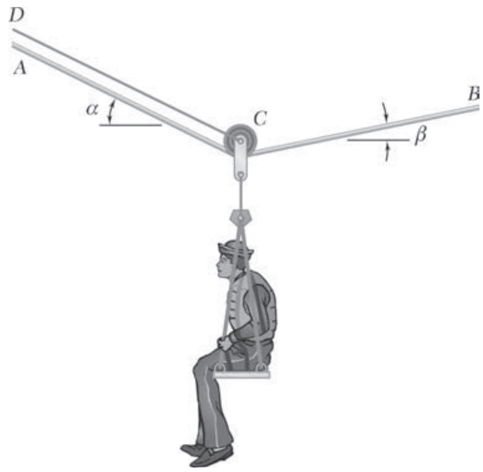
$$+\uparrow \Sigma F_y = 0: \quad T_{ACB}\sin 10° + T_{ACB}\sin 30° + T_{CD}\sin 30° - 900 = 0$$

$$0.67365 T_{ACB} + 0.5 T_{CD} = 900 \qquad (2)$$

(*a*) Substitute (1) into (2): $0.67365 T_{ACB} + 0.5(0.137158 T_{ACB}) = 900$

$$T_{ACB} = 1212.56 \text{ N} \qquad T_{ACB} = 1213 \text{ N} ◀$$

(*b*) From (1): $T_{CD} = 0.137158(1212.56 \text{ N})$ $\qquad T_{CD} = 166.3 \text{ N}$ ◀

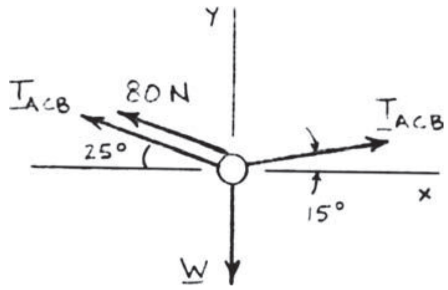


PROBLEM 2.56

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD*. Knowing that $\alpha = 25°$ and $\beta = 15°$ and that the tension in cable *CD* is 80 N, determine (*a*) the combined weight of the boatswain's chair and the sailor, (*b*) in tension in the support cable *ACB*.

SOLUTION

Free-Body Diagram



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}\cos 15° - T_{ACB}\cos 25° - (80 \text{ N})\cos 25° = 0$$

$$T_{ACB} = 1216.15 \text{ N}$$

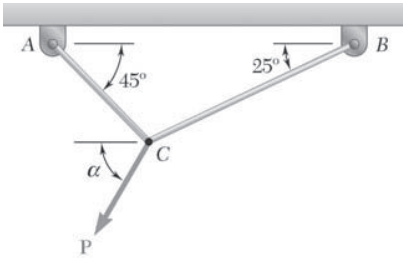
$$+\uparrow \Sigma F_y = 0: \quad (1216.15 \text{ N})\sin 15° + (1216.15 \text{ N})\sin 25°$$

$$+(80 \text{ N})\sin 25° - W = 0$$

$$W = 862.54 \text{ N}$$

(*a*) $W = 863 \text{ N}$ ◀

(*b*) $T_{ACB} = 1216 \text{ N}$ ◀

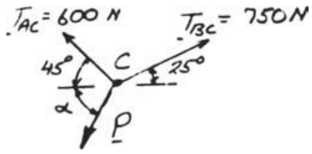


PROBLEM 2.57

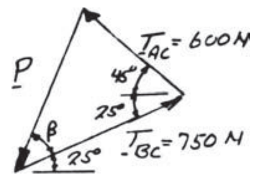
For the cables of Problem 2.45, it is known that the maximum allowable tension is 600 N in cable *AC* and 750 N in cable *BC*. Determine (*a*) the maximum force **P** that can be applied at *C*, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram



Force Triangle



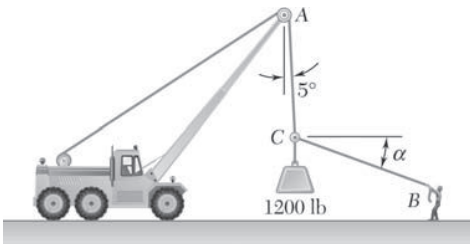
(*a*) Law of cosines $\quad P^2 = (600)^2 + (750)^2 - 2(600)(750)\cos(25° + 45°)$

$P = 784.02$ N $\qquad\qquad P = 784$ N ◀

(*b*) Law of sines

$$\frac{\sin\beta}{600\text{ N}} = \frac{\sin(25° + 45°)}{784.02\text{ N}}$$

$\beta = 46.0°$ $\qquad\qquad \alpha = 46.0° + 25° = 71.0°$ ◀



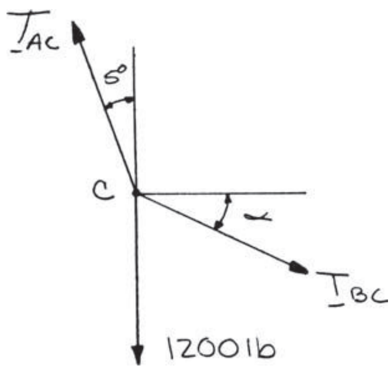
PROBLEM 2.58

For the situation described in Figure P2.47, determine (*a*) the value of α for which the tension in rope *BC* is as small as possible, (*b*) the corresponding value of the tension.

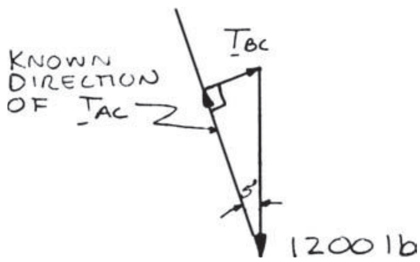
PROBLEM 2.47 Knowing that $\alpha = 20°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

SOLUTION

Free-Body Diagram



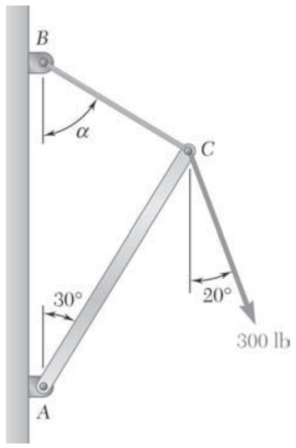
Force Triangle



To be smallest, T_{BC} must be perpendicular to the direction of T_{AC}.

(*a*) Thus, $\alpha = 5°$ $\qquad$ $\alpha = 5.00°$ ◀

(*b*) $T_{BC} = (1200 \text{ lb})\sin 5°$ $\qquad$ $T_{BC} = 104.6 \text{ lb}$ ◀



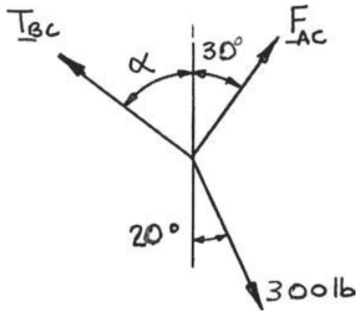
PROBLEM 2.59

For the structure and loading of Problem 2.48, determine (*a*) the value of α for which the tension in cable *BC* is as small as possible, (*b*) the corresponding value of the tension.

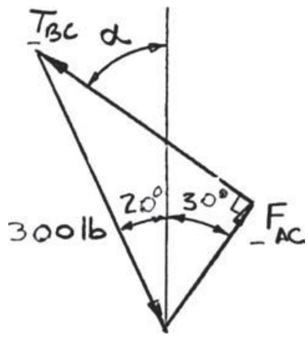
SOLUTION

T_{BC} must be perpendicular to F_{AC} to be as small as possible.

Free-Body Diagram: *C*



Force Triangle is a right triangle

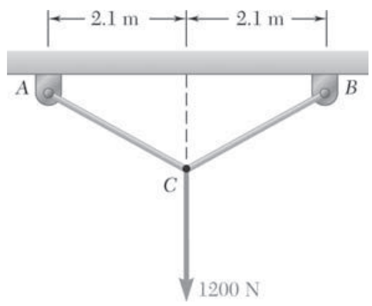


To be a minimum, T_{BC} must be perpendicular to F_{AC}.

(*a*) We observe: $\alpha = 90° - 30°$ $\qquad \alpha = 60.0°$ ◀

(*b*) $T_{BC} = (300 \text{ lb})\sin 50°$

or $T_{BC} = 229.81 \text{ lb}$ $\qquad T_{BC} = 230 \text{ lb}$ ◀



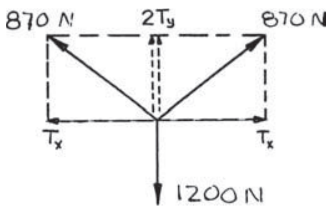
PROBLEM 2.60

Knowing that portions AC and BC of cable ACB must be equal, determine the shortest length of cable that can be used to support the load shown if the tension in the cable is not to exceed 870 N.

SOLUTION

Free-Body Diagram: *C*

(For $T = 725$ N)



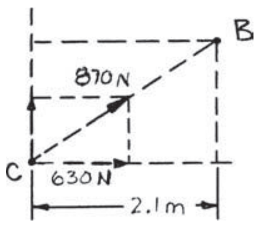
$$+\uparrow \Sigma F_y = 0: \quad 2T_y - 1200 \text{ N} = 0$$

$$T_y = 600 \text{ N}$$

$$T_x^2 + T_y^2 = T^2$$

$$T_x^2 + (600 \text{ N})^2 = (870 \text{ N})^2$$

$$T_x = 630 \text{ N}$$



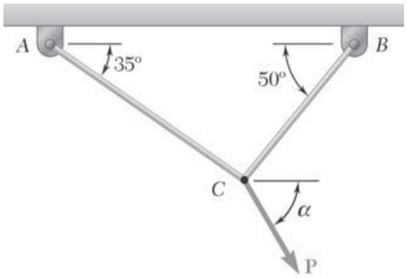
By similar triangles: $$\frac{BC}{870 \text{ N}} = \frac{2.1 \text{ m}}{630 \text{ N}}$$

$$BC = 2.90 \text{ m}$$

$$L = 2(BC)$$

$$= 5.80 \text{ m}$$

$L = 5.80$ m ◀

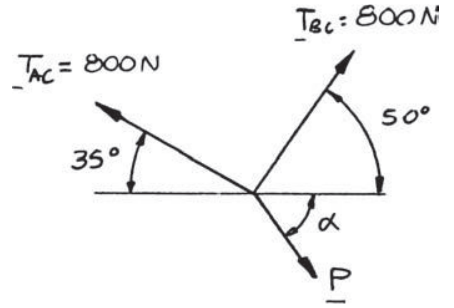


PROBLEM 2.61

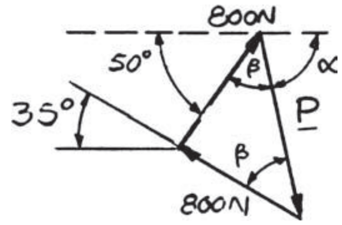
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (a) the magnitude of the largest force **P** that can be applied at C, (b) the corresponding value of α.

SOLUTION

Free-Body Diagram: *C*



Force Triangle



Force triangle is isosceles with

$$2\beta = 180° - 85°$$

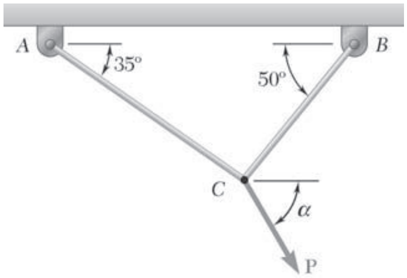
$$\beta = 47.5°$$

(a) $$P = 2(800\text{ N})\cos 47.5° = 1081\text{ N}$$

Since $P > 0$, the solution is correct. $P = 1081\text{ N}$ ◀

(b) $$\alpha = 180° - 50° - 47.5° = 82.5°$$ $\alpha = 82.5°$ ◀

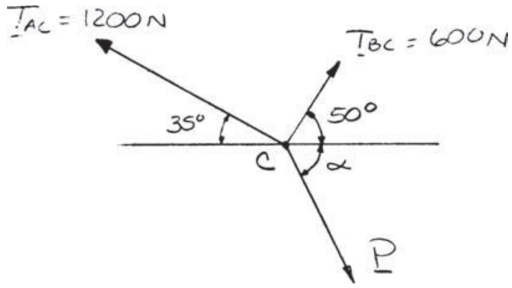
PROBLEM 2.62



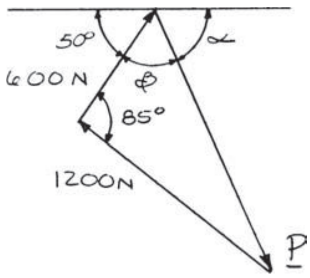
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension is 1200 N in cable AC and 600 N in cable BC, determine (*a*) the magnitude of the largest force **P** that can be applied at C, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram



Force Triangle



(*a*) Law of cosines: $P^2 = (1200\text{ N})^2 + (600\text{ N})^2 - 2(1200\text{ N})(600\text{ N})\cos 85°$

$P = 1294\text{ N}$

Since $P > 1200\text{ N}$, the solution is correct.

$P = 1294\text{ N}$ ◀

(*b*) Law of sines:

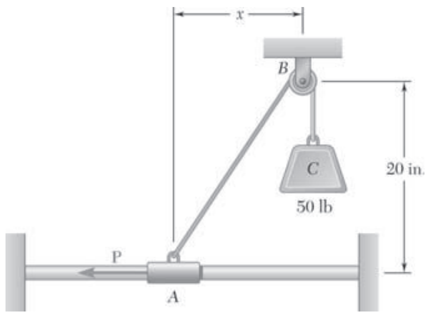
$$\frac{\sin\beta}{1200\text{ N}} = \frac{\sin 85°}{1294\text{ N}}$$

$$\beta = 67.5°$$

$$\alpha = 180° - 50° - 67.5°$$

$\alpha = 62.5°$ ◀

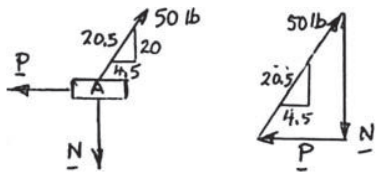
PROBLEM 2.63



Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force **P** required to maintain the equilibrium of the collar when (*a*) $x = 4.5$ in., (*b*) $x = 15$ in.

SOLUTION

(*a*) **Free Body: Collar *A***

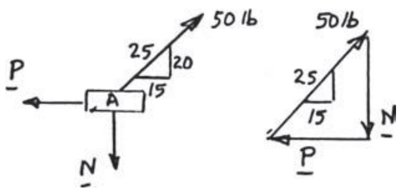


Force Triangle

$$\frac{P}{4.5} = \frac{50 \text{ lb}}{20.5}$$

$P = 10.98$ lb ◀

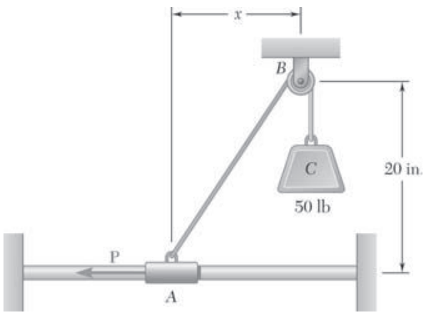
(*b*) **Free Body: Collar *A***



Force Triangle

$$\frac{P}{15} = \frac{50 \text{ lb}}{25}$$

$P = 30.0$ lb ◀

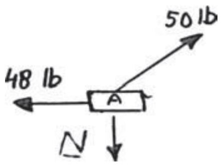


PROBLEM 2.64

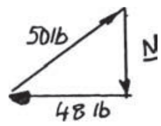
Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the distance x for which the collar is in equilibrium when $P = 48$ lb.

SOLUTION

Free Body: Collar *A*



Force Triangle

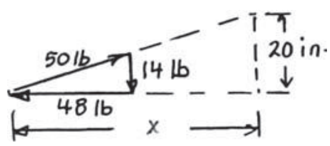


$$N^2 = (50)^2 - (48)^2 = 196$$

$$N = 14.00 \text{ lb}$$

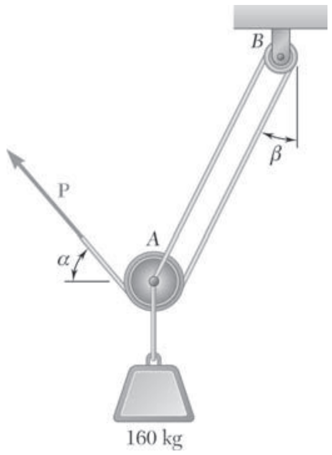
Similar Triangles

$$\frac{x}{20 \text{ in.}} = \frac{48 \text{ lb}}{14 \text{ lb}}$$



$x = 68.6$ in. ◀

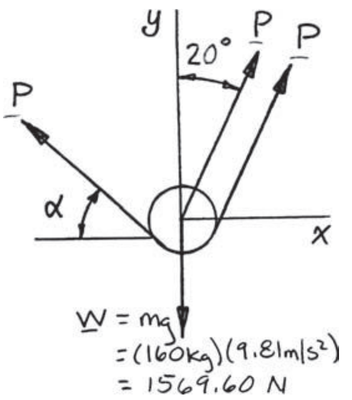
PROBLEM 2.65



A 160-kg load is supported by the rope-and-pulley arrangement shown. Knowing that $\beta = 20°$, determine the magnitude and direction of the force **P** that must be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chapter 4.)

SOLUTION

Free-Body Diagram: Pulley *A*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 2P\sin 20° - P\cos\alpha = 0$$

and $\cos\alpha = 0.8452$ or $\alpha = \pm 46.840°$

$\alpha = +46.840$

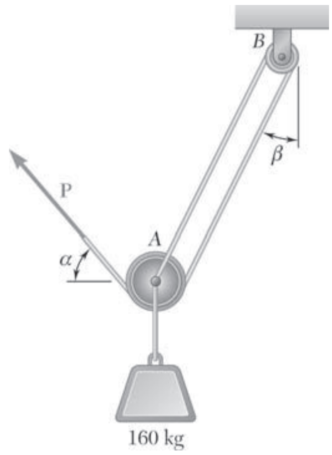
For $+\uparrow \Sigma F_y = 0: \quad 2P\cos 20° + P\sin 46.840° - 1569.60 \text{ N} = 0$

or $\mathbf{P} = 602 \text{ N} \searrow 46.8°$ ◀

For $\alpha = -46.840$

$+\uparrow \Sigma F_y = 0: \quad 2P\cos 20° + P\sin(-46.840°) - 1569.60 \text{ N} = 0$

or $\mathbf{P} = 1365 \text{ N} \nearrow 46.8°$ ◀

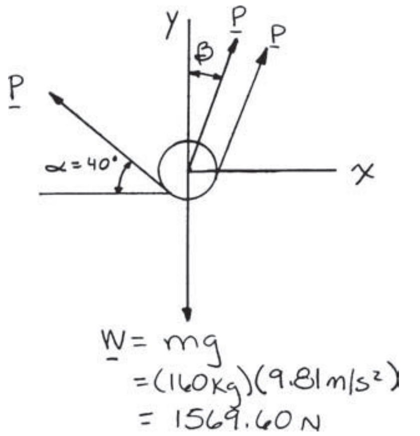


PROBLEM 2.66

A 160-kg load is supported by the rope-and-pulley arrangement shown. Knowing that $\alpha = 40°$, determine (*a*) the angle β, (*b*) the magnitude of the force **P** that must be exerted on the free end of the rope to maintain equilibrium. (See the hint for Problem 2.65.)

SOLUTION

Free-Body Diagram: Pulley *A*



(*a*) $\Sigma F_x = 0\colon \quad 2P\sin\sin\beta - P\cos 40° = 0$

$$\sin\beta = \frac{1}{2}\cos 40°$$

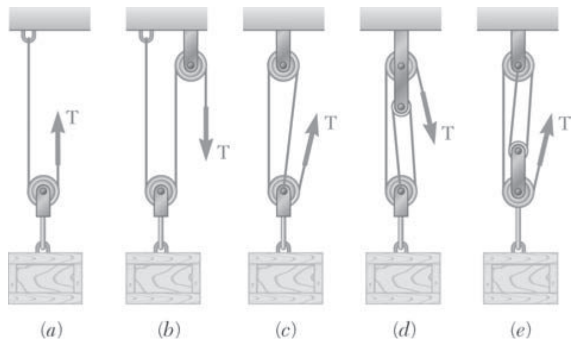
$$\beta = 22.52°$$

$$\beta = 22.5° \blacktriangleleft$$

(*b*) $\Sigma F_y = 0\colon \quad P\sin 40° + 2P\cos 22.52° - 1569.60\text{ N} = 0$

$$P = 630\text{ N} \blacktriangleleft$$

PROBLEM 2.67

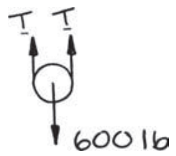


A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.65.)

SOLUTION

Free-Body Diagram of Pulley

(*a*)

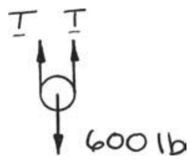


$$+\uparrow \Sigma F_y = 0: \quad 2T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{2}(600 \text{ lb})$$

$$T = 300 \text{ lb} \blacktriangleleft$$

(*b*)

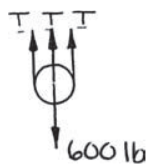


$$+\uparrow \Sigma F_y = 0: \quad 2T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{2}(600 \text{ lb})$$

$$T = 300 \text{ lb} \blacktriangleleft$$

(*c*)

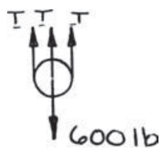


$$+\uparrow \Sigma F_y = 0: \quad 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

$$T = 200 \text{ lb} \blacktriangleleft$$

(*d*)

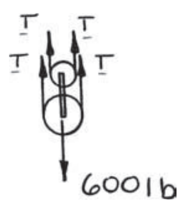


$$+\uparrow \Sigma F_y = 0: \quad 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

$$T = 200 \text{ lb} \blacktriangleleft$$

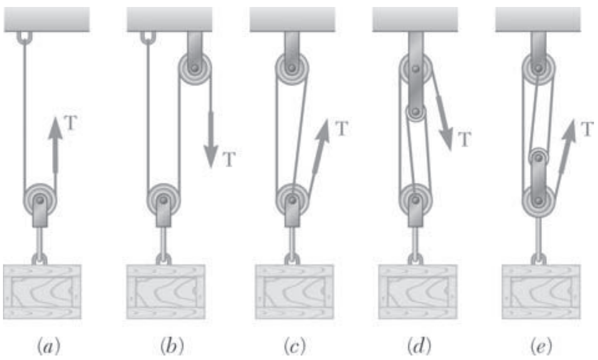
(*e*)



$$+\uparrow \Sigma F_y = 0: \quad 4T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{4}(600 \text{ lb})$$

$$T = 150.0 \text{ lb} \blacktriangleleft$$



PROBLEM 2.68

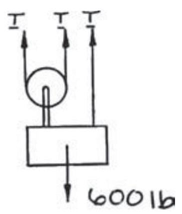
Solve Parts *b* and *d* of Problem 2.67, assuming that the free end of the rope is attached to the crate.

PROBLEM 2.67 A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.65.)

SOLUTION

Free-Body Diagram of Pulley and Crate

(*b*)

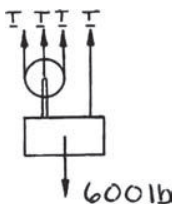


$$+\uparrow \Sigma F_y = 0: \quad 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

$$T = 200 \text{ lb} \blacktriangleleft$$

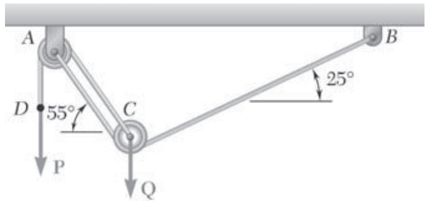
(*d*)



$$+\uparrow \Sigma F_y = 0: \quad 4T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{4}(600 \text{ lb})$$

$$T = 150.0 \text{ lb} \blacktriangleleft$$

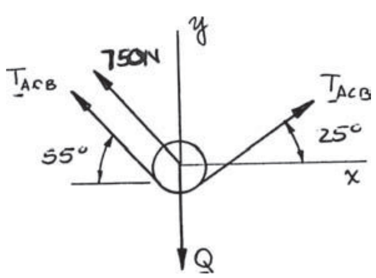


PROBLEM 2.69

A load **Q** is applied to the pulley C, which can roll on the cable ACB. The pulley is held in the position shown by a second cable CAD, which passes over the pulley A and supports a load **P**. Knowing that $P = 750$ N, determine (a) the tension in cable ACB, (b) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley ***C***



(a) $\xrightarrow{+} \Sigma F_x = 0$: $T_{ACB}(\cos 25° - \cos 55°) - (750\text{ N})\cos 55° = 0$

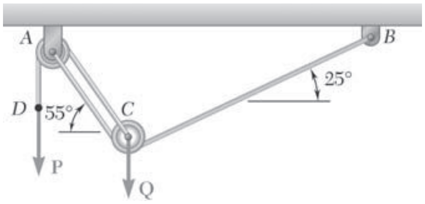
Hence: $T_{ACB} = 1292.88$ N

$T_{ACB} = 1293$ N ◀

(b) $+\uparrow \Sigma F_y = 0$: $T_{ACB}(\sin 25° + \sin 55°) + (750\text{ N})\sin 55° - Q = 0$

$$(1292.88\text{ N})(\sin 25° + \sin 55°) + (750\text{ N})\sin 55° - Q = 0$$

or $Q = 2219.8$ N $\quad Q = 2220$ N ◀

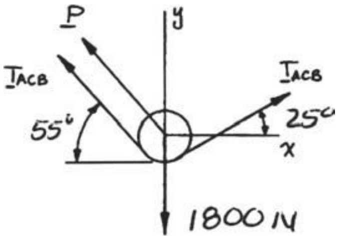


PROBLEM 2.70

An 1800-N load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **P**.

SOLUTION

Free-Body Diagram: Pulley *C*



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}(\cos 25° - \cos 55°) - P\cos 55° = 0$$

or
$$P = 0.58010 T_{ACB} \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 25° + \sin 55°) + P\sin 55° - 1800 \text{ N} = 0$$

or
$$1.24177 T_{ACB} + 0.81915P = 1800 \text{ N} \qquad (2)$$

(*a*) Substitute Equation (1) into Equation (2):

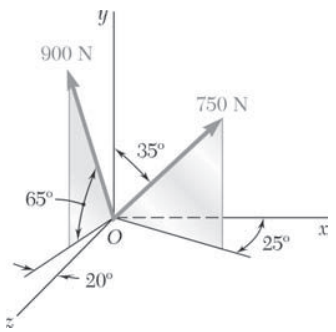
$$1.24177 T_{ACB} + 0.81915(0.58010 T_{ACB}) = 1800 \text{ N}$$

Hence: $T_{ACB} = 1048.37$ N

$$T_{ACB} = 1048 \text{ N} \blacktriangleleft$$

(*b*) Using (1), $P = 0.58010(1048.37 \text{ N}) = 608.16$ N

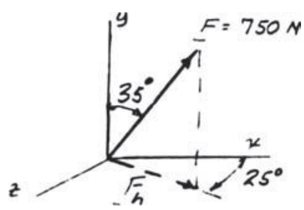
$$P = 608 \text{ N} \blacktriangleleft$$



PROBLEM 2.71

Determine (*a*) the *x*, *y*, and *z* components of the 750-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION



$$F_h = F\sin 35°$$
$$= (750\text{ N})\sin 35°$$
$$F_h = 430.2\text{ N}$$

(*a*)

$$F_x = F_h \cos 25° \qquad F_y = F\cos 35° \qquad F_z = F_h \sin 25°$$

$$= (430.2\text{ N})\cos 25° \qquad = (750\text{ N})\cos 35° \qquad = (430.2\text{ N})\sin 25°$$

$$F_x = +390\text{ N}, \qquad F_y = +614\text{ N}, \qquad F_z = +181.8\text{ N} \blacktriangleleft$$

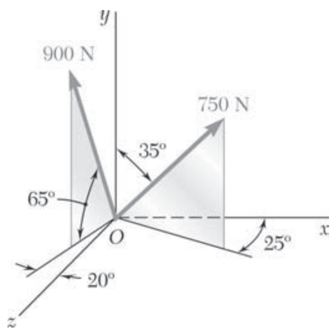
(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{+390\text{ N}}{750\text{ N}} \qquad \theta_x = 58.7° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{+614\text{ N}}{750\text{ N}} \qquad \theta_y = 35.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{+181.8\text{ N}}{750\text{ N}} \qquad \theta_z = 76.0° \blacktriangleleft$$

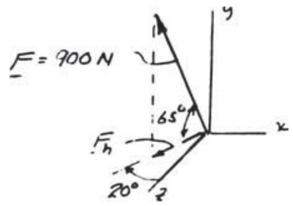
PROBLEM 2.72



Determine (*a*) the *x*, *y*, and *z* components of the 900-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

$$F_h = F\cos 65°$$
$$= (900 \text{ N})\cos 65°$$
$$F_h = 380.4 \text{ N}$$



(*a*)

$F_x = F_h \sin 20°$ $\quad F_y = F\sin 65°$ $\quad F_z = F_h\cos 20°$

$= (380.4 \text{ N})\sin 20°$ $\quad = (900 \text{ N})\sin 65°$ $\quad = (380.4 \text{ N})\cos 20°$

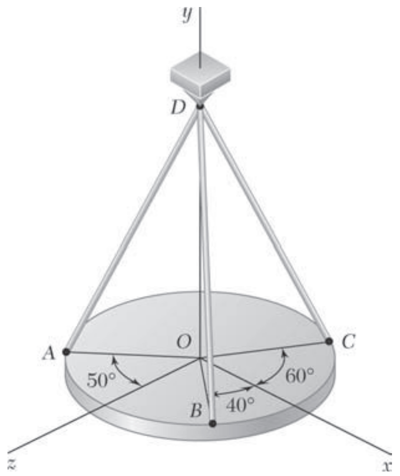
$F_x = -130.1 \text{ N},$ $\quad F_y = +816 \text{ N},$ $\quad F_z = +357 \text{ N}$ ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-130.1 \text{ N}}{900 \text{ N}} \qquad \theta_x = 98.3° \text{ ◀}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{+816 \text{ N}}{900 \text{ N}} \qquad \theta_y = 25.0° \text{ ◀}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{+357 \text{ N}}{900 \text{ N}} \qquad \theta_z = 66.6° \text{ ◀}$$



PROBLEM 2.73

A horizontal circular plate is suspended as shown from three wires that are attached to a support at D and form 30° angles with the vertical. Knowing that the x component of the force exerted by wire AD on the plate is 110.3 N, determine (a) the tension in wire AD, (b) the angles θ_x, θ_y, and θ_z that the force exerted at A forms with the coordinate axes.

SOLUTION

(a)

$$F_x = F\sin 30°\sin 50° = 110.3\text{ N} \quad \text{(Given)}$$

$$F = \frac{110.3\text{ N}}{\sin 30°\sin 50°} = 287.97\text{ N} \qquad F = 288\text{ N} \blacktriangleleft$$

(b)

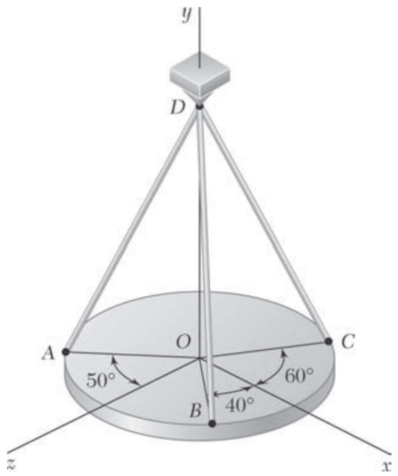
$$\cos\theta_x = \frac{F_x}{F} = \frac{110.3\text{ N}}{287.97\text{ N}} = 0.38303 \qquad \theta_x = 67.5° \blacktriangleleft$$

$$F_y = F\cos 30° = 249.39$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{249.39\text{ N}}{287.97\text{ N}} = 0.86603 \qquad \theta_y = 30.0° \blacktriangleleft$$

$$\begin{aligned} F_z &= -F\sin 30°\cos 50° \\ &= -(287.97\text{ N})\sin 30°\cos 50° \\ &= -92.552\text{ N} \end{aligned}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-92.552\text{ N}}{287.97\text{ N}} = -0.32139 \qquad \theta_z = 108.7° \blacktriangleleft$$



PROBLEM 2.74

A horizontal circular plate is suspended as shown from three wires that are attached to a support at D and form 30° angles with the vertical. Knowing that the z component of the force exerted by wire BD on the plate is –32.14 N, determine (*a*) the tension in wire BD, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at B forms with the coordinate axes.

SOLUTION

(*a*)

$$F_z = -F \sin 30° \sin 40° = 32.14 \text{ N} \quad \text{(Given)}$$

$$F = \frac{32.14}{\sin 30° \sin 40°} = 100.0 \text{ N} \qquad F = 100.0 \text{ N} \blacktriangleleft$$

(*b*)

$$F_x = -F \sin 30° \cos 40°$$
$$= -(100.0 \text{ N}) \sin 30° \cos 40°$$
$$= -38.302 \text{ N}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{38.302 \text{ N}}{100.0 \text{ N}} = -0.38302 \qquad \theta_x = 112.5° \blacktriangleleft$$

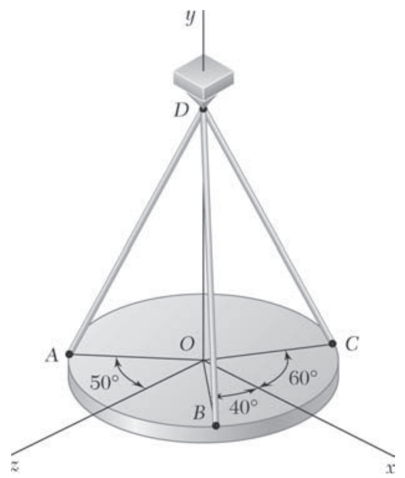
$$F_y = F \cos 30° = 86.603 \text{ N}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{86.603 \text{ N}}{100 \text{ N}} = 0.86603 \qquad \theta_y = 30.0° \blacktriangleleft$$

$$F_z = -32.14 \text{ N}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-32.14 \text{ N}}{100 \text{ N}} = -0.32140 \qquad \theta_z = 108.7° \blacktriangleleft$$

PROBLEM 2.75



A horizontal circular plate is suspended as shown from three wires that are attached to a support at D and form 30° angles with the vertical. Knowing that the tension in wire CD is 60 lb, determine (a) the components of the force exerted by this wire on the plate, (b) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(a)

$$F_x = -(60\text{ lb})\sin 30°\cos 60° = -15\text{ lb} \qquad F_x = -15.00\text{ lb} \blacktriangleleft$$

$$F_y = (60\text{ lb})\cos 30° = 51.96\text{ lb} \qquad F_y = +52.0\text{ lb} \blacktriangleleft$$

$$F_z = (60\text{ lb})\sin 30°\sin 60° = 25.98\text{ lb} \qquad F_z = +26.0\text{ lb} \blacktriangleleft$$

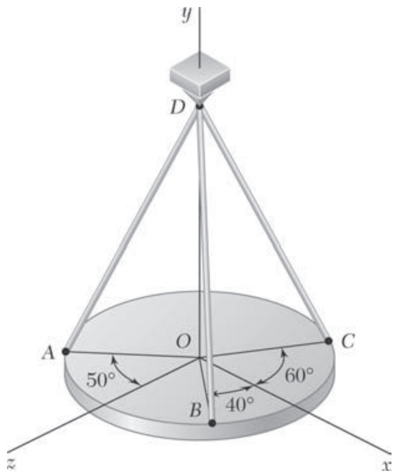
(b)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-15.0\text{ lb}}{60\text{ lb}} = -0.25 \qquad \theta_x = 104.5° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{51.96\text{ lb}}{60\text{ lb}} = 0.866 \qquad \theta_y = 30.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{25.98\text{ lb}}{60\text{ lb}} = 0.433 \qquad \theta_z = 64.3° \blacktriangleleft$$

PROBLEM 2.76



A horizontal circular plate is suspended as shown from three wires that are attached to a support at D and form 30° angles with the vertical. Knowing that the x component of the force exerted by wire CD on the plate is –20.0 lb, determine (*a*) the tension in wire CD, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at C forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = -F \sin 30° \cos 60° = -20 \text{ lb} \quad \text{(Given)}$$

$$F = \frac{20 \text{ lb}}{\sin 30° \cos 60°} = 80 \text{ lb}$$

$F = 80.0 \text{ lb}$ ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-20 \text{ lb}}{80 \text{ lb}} = -0.25$$

$\theta_x = 104.5°$ ◀

$$F_y = (80 \text{ lb})\cos 30° = 69.282 \text{ lb}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{69.282 \text{ lb}}{80 \text{ lb}} = 0.86615$$

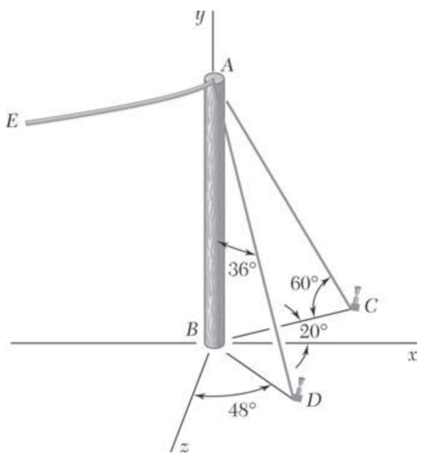
$\theta_y = 30.0°$ ◀

$$F_z = (80 \text{ lb})\sin 30° \sin 60° = 34.641 \text{ lb}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{34.641}{80} = 0.43301$$

$\theta_z = 64.3°$ ◀

PROBLEM 2.77



The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in wire *AC* is 120 lb, determine (*a*) the components of the force exerted by this wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (120 \text{ lb})\cos 60° \cos 20°$$

$$F_x = 56.382 \text{ lb} \qquad F_x = +56.4 \text{ lb} \blacktriangleleft$$

$$F_y = -(120 \text{ lb})\sin 60°$$

$$F_y = -103.923 \text{ lb} \qquad F_y = -103.9 \text{ lb} \blacktriangleleft$$

$$F_z = -(120 \text{ lb})\cos 60° \sin 20°$$

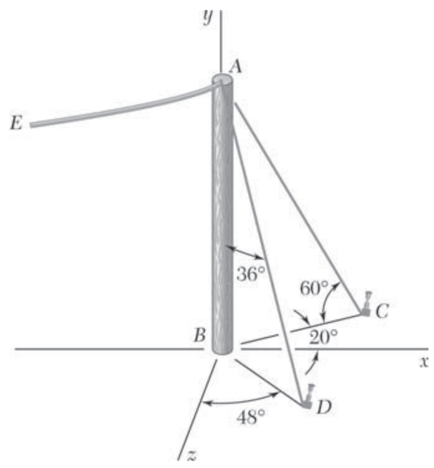
$$F_z = -20.521 \text{ lb} \qquad F_z = -20.5 \text{ lb} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{56.382 \text{ lb}}{120 \text{ lb}} \qquad \theta_x = 62.0° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-103.923 \text{ lb}}{120 \text{ lb}} \qquad \theta_y = 150.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-20.52 \text{ lb}}{120 \text{ lb}} \qquad \theta_z = 99.8° \blacktriangleleft$$



PROBLEM 2.78

The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in wire *AD* is 85 lb, determine (*a*) the components of the force exerted by this wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (85 \text{ lb})\sin 36° \sin 48°$$

$$= 37.129 \text{ lb} \qquad F_x = 37.1 \text{ lb} \blacktriangleleft$$

$$F_y = -(85 \text{ lb})\cos 36°$$

$$= -68.766 \text{ lb} \qquad F_y = -68.8 \text{ lb} \blacktriangleleft$$

$$F_z = (85 \text{ lb})\sin 36° \cos 48°$$

$$= 33.431 \text{ lb} \qquad F_z = 33.4 \text{ lb} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{37.129 \text{ lb}}{85 \text{ lb}} \qquad \theta_x = 64.1° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-68.766 \text{ lb}}{85 \text{ lb}} \qquad \theta_y = 144.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{33.431 \text{ lb}}{85 \text{ lb}} \qquad \theta_z = 66.8° \blacktriangleleft$$

PROBLEM 2.79

Determine the magnitude and direction of the force $\mathbf{F} = (320\text{ N})\mathbf{i} + (400\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k}$.

SOLUTION

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{(320\text{ N})^2 + (400\text{ N})^2 + (-250\text{ N})^2} \qquad F = 570\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{320\text{ N}}{570\text{ N}} \qquad \theta_x = 55.8° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{400\text{ N}}{570\text{ N}} \qquad \theta_y = 45.4° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_z}{F} = \frac{-250\text{ N}}{570\text{ N}} \qquad \theta_z = 116.0° \blacktriangleleft$$

PROBLEM 2.80

Determine the magnitude and direction of the force $\mathbf{F} = (240\text{ N})\mathbf{i} - (270\text{ N})\mathbf{j} + (680\text{ N})\mathbf{k}$.

SOLUTION

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{(240\text{ N})^2 + (-270\text{ N})^2 + (680\text{ N})} \qquad F = 770\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{240\text{ N}}{770\text{ N}} \qquad \theta_x = 71.8° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-270\text{ N}}{770\text{ N}} \qquad \theta_y = 110.5° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{680\text{ N}}{770\text{ N}} \qquad \theta_z = 28.0° \blacktriangleleft$$

PROBLEM 2.81

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 70.9°$ and $\theta_y = 144.9°$. Knowing that the z component of the force is –52.0 lb, determine (*a*) the angle θ_z, (*b*) the other components and the magnitude of the force.

SOLUTION

(*a*) We have

$$(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1 \Rightarrow (\cos\theta_y)^2 = 1 - (\cos\theta_y)^2 - (\cos\theta_z)^2$$

Since $F_z < 0$ we must have $\cos\theta_z < 0$

Thus, taking the negative square root, from above, we have:

$$\cos\theta_z = -\sqrt{1 - (\cos 70.9°)^2 - (\cos 144.9°)^2} = 0.47282 \qquad \theta_z = 118.2° \blacktriangleleft$$

(*b*) Then:

$$F = \frac{F_z}{\cos\theta_z} = \frac{52.0 \text{ lb}}{0.47282} = 109.978 \text{ lb}$$

and

$$F_x = F\cos\theta_x = (109.978 \text{ lb})\cos 70.9° \qquad F_x = 36.0 \text{ lb} \blacktriangleleft$$

$$F_y = F\cos\theta_y = (109.978 \text{ lb})\cos 144.9° \qquad F_y = -90.0 \text{ lb} \blacktriangleleft$$

$$F = 110.0 \text{ lb} \blacktriangleleft$$

PROBLEM 2.82

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_y = 55°$ and $\theta_z = 45°$. Knowing that the x component of the force is -500 lb, determine (a) the angle θ_x, (b) the other components and the magnitude of the force.

SOLUTION

(a) We have

$$(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1 \Rightarrow (\cos\theta_y)^2 = 1 - (\cos\theta_y)^2 - (\cos\theta_z)^2$$

Since $F_x < 0$ we must have $\cos\theta_x < 0$

Thus, taking the negative square root, from above, we have:

$$\cos\theta_x = -\sqrt{1 - (\cos 55)^2 - (\cos 45)^2} = 0.41353 \qquad \theta_x = 114.4° \blacktriangleleft$$

(b) Then:

$$F = \frac{F_x}{\cos\theta_x} = \frac{500 \text{ lb}}{0.41353} = 1209.10 \text{ lb} \qquad F = 1209 \text{ lb} \blacktriangleleft$$

and $$F_y = F\cos\theta_y = (1209.10 \text{ lb})\cos 55° \qquad F_y = 694 \text{ lb} \blacktriangleleft$$

$$F_z = F\cos\theta_z = (1209.10 \text{ lb})\cos 45° \qquad F_z = 855 \text{ lb} \blacktriangleleft$$

PROBLEM 2.83

A force **F** of magnitude 210 N acts at the origin of a coordinate system. Knowing that $F_x = 80$ N, $\theta_z = 151.2°$, and $F_y < 0$, determine (*a*) the components F_y and F_z, (*b*) the angles θ_x and θ_y.

SOLUTION

(*a*) $$F_z = F\cos\theta_z = (210\text{ N})\cos 151.2°$$

$$= -184.024\text{ N} \qquad F_z = -184.0\text{ N} \blacktriangleleft$$

Then: $$F^2 = F_x^2 + F_y^2 + F_z^2$$

So: $$(210\text{ N})^2 = (80\text{ N})^2 + (F_y)^2 + (184.024\text{ N})^2$$

Hence: $$F_y = -\sqrt{(210\text{ N})^2 - (80\text{ N})^2 - (184.024\text{ N})^2}$$

$$= -61.929\text{ N} \qquad F_y = -62.0\text{ lb} \blacktriangleleft$$

(*b*) $$\cos\theta_x = \frac{F_x}{F} = \frac{80\text{ N}}{210\text{ N}} = 0.38095 \qquad \theta_x = 67.6° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{61.929\text{ N}}{210\text{ N}} = -0.29490 \qquad \theta_y = 107.2 \blacktriangleleft$$

PROBLEM 2.84

A force **F** of magnitude 230 N acts at the origin of a coordinate system. Knowing that $\theta_x = 32.5°$, $F_y = -60$ N, and $F_z > 0$, determine (*a*) the components F_x and F_z, (*b*) the angles θ_y and θ_z.

SOLUTION

(*a*) We have

$$F_x = F\cos\theta_x = (230\text{ N})\cos 32.5° \qquad F_x = -194.0\text{ N} ◀$$

Then:

$$F_x = 193.980\text{ N}$$

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

So:

$$(230\text{ N})^2 = (193.980\text{ N})^2 + (-60\text{ N})^2 + F_z^2$$

Hence:

$$F_z = +\sqrt{(230\text{ N})^2 - (193.980\text{ N})^2 - (-60\text{ N})^2} \qquad F_z = 108.0\text{ N} ◀$$

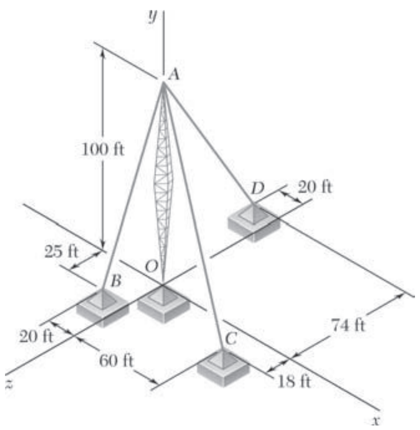
(*b*)

$$F_z = 108.036\text{ N}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-60\text{ N}}{230\text{ N}} = -0.26087 \qquad \theta_y = 105.1° ◀$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{108.036\text{ N}}{230\text{ N}} = 0.46972 \qquad \theta_z = 62.0° ◀$$

PROBLEM 2.85



A transmission tower is held by three guy wires anchored by bolts at B, C, and D. If the tension in wire AB is 525 lb, determine the components of the force exerted by the wire on the bolt at B.

SOLUTION

$$\overrightarrow{BA} = (20\text{ ft})\mathbf{i} + (100\text{ ft})\mathbf{j} - (25\text{ ft})\mathbf{k}$$

$$BA = \sqrt{(20\text{ ft})^2 + (100\text{ ft})^2 + (-25\text{ ft})^2}$$

$$= 105\text{ ft}$$

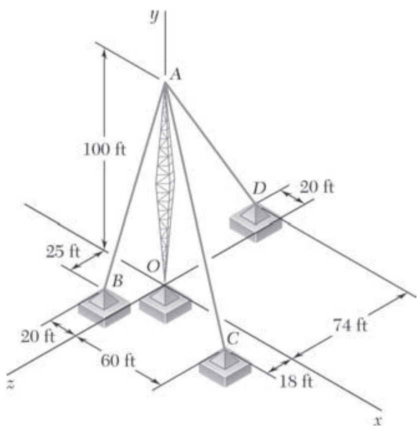
$$\mathbf{F} = F\boldsymbol{\lambda}_{BA}$$

$$= F\frac{\overrightarrow{BA}}{BA}$$

$$= \frac{525\text{ lb}}{105\text{ ft}}[(20\text{ ft})\mathbf{i} + (100\text{ ft})\mathbf{j} - (25\text{ ft})\mathbf{k}]$$

$$\mathbf{F} = (100.0\text{ lb})\mathbf{i} + (500\text{ lb})\mathbf{j} - (125.0\text{ lb})\mathbf{k}$$

$$F_x = +100.0\text{ lb}, \quad F_y = +500\text{ lb}, \quad F_z = -125.0\text{ lb} \blacktriangleleft$$



PROBLEM 2.86

A transmission tower is held by three guy wires anchored by bolts at B, C, and D. If the tension in wire AD is 315 lb, determine the components of the force exerted by the wire on the bolt at D.

SOLUTION

$$\overrightarrow{DA} = (20\text{ ft})\mathbf{i} + (100\text{ ft})\mathbf{j} + (70\text{ ft})\mathbf{k}$$

$$DA = \sqrt{(20\text{ ft})^2 + (100\text{ ft})^2 + (+70\text{ ft})^2}$$

$$= 126\text{ ft}$$

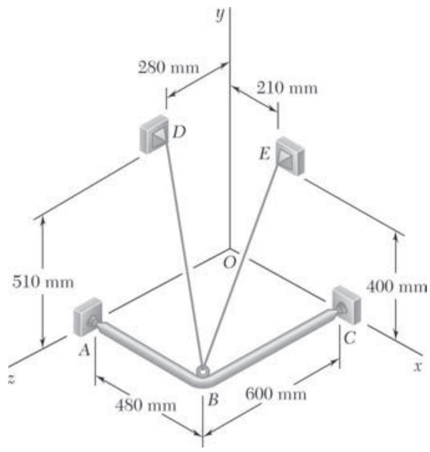
$$\mathbf{F} = F\boldsymbol{\lambda}_{DA}$$

$$= F\frac{\overrightarrow{DA}}{DA}$$

$$= \frac{315\text{ lb}}{126\text{ ft}}[(20\text{ ft})\mathbf{i} + (100\text{ ft})\mathbf{j} + (74\text{ ft})\mathbf{k}]$$

$$\mathbf{F} = (50\text{ lb})\mathbf{i} + (250\text{ lb})\mathbf{j} + (185\text{ lb})\mathbf{k}$$

$$F_x = +50\text{ lb},\quad F_y = +250\text{ lb},\quad F_z = +185.0\text{ lb} \blacktriangleleft$$



PROBLEM 2.87

A frame *ABC* is supported in part by cable *DBE* that passes through a frictionless ring at *B*. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at *D*.

SOLUTION

$$\overrightarrow{DB} = (480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}$$

$$DB = \sqrt{(480 \text{ mm})^2 + (510 \text{ mm}^2) + (320 \text{ mm})^2}$$

$$= 770 \text{ mm}$$

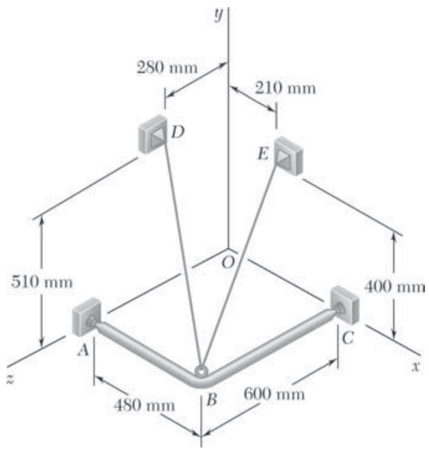
$$\mathbf{F} = F\boldsymbol{\lambda}_{DB}$$

$$= F\frac{\overrightarrow{DB}}{DB}$$

$$= \frac{385 \text{ N}}{770 \text{ mm}}[(480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}]$$

$$= (240 \text{ N})\mathbf{i} - (255 \text{ N})\mathbf{j} + (160 \text{ N})\mathbf{k}$$

$$F_x = +240 \text{ N}, \quad F_y = -255 \text{ N}, \quad F_z = +160.0 \text{ N} \blacktriangleleft$$



PROBLEM 2.88

For the frame and cable of Problem 2.87, determine the components of the force exerted by the cable on the support at E.

PROBLEM 2.87 A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D.

SOLUTION

$$\overrightarrow{EB} = (270 \text{ mm})\mathbf{i} - (400 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$EB = \sqrt{(270 \text{ mm})^2 + (400 \text{ mm})^2 + (600 \text{ mm})^2}$$

$$= 770 \text{ mm}$$

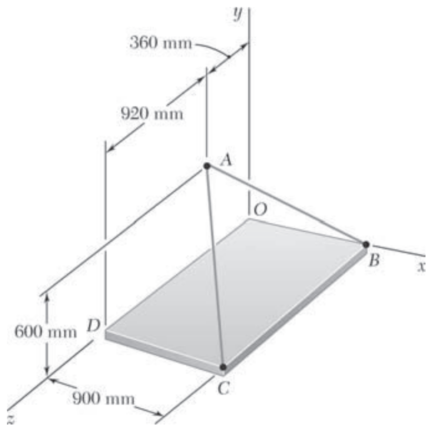
$$\mathbf{F} = F\boldsymbol{\lambda}_{EB}$$

$$= F\frac{\overrightarrow{EB}}{EB}$$

$$= \frac{385 \text{ N}}{770 \text{ mm}}[(270 \text{ mm})\mathbf{i} - (400 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}]$$

$$\mathbf{F} = (135 \text{ N})\mathbf{i} - (200 \text{ N})\mathbf{j} + (300 \text{ N})\mathbf{k}$$

$F_x = +135.0 \text{ N}, \quad F_y = -200 \text{ N}, \quad F_z = +300 \text{ N}$ ◀



PROBLEM 2.89

Knowing that the tension in cable AB is 1425 N, determine the components of the force exerted on the plate at B.

SOLUTION

$$\overrightarrow{BA} = -(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}$$

$$BA = \sqrt{(900\text{ mm})^2 + (600\text{ mm})^2 + (360\text{ mm})^2}$$

$$= 1140\text{ mm}$$

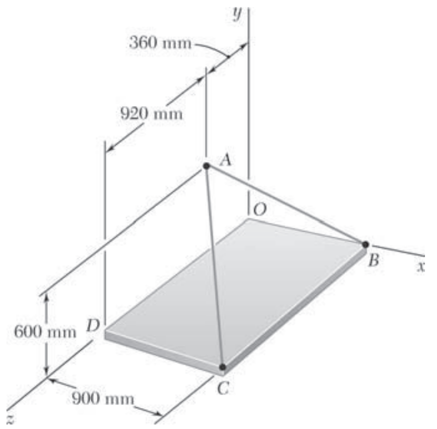
$$\mathbf{T}_{BA} = T_{BA}\boldsymbol{\lambda}_{BA}$$

$$= T_{BA}\frac{\overrightarrow{BA}}{BA}$$

$$\mathbf{T}_{BA} = \frac{1425\text{ N}}{1140\text{ mm}}[-(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}]$$

$$= -(1125\text{ N})\mathbf{i} + (750\text{ N})\mathbf{j} + (450\text{ N})\mathbf{k}$$

$$(T_{BA})_x = -1125\text{ N},\quad (T_{BA})_y = 750\text{ N},\quad (T_{BA})_z = 450\text{ N} \blacktriangleleft$$



PROBLEM 2.90

Knowing that the tension in cable AC is 2130 N, determine the components of the force exerted on the plate at C.

SOLUTION

$$\overrightarrow{CA} = -(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k}$$

$$CA = \sqrt{(900 \text{ mm})^2 + (600 \text{ mm})^2 + (920 \text{ mm})^2}$$

$$= 1420 \text{ mm}$$

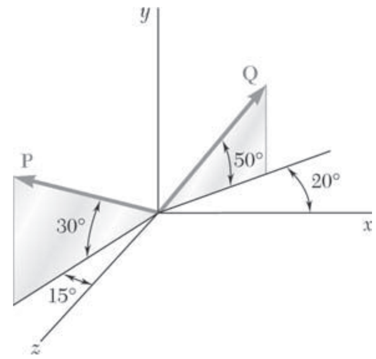
$$\mathbf{T}_{CA} = T_{CA}\lambda_{CA}$$

$$= T_{CA}\frac{\overrightarrow{CA}}{CA}$$

$$\mathbf{T}_{CA} = \frac{2130 \text{ N}}{1420 \text{ mm}}[-(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k}]$$

$$= -(1350 \text{ N})\mathbf{i} + (900 \text{ N})\mathbf{j} - (1380 \text{ N})\mathbf{k}$$

$$(T_{CA})_x = -1350 \text{ N}, \quad (T_{CA})_y = 900 \text{ N}, \quad (T_{CA})_z = -1380 \text{ N} \blacktriangleleft$$



PROBLEM 2.91

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 300$ N and $Q = 400$ N.

SOLUTION

$$\mathbf{P} = (300\text{ N})[-\cos 30°\sin 15°\mathbf{i} + \sin 30°\mathbf{j} + \cos 30°\cos 15°\mathbf{k}]$$
$$= -(67.243\text{ N})\mathbf{i} + (150\text{ N})\mathbf{j} + (250.95\text{ N})\mathbf{k}$$
$$\mathbf{Q} = (400\text{ N})[\cos 50°\cos 20°\mathbf{i} + \sin 50°\mathbf{j} - \cos 50°\sin 20°\mathbf{k}]$$
$$= (400\text{ N})[0.60402\mathbf{i} + 0.76604\mathbf{j} - 0.21985]$$
$$= (241.61\text{ N})\mathbf{i} + (306.42\text{ N})\mathbf{j} - (87.939\text{ N})\mathbf{k}$$
$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$
$$= (174.367\text{ N})\mathbf{i} + (456.42\text{ N})\mathbf{j} + (163.011\text{ N})\mathbf{k}$$
$$R = \sqrt{(174.367\text{ N})^2 + (456.42\text{ N})^2 + (163.011\text{ N})^2}$$
$$= 515.07\text{ N}$$

$R = 515\text{ N}$ ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{174.367\text{ N}}{515.07\text{ N}} = 0.33853$$

$\theta_x = 70.2°$ ◀

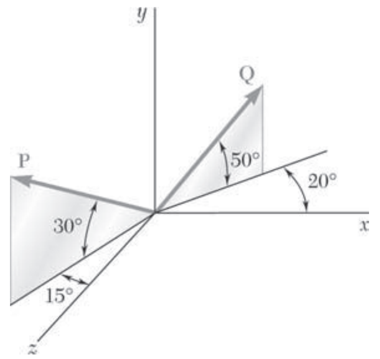
$$\cos\theta_y = \frac{R_y}{R} = \frac{456.42\text{ N}}{515.07\text{ N}} = 0.88613$$

$\theta_y = 27.6°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{163.011\text{ N}}{515.07\text{ N}} = 0.31648$$

$\theta_z = 71.5°$ ◀

PROBLEM 2.92



Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 400$ N and $Q = 300$ N.

SOLUTION

$$\mathbf{P} = (400\text{ N})[-\cos 30°\sin 15°\mathbf{i} + \sin 30°\mathbf{j} + \cos 30°\cos 15°\mathbf{k}]$$
$$= -(89.678\text{ N})\mathbf{i} + (200\text{ N})\mathbf{j} + (334.61\text{ N})\mathbf{k}$$
$$\mathbf{Q} = (300\text{ N})[\cos 50°\cos 20°\mathbf{i} + \sin 50°\mathbf{j} - \cos 50°\sin 20°\mathbf{k}]$$
$$= (181.21\text{ N})\mathbf{i} + (229.81\text{ N})\mathbf{j} - (65.954\text{ N})\mathbf{k}$$
$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$
$$= (91.532\text{ N})\mathbf{i} + (429.81\text{ N})\mathbf{j} + (268.66\text{ N})\mathbf{k}$$
$$R = \sqrt{(91.532\text{ N})^2 + (429.81\text{ N})^2 + (268.66\text{ N})^2}$$
$$= 515.07\text{ N}$$

$R = 515$ N ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{91.532\text{ N}}{515.07\text{ N}} = 0.177708$$

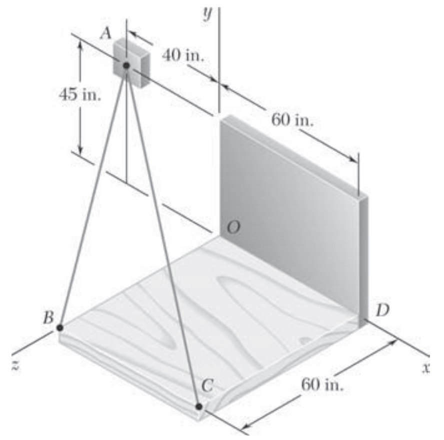
$\theta_x = 79.8°$ ◀

$$\cos\theta_y = \frac{R_y}{R} = \frac{429.81\text{ N}}{515.07\text{ N}} = 0.83447$$

$\theta_y = 33.4°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{268.66\text{ N}}{515.07\text{ N}} = 0.52160$$

$\theta_z = 58.6°$ ◀



PROBLEM 2.93

Knowing that the tension is 425 lb in cable AB and 510 lb in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION

$$\overrightarrow{AB} = (40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 85\text{ in.}$$

$$\overrightarrow{AC} = (100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 125\text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (425\text{ lb})\left[\frac{(40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{85\text{ in.}}\right]$$

$$\mathbf{T}_{AB} = (200\text{ lb})\mathbf{i} - (225\text{ lb})\mathbf{j} + (300\text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (510\text{ lb})\left[\frac{(100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{125\text{ in.}}\right]$$

$$\mathbf{T}_{AC} = (408\text{ lb})\mathbf{i} - (183.6\text{ lb})\mathbf{j} + (244.8\text{ lb})\mathbf{k}$$

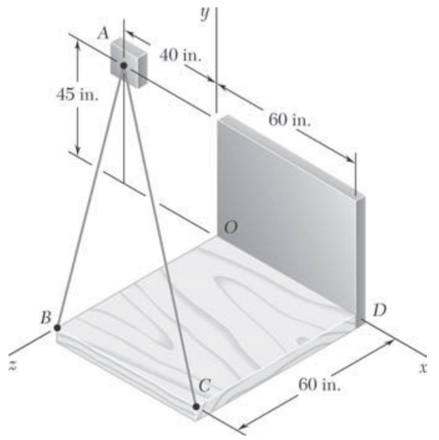
$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (608)\mathbf{i} - (408.6\text{ lb})\mathbf{j} + (544.8\text{ lb})\mathbf{k}$$

Then: $R = 912.92\text{ lb}$ $\qquad R = 913\text{ lb}$ ◀

and $\cos\theta_x = \dfrac{608\text{ lb}}{912.92\text{ lb}} = 0.66599$ $\qquad \theta_x = 48.2°$ ◀

$\cos\theta_y = \dfrac{408.6\text{ lb}}{912.92\text{ lb}} = -0.44757$ $\qquad \theta_y = 116.6°$ ◀

$\cos\theta_z = \dfrac{544.8\text{ lb}}{912.92\text{ lb}} = 0.59677$ $\qquad \theta_z = 53.4°$ ◀



PROBLEM 2.94

Knowing that the tension is 510 lb in cable AB and 425 lb in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION

$$\overrightarrow{AB} = (40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 85 \text{ in.}$$

$$\overrightarrow{AC} = (100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 125 \text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (510 \text{ lb})\left[\frac{(40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{85 \text{ in.}}\right]$$

$$\mathbf{T}_{AB} = (240 \text{ lb})\mathbf{i} - (270 \text{ lb})\mathbf{j} + (360 \text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (425 \text{ lb})\left[\frac{(100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{125 \text{ in.}}\right]$$

$$\mathbf{T}_{AC} = (340 \text{ lb})\mathbf{i} - (153 \text{ lb})\mathbf{j} + (204 \text{ lb})\mathbf{k}$$

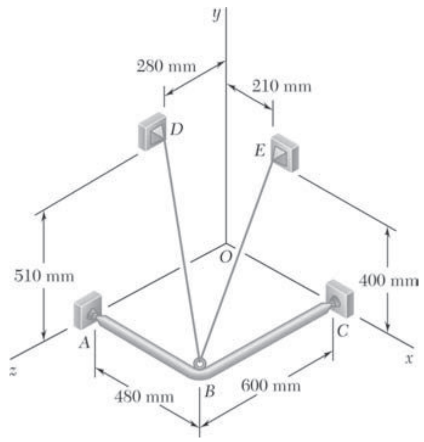
$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (580 \text{ lb})\mathbf{i} - (423 \text{ lb})\mathbf{j} + (564 \text{ lb})\mathbf{k}$$

Then: $R = 912.92 \text{ lb}$ $\qquad$ $R = 913 \text{ lb}$ ◀

and $\cos\theta_x = \dfrac{580 \text{ lb}}{912.92 \text{ lb}} = 0.63532$ $\qquad$ $\theta_x = 50.6°$ ◀

$\cos\theta_y = \dfrac{-423 \text{ lb}}{912.92 \text{ lb}} = -0.46335$ $\qquad$ $\theta_y = 117.6°$ ◀

$\cos\theta_z = \dfrac{564 \text{ lb}}{912.92 \text{ lb}} = 0.61780$ $\qquad$ $\theta_z = 51.8°$ ◀



PROBLEM 2.95

For the frame of Problem 2.87, determine the magnitude and direction of the resultant of the forces exerted by the cable at B knowing that the tension in the cable is 385 N.

PROBLEM 2.87 A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D.

SOLUTION

$$\overrightarrow{BD} = -(480\text{ mm})\mathbf{i} + (510\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k}$$

$$BD = \sqrt{(480\text{ mm})^2 + (510\text{ mm})^2 + (320\text{ mm})^2} = 770\text{ mm}$$

$$\begin{aligned}\mathbf{F}_{BD} &= T_{BD}\boldsymbol{\lambda}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD}\\ &= \frac{(385\text{ N})}{(770\text{ mm})}[-(480\text{ mm})\mathbf{i} + (510\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k}]\\ &= -(240\text{ N})\mathbf{i} + (255\text{ N})\mathbf{j} - (160\text{ N})\mathbf{k}\end{aligned}$$

$$\overrightarrow{BE} = -(270\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} - (600\text{ mm})\mathbf{k}$$

$$BE = \sqrt{(270\text{ mm})^2 + (400\text{ mm})^2 + (600\text{ mm})^2} = 770\text{ mm}$$

$$\begin{aligned}\mathbf{F}_{BE} &= T_{BE}\boldsymbol{\lambda}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE}\\ &= \frac{(385\text{ N})}{(770\text{ mm})}[-(270\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} - (600\text{ mm})\mathbf{k}]\\ &= -(135\text{ N})\mathbf{i} + (200\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}\end{aligned}$$

$$\mathbf{R} = \mathbf{F}_{BD} + \mathbf{F}_{BE} = -(375\text{ N})\mathbf{i} + (455\text{ N})\mathbf{j} - (460\text{ N})\mathbf{k}$$

$$R = \sqrt{(375\text{ N})^2 + (455\text{ N})^2 + (460\text{ N})^2} = 747.83\text{ N}$$

$R = 748\text{ N}$ ◀

$$\cos\theta_x = \frac{-375\text{ N}}{747.83\text{ N}}$$

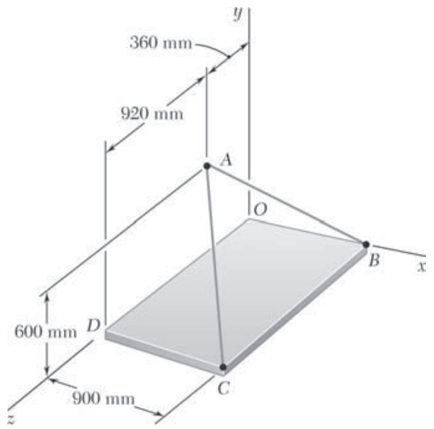
$\theta_x = 120.1°$ ◀

$$\cos\theta_y = \frac{455\text{ N}}{747.83\text{ N}}$$

$\theta_y = 52.5°$ ◀

$$\cos\theta_z = \frac{-460\text{ N}}{747.83\text{ N}}$$

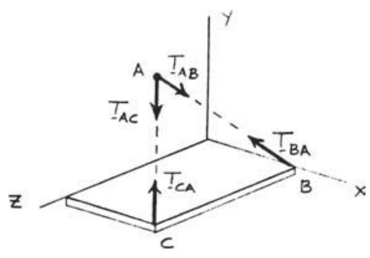
$\theta_z = 128.0°$ ◀



PROBLEM 2.96

For the cables of Problem 2.89, knowing that the tension is 1425 N in cable AB and 2130 N in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION



$$T_{AB} = -T_{BA} \quad \text{(use results of Problem 2.89)}$$

$$(T_{AB})_x = +1125 \text{ N} \quad (T_{AB})_y = -750 \text{ N} \quad (T_{AB})_z = -450 \text{ N}$$

$$T_{AC} = -T_{CA} \quad \text{(use results of Problem 2.90)}$$

$$(T_{AC})_x = +1350 \text{ N} \quad (T_{AC})_y = -900 \text{ N} \quad (T_{AC})_z = +1380 \text{ N}$$

Resultant:

$$R_x = \Sigma F_x = +1125 + 1350 = +2475 \text{ N}$$

$$R_y = \Sigma F_y = -750 - 900 = -1650 \text{ N}$$

$$R_z = \Sigma F_z = -450 + 1380 = +930 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{(+2475)^2 + (-1650)^2 + (+930)^2}$$

$$= 3116.6 \text{ N}$$

$R = 3120 \text{ N}$ ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{+2475}{3116.6}$$

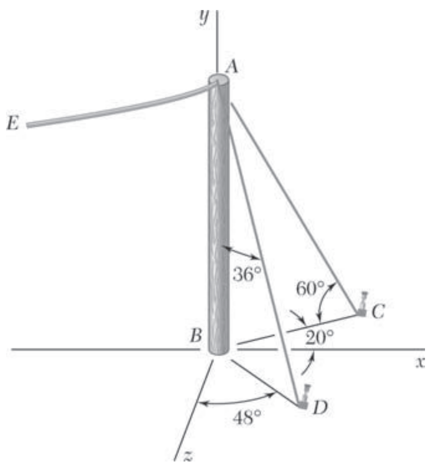
$\theta_x = 37.4°$ ◀

$$\cos\theta_y = \frac{R_y}{R} = \frac{-1650}{3116.6}$$

$\theta_y = 122.0°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{+930}{3116.6}$$

$\theta_z = 72.6°$ ◀



PROBLEM 2.97

The end of the coaxial cable AE is attached to the pole AB, which is strengthened by the guy wires AC and AD. Knowing that the tension in AC is 150 lb and that the resultant of the forces exerted at A by wires AC and AD must be contained in the xy plane, determine (*a*) the tension in AD, (*b*) the magnitude and direction of the resultant of the two forces.

SOLUTION

$$\begin{aligned}\mathbf{R} &= \mathbf{T}_{AC} + \mathbf{T}_{AD}\\ &= (150\text{ lb})(\cos 60°\cos 20°\mathbf{i} - \sin 60°\mathbf{j} - \cos 60°\sin 20°\mathbf{k})\\ &\quad + T_{AD}(\sin 36°\sin 48°\mathbf{i} - \cos 36°\mathbf{j} + \sin 36°\cos 48°\mathbf{k})\end{aligned} \tag{1}$$

(*a*) Since $R_z = 0$, The coefficient of **k** must be zero.

$$(150\text{ lb})(-\cos 60°\sin 20°) + T_{AD}(\sin 36°\cos 48°) = 0$$

$$T_{AD} = 65.220\text{ lb} \qquad T_{AD} = 65.2\text{ lb} \blacktriangleleft$$

(*b*) Substituting for T_{AD} into Eq. (1) gives:

$$\begin{aligned}\mathbf{R} &= [(150\text{ lb})\cos 60°\cos 20° + (65.220\text{ lb})\sin 36°\sin 48°)]\mathbf{i}\\ &\quad -[(150\text{ lb})\sin 60° + (65.220\text{ lb})\cos 36°]\mathbf{j} + 0\end{aligned}$$

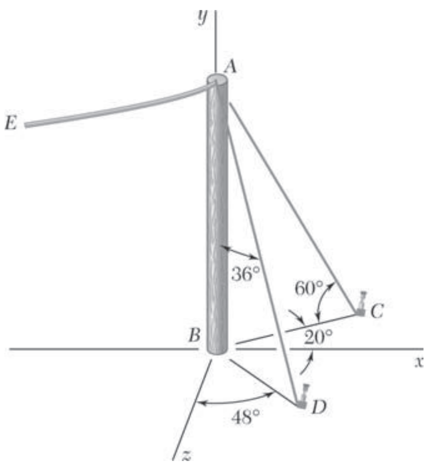
$$\mathbf{R} = (98.966\text{ lb})\mathbf{i} - (182.668\text{ lb})\mathbf{j}$$

$$\begin{aligned}R &= \sqrt{(98.966\text{ lb})^2 + (182.668\text{ lb})^2}\\ &= 207.76\text{ lb}\end{aligned} \qquad R = 208\text{ lb} \blacktriangleleft$$

$$\cos\theta_x = \frac{98.966\text{ lb}}{207.76\text{ lb}} \qquad \theta_x = 61.6° \blacktriangleleft$$

$$\cos\theta_y = \frac{182.668\text{ lb}}{207.76\text{ lb}} \qquad \theta_y = 151.6° \blacktriangleleft$$

$$\cos\theta_z = 0 \qquad \theta_z = 90.0° \blacktriangleleft$$



PROBLEM 2.98

The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in *AD* is 125 lb and that the resultant of the forces exerted at *A* by wires *AC* and *AD* must be contained in the *xy* plane, determine (*a*) the tension in *AC*, (*b*) the magnitude and direction of the resultant of the two forces.

SOLUTION

$$\begin{aligned}\mathbf{R} &= \mathbf{T}_{AC} + \mathbf{T}_{AD} \\ &= T_{AC}(\cos 60°\cos 20°\mathbf{i} - \sin 60°\mathbf{j} - \cos 60°\sin 20°\mathbf{k}) \\ &\quad + (125 \text{ lb})(\sin 36°\sin 48°\mathbf{i} - \cos 36°\mathbf{j} + \sin 36°\cos 48°\mathbf{k}) \qquad (1)\end{aligned}$$

(*a*) Since $R_z = 0$, The coefficient of **k** must be zero.

$$T_{AC}(-\cos 60°\sin 20°) + (125 \text{ lb})(\sin 36°\cos 48°) = 0$$

$$T_{AC} = 287.49 \text{ lb} \qquad T_{AC} = 287 \text{ lb} \blacktriangleleft$$

(*b*) Substituting for T_{AC} into Eq. (1) gives:

$$\begin{aligned}\mathbf{R} &= [(287.49 \text{ lb})\cos 60°\cos 20° + (125 \text{ lb})\sin 36°\sin 48°]\mathbf{i} \\ &\quad - [(287.49 \text{ lb})\sin 60° + (125 \text{ lb})\cos 36°]\mathbf{j} + 0\end{aligned}$$

$$\mathbf{R} = (189.677 \text{ lb})\mathbf{i} - (350.10 \text{ lb})\mathbf{j}$$

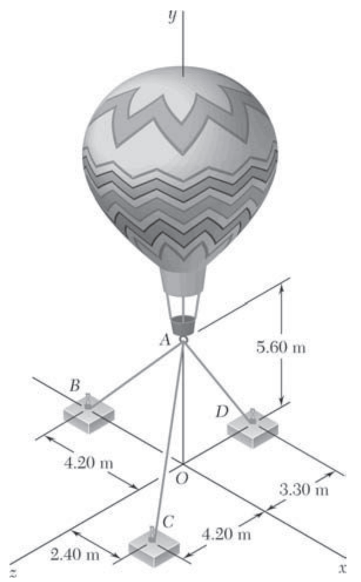
$$R = \sqrt{(189.677 \text{ lb})^2 + (350.10 \text{ lb})^2}$$

$$= 398.18 \text{ lb} \qquad R = 398 \text{ lb} \blacktriangleleft$$

$$\cos\theta_x = \frac{189.677 \text{ lb}}{398.18 \text{ lb}} \qquad \theta_x = 61.6° \blacktriangleleft$$

$$\cos\theta_y = \frac{350.10 \text{ lb}}{398.18 \text{ lb}} \qquad \theta_y = 151.6° \blacktriangleleft$$

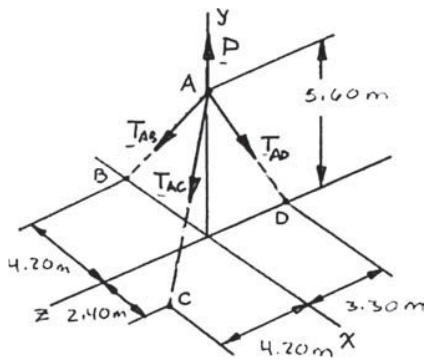
$$\cos\theta_z = 0 \qquad \theta_z = 90.0° \blacktriangleleft$$



PROBLEM 2.99

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AB is 259 N.

SOLUTION



The forces applied at A are: $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD}$, and $\mathbf{P}$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write

$$\overrightarrow{AB} = -(4.20 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} \qquad AB = 7.00 \text{ m}$$
$$\overrightarrow{AC} = (2.40 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} + (4.20 \text{ m})\mathbf{k} \qquad AC = 7.40 \text{ m}$$
$$\overrightarrow{AD} = -(5.60 \text{ m})\mathbf{j} - (3.30 \text{ m})\mathbf{k} \qquad AD = 6.50 \text{ m}$$

and

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.32432 - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (-0.86154\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

PROBLEM 2.99 (Continued)

Equilibrium condition $\Sigma F = 0$: $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$(-0.6T_{AB} + 0.32432T_{AC})\mathbf{i} + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)\mathbf{j}$$
$$+(0.56757T_{AC} - 0.50769T_{AD})\mathbf{k} = 0$$

Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

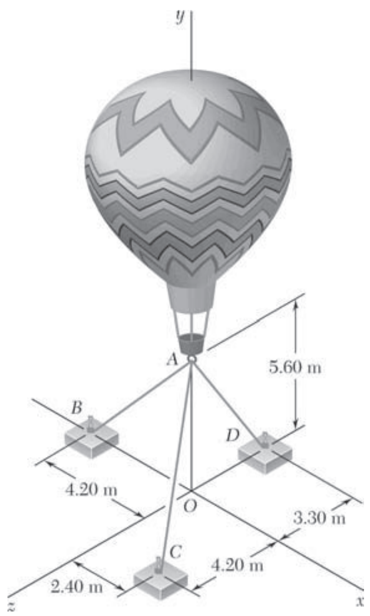
$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

Setting $T_{AB} = 259 \text{ N}$ in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 479.15 \text{ N}$$
$$T_{AD} = 535.66 \text{ N}$$

$\mathbf{P} = 1031 \text{ N} \uparrow$ ◀



PROBLEM 2.100

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at *A* knowing that the tension in cable *AC* is 444 N.

SOLUTION

See Problem 2.99 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \tag{1}$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \tag{2}$$

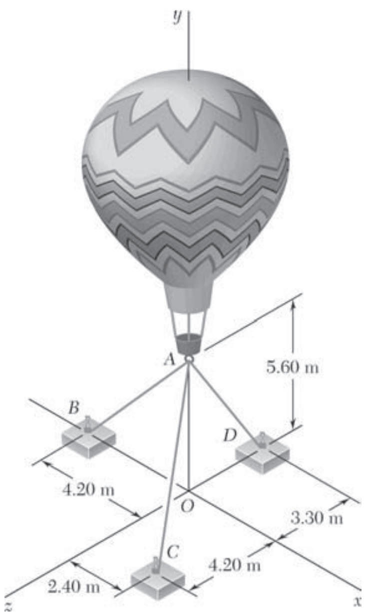
$$0.56757T_{AC} - 0.50769T_{AD} = 0 \tag{3}$$

Substituting $T_{AC} = 444$ N in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AB} = 240 \text{ N}$$

$$T_{AD} = 496.36 \text{ N}$$

$\mathbf{P} = 956 \text{ N} \uparrow$ ◀



PROBLEM 2.101

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AD is 481 N.

SOLUTION

See Problem 2.99 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3).

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \tag{1}$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \tag{2}$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \tag{3}$$

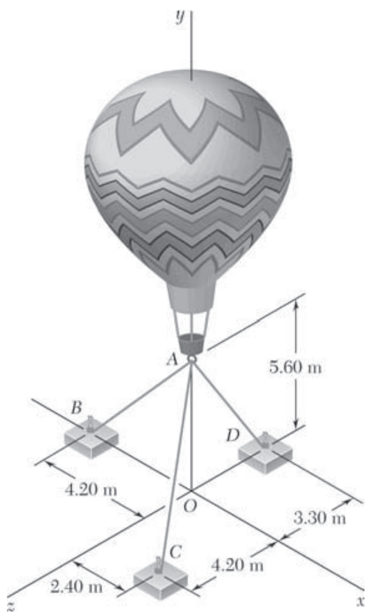
Substituting $T_{AD} = 481$ N in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AC} = 430.26 \text{ N}$$

$$T_{AB} = 232.57 \text{ N}$$

$\mathbf{P} = 926 \text{ N}\uparrow$ ◀

PROBLEM 2.102



Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at A, determine the tension in each cable.

SOLUTION

See Problem 2.99 for the figure and analysis leading to the linear algebraic Equations (1), (2), and (3).

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \qquad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \qquad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \qquad (3)$$

From Eq. (1) $\quad T_{AB} = 0.54053T_{AC}$

From Eq. (3) $\quad T_{AD} = 1.11795T_{AC}$

Substituting for T_{AB} and T_{AD} in terms of T_{AC} into Eq. (2) gives:

$$-0.8(0.54053T_{AC}) - 0.75676T_{AC} - 0.86154(1.11795T_{AC}) + P = 0$$

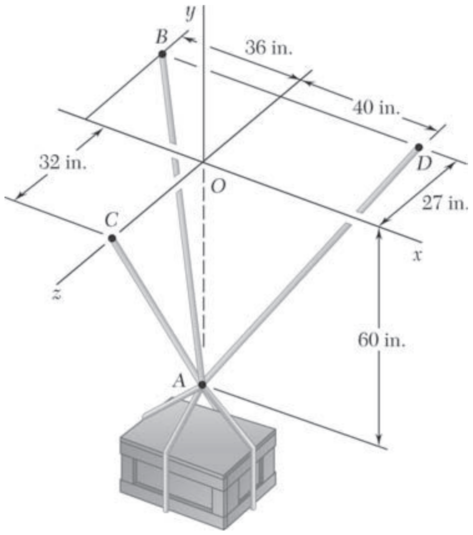
$$2.1523T_{AC} = P; \quad P = 800 \text{ N}$$

$$T_{AC} = \frac{800 \text{ N}}{2.1523}$$
$$= 371.69 \text{ N}$$

Substituting into expressions for T_{AB} and T_{AD} gives:

$$T_{AB} = 0.54053(371.69 \text{ N})$$
$$T_{AD} = 1.11795(371.69 \text{ N})$$

$$T_{AB} = 201 \text{ N}, \quad T_{AC} = 372 \text{ N}, \quad T_{AD} = 416 \text{ N} \blacktriangleleft$$



PROBLEM 2.103

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AB is 750 lb.

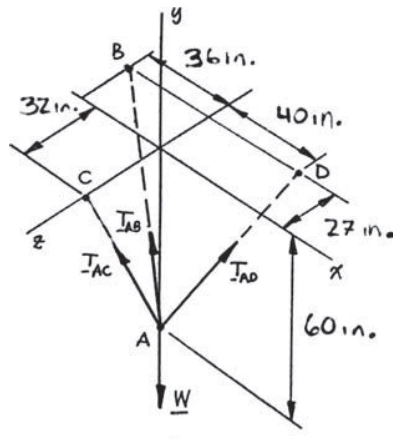
SOLUTION

The forces applied at A are:

$$\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD} \text{ and } \mathbf{W}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write

$$\overrightarrow{AB} = -(36 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (27 \text{ in.})\mathbf{k}$$
$$AB = 75 \text{ in.}$$
$$\overrightarrow{AC} = (60 \text{ in.})\mathbf{j} + (32 \text{ in.})\mathbf{k}$$
$$AC = 68 \text{ in.}$$
$$\overrightarrow{AD} = (40 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (27 \text{ in.})\mathbf{k}$$
$$AD = 77 \text{ in.}$$



and

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$
$$= (-0.48\mathbf{i} + 0.8\mathbf{j} - 0.36\mathbf{k})T_{AB}$$
$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$
$$= (0.88235\mathbf{j} + 0.47059\mathbf{k})T_{AC}$$
$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD}$$
$$= (0.51948\mathbf{i} + 0.77922\mathbf{j} - 0.35065\mathbf{k})T_{AD}$$

Equilibrium Condition with $\mathbf{W} = -W\mathbf{j}$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

PROBLEM 2.103 (Continued)

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$(-0.48T_{AB} + 0.51948T_{AD})\mathbf{i} + (0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W)\mathbf{j}$$
$$+(-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD})\mathbf{k} = 0$$

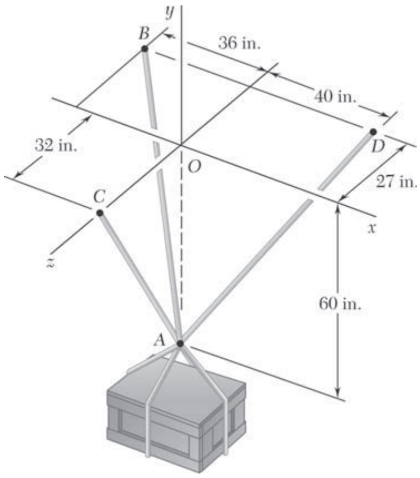
Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $T_{AB} = 750$ lb in Equations (1), (2), and (3) and solving the resulting set of equations, using conventional algorithms for solving linear algebraic equations, gives:

$$T_{AC} = 1090.1 \text{ lb}$$
$$T_{AD} = 693 \text{ lb}$$

$W = 2100$ lb ◀



PROBLEM 2.104

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable *AD* is 616 lb.

SOLUTION

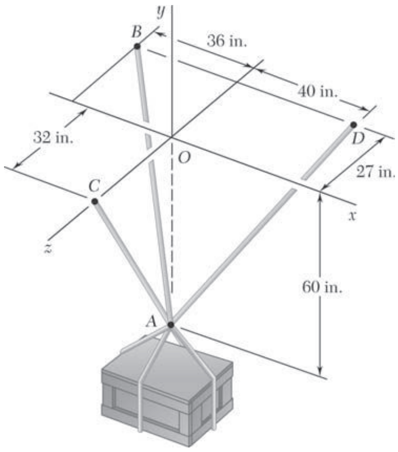
See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $T_{AD} = 616$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB} = 667.67 \text{ lb}$$
$$T_{AC} = 969.00 \text{ lb}$$

$W = 1868$ lb ◀



PROBLEM 2.105

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AC is 544 lb.

SOLUTION

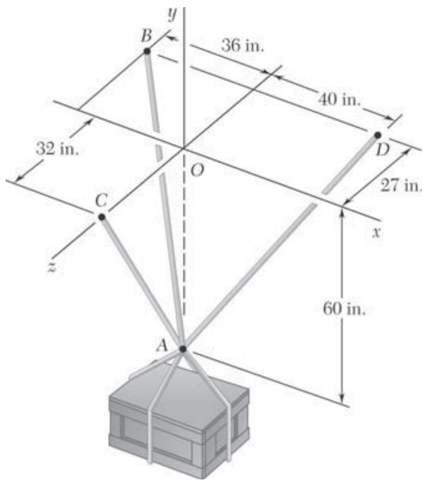
See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $T_{AC} = 544$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB} = 374.27 \text{ lb}$$
$$T_{AD} = 345.82 \text{ lb}$$

$W = 1049$ lb ◀



PROBLEM 2.106

A 1600-lb crate is supported by three cables as shown. Determine the tension in each cable.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

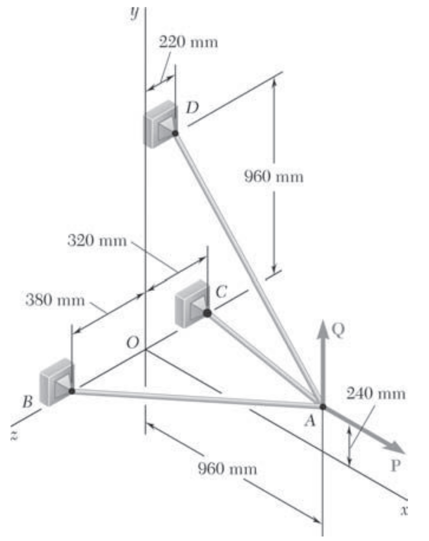
$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $W = 1600$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$T_{AB} = 571$ lb ◀

$T_{AC} = 830$ lb ◀

$T_{AD} = 528$ lb ◀



PROBLEM 2.107

Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that $Q = 0$, find the value of P for which the tension in cable AD is 305 N.

SOLUTION

$$\Sigma\mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0 \quad \text{where} \quad \mathbf{P} = P\mathbf{i}$$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \qquad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \qquad AC = 1040 \text{ mm}$$

$$\overrightarrow{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \qquad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = \frac{305 \text{ N}}{1220 \text{ mm}}[(-960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k}]$$

$$= -(240 \text{ N})\mathbf{i} + (180 \text{ N})\mathbf{j} - (55 \text{ N})\mathbf{k}$$

Substituting into $\Sigma\mathbf{F}_A = 0$, factoring **i**, **j**, **k**, and setting each coefficient equal to ϕ gives:

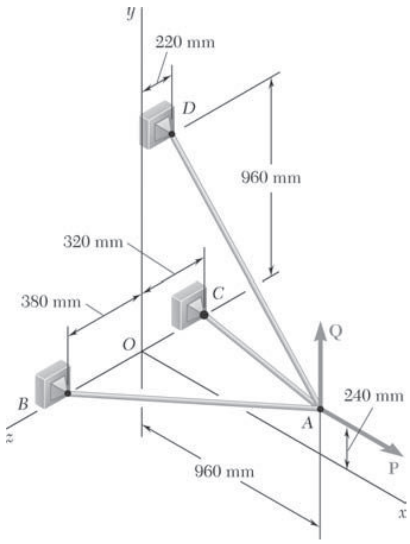
$$\mathbf{i}: \quad P = \frac{48}{53}T_{AB} + \frac{12}{13}T_{AC} + 240 \text{ N} \tag{1}$$

$$\mathbf{j}: \quad \frac{12}{53}T_{AB} + \frac{3}{13}T_{AC} = 180 \text{ N} \tag{2}$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 55 \text{ N} \tag{3}$$

Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 446.71 \text{ N}$$
$$T_{AC} = 341.71 \text{ N} \qquad P = 960 \text{ N} \blacktriangleleft$$



PROBLEM 2.108

Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that $P = 1200$ N, determine the values of Q for which cable AD is taut.

SOLUTION

We assume that $T_{AD} = 0$ and write $\qquad \Sigma\mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + Q\mathbf{j} + (1200\text{ N})\mathbf{i} = 0$

$$\overrightarrow{AB} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} + (380\text{ mm})\mathbf{k} \quad AB = 1060\text{ mm}$$

$$\overrightarrow{AC} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad AC = 1040\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)T_{AC}$$

Substituting into $\Sigma\mathbf{F}_A = 0$, factoring $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200\text{ N} = 0 \tag{1}$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q = 0 \tag{2}$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 0 \tag{3}$$

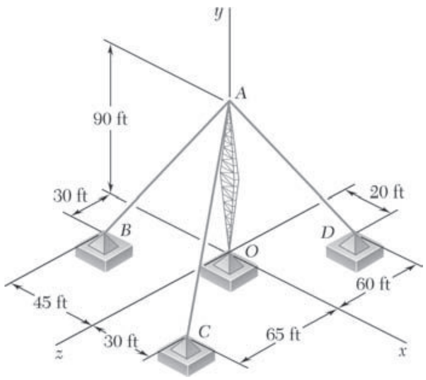
Solving the resulting system of linear equations using conventional algorithms gives:

$$T_{AB} = 605.71\text{ N}$$

$$T_{AC} = 705.71\text{ N}$$

$$Q = 300.00\text{ N} \qquad\qquad 0 \le Q < 300\text{ N} ◀$$

Note: This solution assumes that Q is directed upward as shown $(Q \ge 0)$, if negative values of Q are considered, cable AD remains taut, but AC becomes slack for $Q = -460$ N.

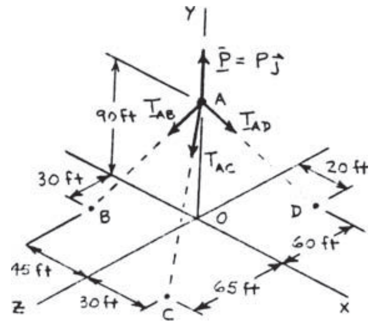


PROBLEM 2.109

A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AB is 630 lb, determine the vertical force **P** exerted by the tower on the pin at A.

SOLUTION

Free Body A:



$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

$$\overrightarrow{AB} = -45\mathbf{i} - 90\mathbf{j} + 30\mathbf{k} \qquad AB = 105 \text{ ft}$$

$$\overrightarrow{AC} = 30\mathbf{i} - 90\mathbf{j} + 65\mathbf{k} \qquad AC = 115 \text{ ft}$$

$$\overrightarrow{AD} = 20\mathbf{i} - 90\mathbf{j} - 60\mathbf{k} \qquad AD = 110 \text{ ft}$$

We write

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(\frac{6}{23}\mathbf{i} - \frac{18}{23}\mathbf{j} + \frac{13}{23}\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \left(\frac{2}{11}\mathbf{i} - \frac{9}{11}\mathbf{j} - \frac{6}{11}\mathbf{k}\right)T_{AD}$$

Substituting into the Eq. $\Sigma \mathbf{F} = 0$ and factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\left(-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD}\right)\mathbf{i} + \left(-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P\right)\mathbf{j} + \left(\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD}\right)\mathbf{k} = 0$$

PROBLEM 2.109 (Continued)

Setting the coefficients of **i**, **j**, **k**, equal to zero:

$$\mathbf{i}: \quad -\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \quad -\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \quad \frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \tag{3}$$

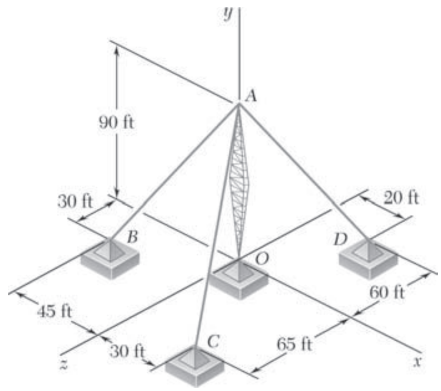
Set $T_{AB} = 630$ lb in Eqs. (1) – (3):

$$-270 \text{ lb} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \tag{1$'$}$$

$$-540 \text{ lb} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \tag{2$'$}$$

$$180 \text{ lb} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \tag{3$'$}$$

Solving, $T_{AC} = 467.42$ lb $\quad T_{AD} = 814.35$ lb $\quad P = 1572.10$ lb $\qquad P = 1572$ lb ◀



PROBLEM 2.110

A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AC* is 920 lb, determine the vertical force **P** exerted by the tower on the pin at *A*.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \tag{1}$$

$$-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \tag{2}$$

$$\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \tag{3}$$

Substituting for $T_{AC} = 920$ lb in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-\frac{3}{7}T_{AB} + 240\text{ lb} + \frac{2}{11}T_{AD} = 0 \tag{1′}$$

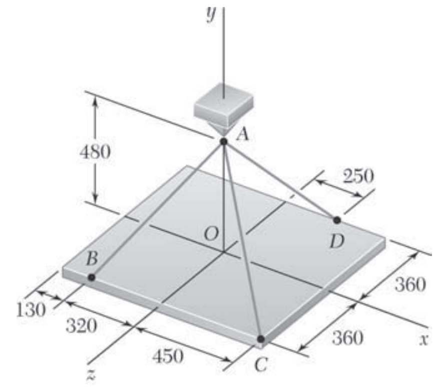
$$-\frac{6}{7}T_{AB} - 720\text{ lb} - \frac{9}{11}T_{AD} + P = 0 \tag{2′}$$

$$\frac{2}{7}T_{AB} + 520\text{ lb} - \frac{6}{11}T_{AD} = 0 \tag{3′}$$

Solving,

$$T_{AB} = 1240.00\text{ lb}$$
$$T_{AD} = 1602.86\text{ lb}$$
$$P = 3094.3\text{ lb}$$

$P = 3090$ lb ◀



Dimensions in mm

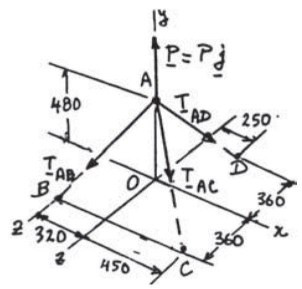
PROBLEM 2.111

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the weight of the plate.

SOLUTION

We note that the weight of the plate is equal in magnitude to the force **P** exerted by the support on Point A.

Free Body A:



Dimensions in mm

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

We have:

$$\overrightarrow{AB} = -(320 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \qquad AB = 680 \text{ mm}$$
$$\overrightarrow{AC} = (450 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \qquad AC = 750 \text{ mm}$$
$$\overrightarrow{AD} = (250 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} - (360 \text{ mm})\mathbf{k} \qquad AD = 650 \text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{8}{17}\mathbf{i} - \frac{12}{17}\mathbf{j} + \frac{9}{17}\mathbf{k}\right)T_{AB}$$
$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(0.6\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}\right)T_{AC}$$
$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \left(\frac{5}{13}\mathbf{i} - \frac{9.6}{13}\mathbf{j} - \frac{7.2}{13}\mathbf{k}\right)T_{AD}$$

Substituting into the Eq. $\Sigma F = 0$ and factoring $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$$\left(-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD}\right)\mathbf{i}$$
$$+\left(-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P\right)\mathbf{j}$$
$$+\left(\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD}\right)\mathbf{k} = 0$$

PROBLEM 2.111 (Continued)

Setting the coefficient of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \qquad -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \qquad (1)$$

$$\mathbf{j}: \qquad -\frac{12}{7}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \qquad (2)$$

$$\mathbf{k}: \qquad \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \qquad (3)$$

Making $T_{AC} = 60$ N in (1) and (3):

$$-\frac{8}{17}T_{AB} + 36 \text{ N} + \frac{5}{13}T_{AD} = 0 \qquad (1')$$

$$\frac{9}{17}T_{AB} + 28.8 \text{ N} - \frac{7.2}{13}T_{AD} = 0 \qquad (3')$$

Multiply (1′) by 9, (3′) by 8, and add:

$$554.4 \text{ N} - \frac{12.6}{13}T_{AD} = 0 \quad T_{AD} = 572.0 \text{ N}$$

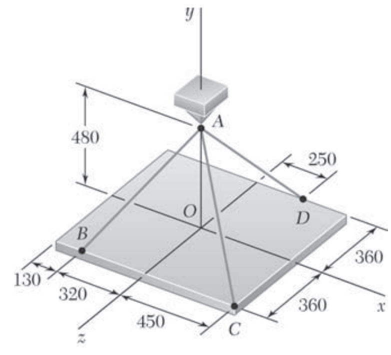
Substitute into (1′) and solve for T_{AB}:

$$T_{AB} = \frac{17}{8}\left(36 + \frac{5}{13}\times 572\right) \qquad T_{AB} = 544.0 \text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P:

$$P = \frac{12}{17}(544 \text{ N}) + 0.64(60 \text{ N}) + \frac{9.6}{13}(572 \text{ N})$$
$$= 844.8 \text{ N}$$

Weight of plate $= P = 845$ N ◀



PROBLEM 2.112

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 520 N, determine the weight of the plate.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \tag{1}$$

$$-\frac{12}{17}T_{AB} + 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \tag{2}$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \tag{3}$$

Making $T_{AD} = 520\text{ N}$ in Eqs. (1) and (3):

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + 200\text{ N} = 0 \tag{1$'$}$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - 288\text{ N} = 0 \tag{3$'$}$$

Multiply (1′) by 9, (3′) by 8, and add:

$$9.24T_{AC} - 504\text{ N} = 0 \quad T_{AC} = 54.5455\text{ N}$$

Substitute into (1′) and solve for T_{AB}:

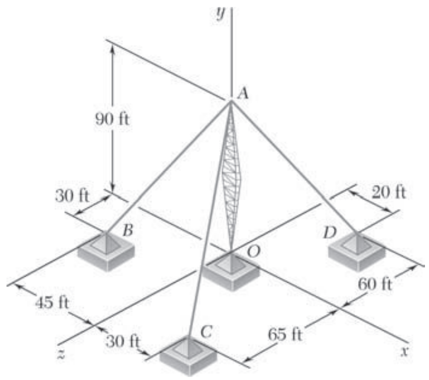
$$T_{AB} = \frac{17}{8}(0.6\times 54.5455 + 200) \quad T_{AB} = 494.545\text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P:

$$P = \frac{12}{17}(494.545\text{ N}) + 0.64(54.5455\text{ N}) + \frac{9.6}{13}(520\text{ N})$$

$$= 768.00\text{ N}$$

Weight of plate $= P = 768\text{ N}$ ◀



PROBLEM 2.113

For the transmission tower of Problems 2.109 and 2.110, determine the tension in each guy wire knowing that the tower exerts on the pin at A an upward vertical force of 2100 lb.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{3}{7}T_{AB}+\frac{6}{23}T_{AC}+\frac{2}{11}T_{AD}=0 \qquad (1)$$

$$-\frac{6}{7}T_{AB}-\frac{18}{23}T_{AC}-\frac{9}{11}T_{AD}+P=0 \qquad (2)$$

$$\frac{2}{7}T_{AB}+\frac{13}{23}T_{AC}-\frac{6}{11}T_{AD}=0 \qquad (3)$$

Substituting for $P=2100$ lb in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-\frac{3}{7}T_{AB}+\frac{6}{23}T_{AC}+\frac{2}{11}T_{AD}=0 \qquad (1')$$

$$-\frac{6}{7}T_{AB}-\frac{18}{23}T_{AC}-\frac{9}{11}T_{AD}+2100\text{ lb}=0 \qquad (2')$$

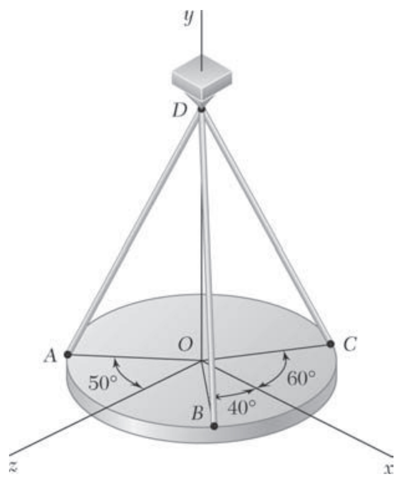
$$\frac{2}{7}T_{AB}+\frac{13}{23}T_{AC}-\frac{6}{11}T_{AD}=0 \qquad (3')$$

$$T_{AB}=841.55\text{ lb}$$
$$T_{AC}=624.38\text{ lb}$$
$$T_{AD}=1087.81\text{ lb}$$

$T_{AB}=842$ lb ◀

$T_{AC}=624$ lb ◀

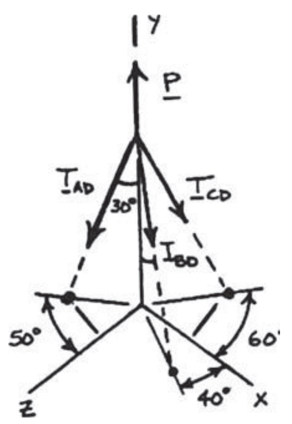
$T_{AD}=1088$ lb ◀



PROBLEM 2.114

A horizontal circular plate weighing 60 lb is suspended as shown from three wires that are attached to a support at *D* and form 30° angles with the vertical. Determine the tension in each wire.

SOLUTION



$\Sigma F_x = 0$:

$$-T_{AD}(\sin 30°)(\sin 50°) + T_{BD}(\sin 30°)(\cos 40°) + T_{CD}(\sin 30°)(\cos 60°) = 0$$

Dividing through by $\sin 30°$ and evaluating:

$$-0.76604T_{AD} + 0.76604T_{BD} + 0.5T_{CD} = 0 \quad (1)$$

$\Sigma F_y = 0$: $\quad -T_{AD}(\cos 30°) - T_{BD}(\cos 30°) - T_{CD}(\cos 30°) + 60 \text{ lb} = 0$

or $\quad T_{AD} + T_{BD} + T_{CD} = 69.282 \text{ lb} \quad (2)$

$\Sigma F_z = 0$: $\quad T_{AD}\sin 30°\cos 50° + T_{BD}\sin 30°\sin 40° - T_{CD}\sin 30°\sin 60° = 0$

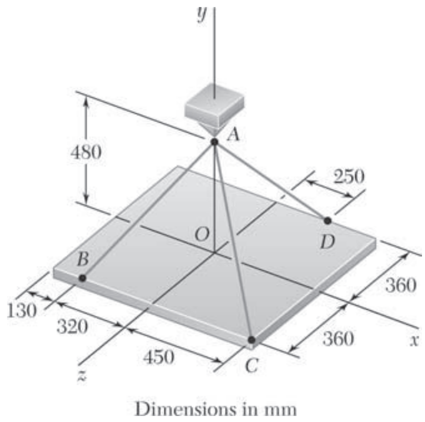
or $\quad 0.64279T_{AD} + 0.64279T_{BD} - 0.86603T_{CD} = 0 \quad (3)$

Solving Equations (1), (2), and (3) simultaneously:

$T_{AD} = 29.5 \text{ lb}$ ◀

$T_{BD} = 10.25 \text{ lb}$ ◀

$T_{CD} = 29.5 \text{ lb}$ ◀



PROBLEM 2.115

For the rectangular plate of Problems 2.111 and 2.112, determine the tension in each of the three cables knowing that the weight of the plate is 792 N.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below. Setting $P = 792$ N gives:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \qquad (1)$$

$$-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + 792 \text{ N} = 0 \qquad (2)$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \qquad (3)$$

Solving Equations (1), (2), and (3) by conventional algorithms gives

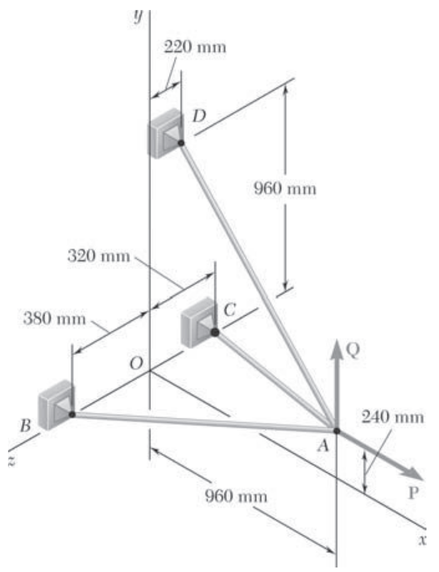
$T_{AB} = 510.00$ N $\qquad T_{AB} = 510$ N ◀

$T_{AC} = 56.250$ N $\qquad T_{AC} = 56.2$ N ◀

$T_{AD} = 536.25$ N $\qquad T_{AD} = 536$ N ◀

PROBLEM 2.116

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = 0$.



SOLUTION

$$\Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} + \mathbf{Q} = 0$$

Where $\quad \mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{j}$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\overrightarrow{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \quad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = T_{AD}\left(-\frac{48}{61}\mathbf{i} + \frac{36}{61}\mathbf{j} - \frac{11}{61}\mathbf{k}\right)$$

Substituting into $\Sigma \mathbf{F}_A = 0$, setting $P = (2880 \text{ N})\mathbf{i}$ and $Q = 0$, and setting the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ equal to 0, we obtain the following three equilibrium equations:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \qquad (3)$$

PROBLEM 2.116 (Continued)

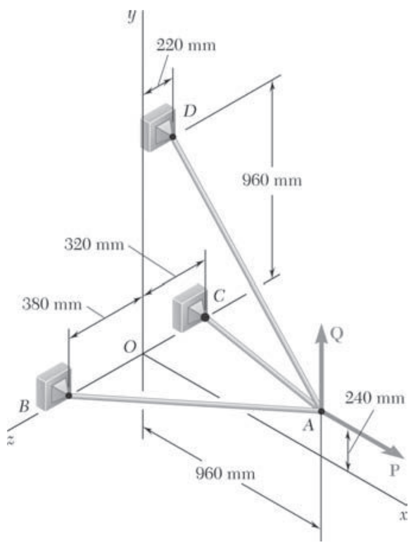
Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 1340.14 \text{ N}$$
$$T_{AC} = 1025.12 \text{ N}$$
$$T_{AD} = 915.03 \text{ N}$$

$T_{AB} = 1340 \text{ N}$ ◀

$T_{AC} = 1025 \text{ N}$ ◀

$T_{AD} = 915 \text{ N}$ ◀



PROBLEM 2.117

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = 576$ N.

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \tag{1}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \tag{2}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3}$$

Setting $P = 2880$ N and $Q = 576$ N gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \tag{1$'$}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + 576 \text{ N} = 0 \tag{2$'$}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3$'$}$$

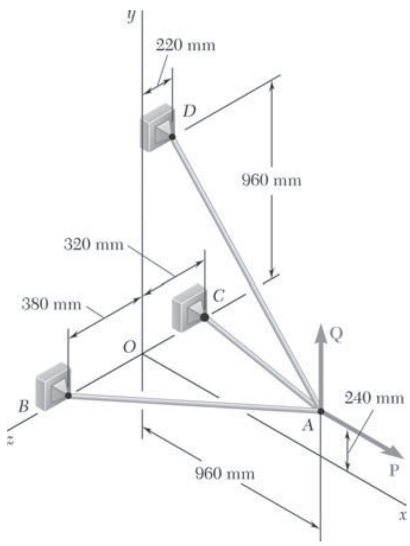
Solving the resulting set of equations using conventional algorithms gives:

$$T_{AB} = 1431.00 \text{ N}$$
$$T_{AC} = 1560.00 \text{ N}$$
$$T_{AD} = 183.010 \text{ N}$$

$T_{AB} = 1431$ N ◀

$T_{AC} = 1560$ N ◀

$T_{AD} = 183.0$ N ◀



PROBLEM 2.118

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = -576$ N. (**Q** is directed downward).

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \tag{1}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \tag{2}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3}$$

Setting $P = 2880$ N and $Q = -576$ N gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \tag{1'}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} - 576 \text{ N} = 0 \tag{2'}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3'}$$

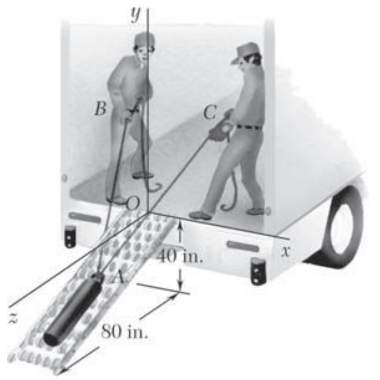
Solving the resulting set of equations using conventional algorithms gives:

$$T_{AB} = 1249.29 \text{ N}$$
$$T_{AC} = 490.31 \text{ N}$$
$$T_{AD} = 1646.97 \text{ N}$$

$T_{AB} = 1249$ N ◀

$T_{AC} = 490$ N ◀

$T_{AD} = 1647$ N ◀

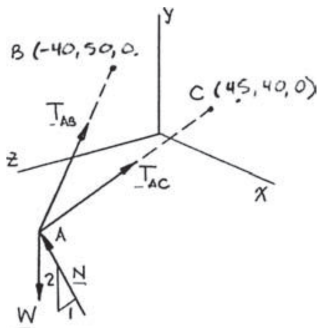


PROBLEM 2.119

Using two ropes and a roller chute, two workers are unloading a 200-lb cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of Points *A*, *B*, and *C* are, respectively, *A*(0, –20 in., 40 in.), *B*(–40 in., 50 in., 0), and *C*(45 in., 40 in., 0), and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint*: Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.)

SOLUTION

From the geometry of the chute:



$$\mathbf{N} = \frac{N}{\sqrt{5}}(2\mathbf{j} + \mathbf{k})$$
$$= N(0.8944\mathbf{j} + 0.4472\mathbf{k})$$

The force in each rope can be written as the product of the magnitude of the force and the unit vector along the cable. Thus, with

$$\overrightarrow{AB} = (40 \text{ in.})\mathbf{i} + (70 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k}$$
$$AB = \sqrt{(40 \text{ in.})^2 + (70 \text{ in.})^2 + (40 \text{ in.})^2}$$
$$= 90 \text{ in.}$$
$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$
$$= \frac{T_{AB}}{90 \text{ in.}}[(-40 \text{ in.})\mathbf{i} + (70 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k}]$$
$$\mathbf{T}_{AB} = T_{AB}\left(-\frac{4}{9}\mathbf{i} + \frac{7}{9}\mathbf{j} - \frac{4}{9}\mathbf{k}\right)$$

and

$$\overrightarrow{AC} = (45 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k}$$
$$AC = \sqrt{(45 \text{ in.})^2 + (60 \text{ in.})^2 + (40 \text{ in.})^2} = 85 \text{ in.}$$
$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$
$$= \frac{T_{AC}}{85 \text{ in.}}[(45 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k}]$$
$$\mathbf{T}_{AC} = T_{AC}\left(\frac{9}{17}\mathbf{i} + \frac{12}{17}\mathbf{j} - \frac{8}{17}\mathbf{k}\right)$$

Then: $\Sigma\mathbf{F} = 0\!: \quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{W} = 0$

PROBLEM 2.119 (Continued)

With $W = 200$ lb, and equating the factors of **i**, **j**, and **k** to zero, we obtain the linear algebraic equations:

$$\mathbf{i}: \quad -\frac{4}{9}T_{AB} + \frac{9}{17}T_{AC} = 0 \tag{1}$$

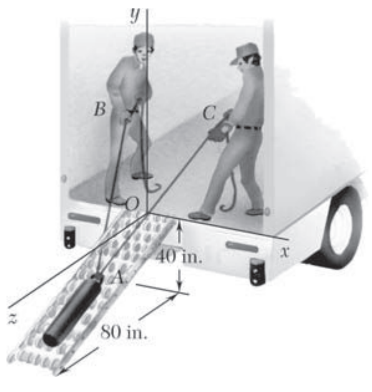
$$\mathbf{j}: \quad \frac{7}{9}T_{AB} + \frac{12}{17}T_{AC} + \frac{2}{\sqrt{5}} - 200 \text{ lb} = 0 \tag{2}$$

$$\mathbf{k}: \quad -\frac{4}{9}T_{AB} - \frac{8}{17}T_{AC} + \frac{1}{\sqrt{5}}N = 0 \tag{3}$$

Using conventional methods for solving linear algebraic equations we obtain:

$T_{AB} = 65.6$ lb ◀

$T_{AC} = 55.1$ lb ◀



PROBLEM 2.120

Solve Problem 2.119 assuming that a third worker is exerting a force $\mathbf{P} = -(40 \text{ lb})\mathbf{i}$ on the counterweight.

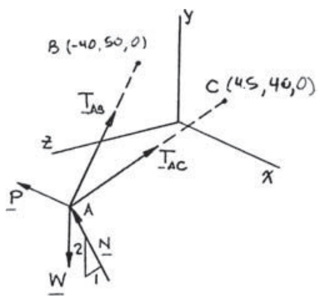
PROBLEM 2.119 Using two ropes and a roller chute, two workers are unloading a 200-lb cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of Points A, B, and C are, respectively, A(0, –20 in., 40 in.), B(–40 in., 50 in., 0), and C(45 in., 40 in., 0), and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint:* Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.)

SOLUTION

See Problem 2.119 for the analysis leading to the vectors describing the tension in each rope.

$$\mathbf{T}_{AB} = T_{AB}\left(-\frac{4}{9}\mathbf{i} + \frac{7}{9}\mathbf{j} - \frac{4}{9}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\left(\frac{9}{17}\mathbf{i} + \frac{12}{17}\mathbf{j} - \frac{8}{17}\mathbf{k}\right)$$



Then: $\qquad \Sigma \mathbf{F}_A = 0: \quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{W} = 0$

Where $\qquad \mathbf{P} = -(40 \text{ lb})\mathbf{i}$

and $\qquad \mathbf{W} = (200 \text{ lb})\mathbf{j}$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the linear equations:

$$\mathbf{i}: \quad -\frac{4}{9}T_{AB} + \frac{9}{17}T_{AC} - 40 \text{ lb} = 0$$

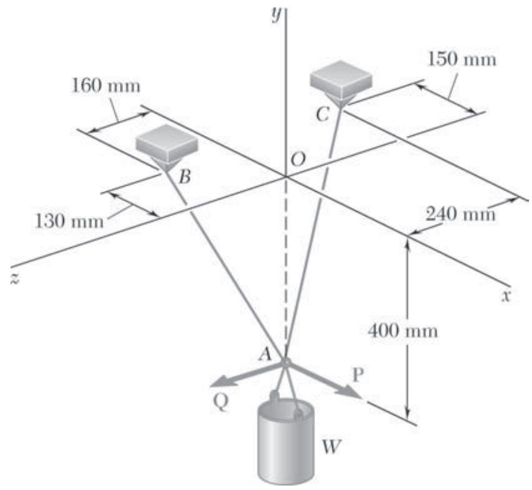
$$\mathbf{j}: \quad \frac{2}{\sqrt{5}}N + \frac{7}{9}T_{AB} + \frac{12}{17}T_{AC} - 200 \text{ lb} = 0$$

$$\mathbf{k}: \quad \frac{1}{\sqrt{5}}N - \frac{4}{9}T_{AB} - \frac{8}{17}T_{AC} = 0$$

Using conventional methods for solving linear algebraic equations we obtain

$T_{AB} = 24.8 \text{ lb}$ ◀

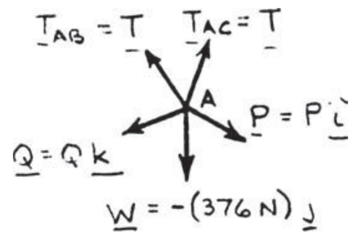
$T_{AC} = 96.4 \text{ lb}$ ◀



PROBLEM 2.121

A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

SOLUTION



Free Body A:

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB}$$

$$= T\frac{\overrightarrow{AB}}{AB}$$

$$= T\frac{(-130\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} + (160\text{ mm})\mathbf{k}}{450\text{ mm}}$$

$$= T\left(-\frac{13}{45}\mathbf{i} + \frac{40}{45}\mathbf{j} + \frac{16}{45}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC}$$

$$= T\frac{\overrightarrow{AC}}{AC}$$

$$= T\frac{(-150\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} + (-240\text{ mm})\mathbf{k}}{490\text{ mm}}$$

$$= T\left(-\frac{15}{49}\mathbf{i} + \frac{40}{49}\mathbf{j} - \frac{24}{49}\mathbf{k}\right)$$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{Q} + \mathbf{P} + \mathbf{W} = 0$$

Setting coefficients of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{13}{45}T - \frac{15}{49}T + P = 0 \qquad 0.59501T = P \tag{1}$$

$$\mathbf{j}: \quad +\frac{40}{45}T + \frac{40}{49}T - W = 0 \qquad 1.70521T = W \tag{2}$$

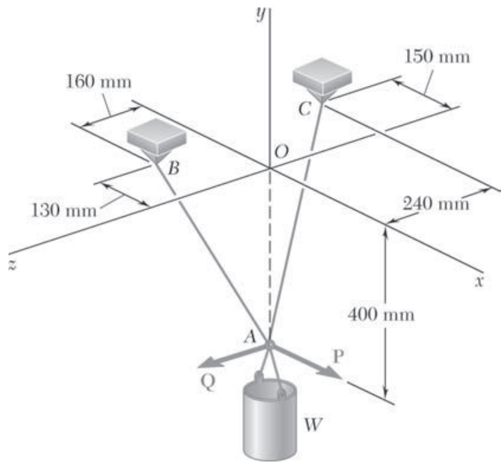
$$\mathbf{k}: \quad +\frac{16}{45}T - \frac{24}{49}T + Q = 0 \qquad 0.134240T = Q \tag{3}$$

PROBLEM 2.121 (Continued)

Data: $$W = 376 \text{ N} \quad 1.70521T = 376 \text{ N} \quad T = 220.50 \text{ N}$$

$$0.59501(220.50 \text{ N}) = P \qquad P = 131.2 \text{ N} \blacktriangleleft$$

$$0.134240(220.50 \text{ N}) = Q \qquad Q = 29.6 \text{ N} \blacktriangleleft$$



PROBLEM 2.122

For the system of Problem 2.121, determine W and Q knowing that $P = 164$ N.

PROBLEM 2.121 A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

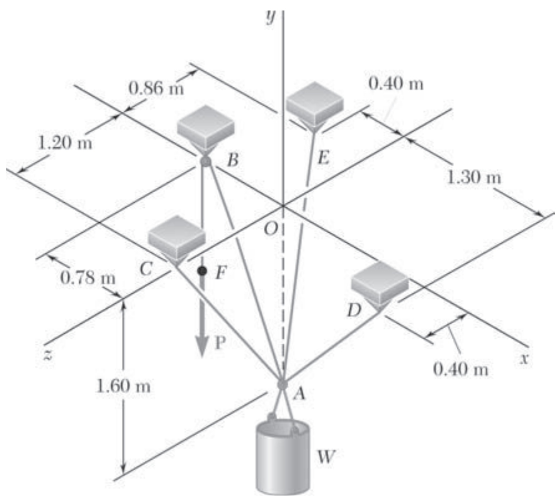
SOLUTION

Refer to Problem 2.121 for the figure and analysis resulting in Equations (1), (2), and (3) for P, W, and Q in terms of T below. Setting $P = 164$ N we have:

Eq. (1): $0.59501T = 164 \text{ N}$ $\quad T = 275.63 \text{ N}$

Eq. (2): $1.70521(275.63 \text{ N}) = W$ $\quad W = 470 \text{ N}$ ◀

Eq. (3): $0.134240(275.63 \text{ N}) = Q$ $\quad Q = 37.0 \text{ N}$ ◀



PROBLEM 2.123

A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force $\mathbf{P}$ is applied to the end F of a third cable that passes over a pulley at B and through ring A and that is attached to a support at D. Knowing that $W = 1000$ N, determine the magnitude of P. (*Hint:* The tension is the same in all portions of cable $FBAD$.)

SOLUTION

The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\overrightarrow{AB} = -(0.78\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} + (0\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.78\text{ m})^2 + (1.6\text{ m})^2 + (0)^2}$$

$$= 1.78\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= \frac{T_{AB}}{1.78\text{ m}}[-(0.78\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} + (0\text{ m})\mathbf{k}]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.4382\mathbf{i} + 0.8989\mathbf{j} + 0\mathbf{k})$$

and

$$\overrightarrow{AC} = (0)\mathbf{i} + (1.6\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}$$

$$AC = \sqrt{(0\text{ m})^2 + (1.6\text{ m})^2 + (1.2\text{ m})^2} = 2\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{2\text{ m}}[(0)\mathbf{i} + (1.6\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}]$$

$$\mathbf{T}_{AC} = T_{AC}(0.8\mathbf{j} + 0.6\mathbf{k})$$

and

$$\overrightarrow{AD} = (1.3\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} + (0.4\text{ m})\mathbf{k}$$

$$AD = \sqrt{(1.3\text{ m})^2 + (1.6\text{ m})^2 + (0.4\text{ m})^2} = 2.1\text{ m}$$

$$\mathbf{T}_{AD} = T\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.1\text{ m}}[(1.3\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} + (0.4\text{ m})\mathbf{k}]$$

$$\mathbf{T}_{AD} = T_{AD}(0.6190\mathbf{i} + 0.7619\mathbf{j} + 0.1905\mathbf{k})$$

PROBLEM 2.123 (Continued)

Finally,
$$\overrightarrow{AE} = -(0.4\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} - (0.86\text{ m})\mathbf{k}$$
$$AE = \sqrt{(-0.4\text{ m})^2 + (1.6\text{ m})^2 + (-0.86\text{ m})^2} = 1.86\text{ m}$$
$$\mathbf{T}_{AE} = T\boldsymbol{\lambda}_{AE} = T_{AE}\frac{\overrightarrow{AE}}{AE}$$
$$= \frac{T_{AE}}{1.86\text{ m}}[-(0.4\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} - (0.86\text{ m})\mathbf{k}]$$
$$\mathbf{T}_{AE} = T_{AE}(-0.2151\mathbf{i} + 0.8602\mathbf{j} - 0.4624\mathbf{k})$$

With the weight of the container $\mathbf{W} = -W\mathbf{j}$, at A we have:

$$\Sigma\mathbf{F} = 0\text{:}\quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the following linear algebraic equations:

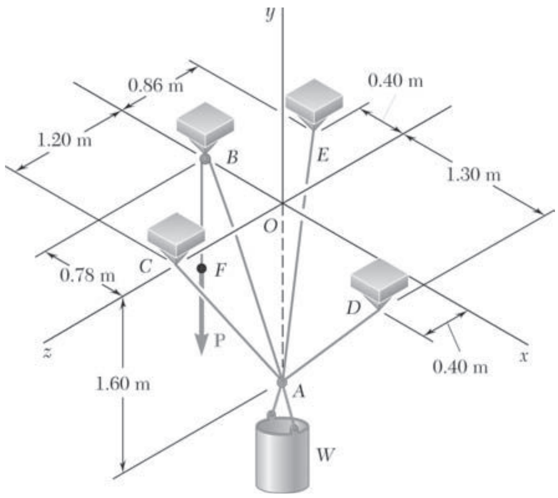
$$-0.4382T_{AB} + 0.6190T_{AD} - 0.2151T_{AE} = 0 \qquad (1)$$

$$0.8989T_{AB} + 0.8T_{AC} + 0.7619T_{AD} + 0.8602T_{AE} - W = 0 \qquad (2)$$

$$0.6T_{AC} + 0.1905T_{AD} - 0.4624T_{AE} = 0 \qquad (3)$$

Knowing that $W = 1000$ N and that because of the pulley system at B $T_{AB} = T_{AD} = P$, where P is the externally applied (unknown) force, we can solve the system of linear Equations (1), (2) and (3) uniquely for P.

$P = 378$ N ◀



PROBLEM 2.124

Knowing that the tension in cable AC of the system described in Problem 2.123 is 150 N, determine (*a*) the magnitude of the force **P**, (*b*) the weight W of the container.

PROBLEM 2.123 A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force **P** is applied to the end F of a third cable that passes over a pulley at B and through ring A and that is attached to a support at D. Knowing that $W = 1000$ N, determine the magnitude of P. (*Hint:* The tension is the same in all portions of cable $FBAD$.)

SOLUTION

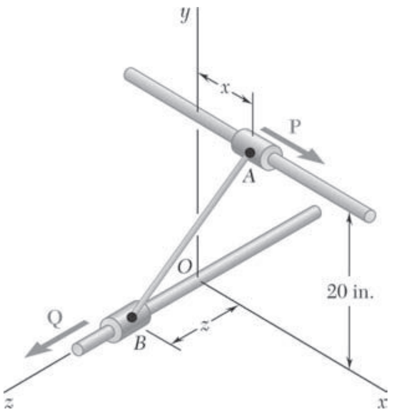
Here, as in Problem 2.123, the support of the container consists of the four cables AE, AC, AD, and AB, with the condition that the force in cables AB and AD is equal to the externally applied force P. Thus, with the condition

$$T_{AB} = T_{AD} = P$$

and using the linear algebraic equations of Problem 2.131 with $T_{AC} = 150$ N, we obtain

(*a*) $P = 454$ N ◀

(*b*) $W = 1202$ N ◀



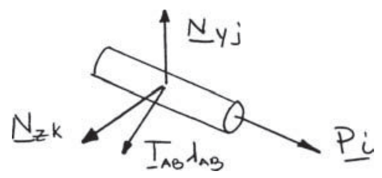
PROBLEM 2.125

Collars A and B are connected by a 25-in.-long wire and can slide freely on frictionless rods. If a 60-lb force **Q** is applied to collar B as shown, determine (*a*) the tension in the wire when $x = 9$ in., (*b*) the corresponding magnitude of the force **P** required to maintain the equilibrium of the system.

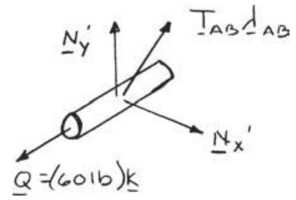
SOLUTION

Free Body Diagrams of Collars:

***A*:**



***B*:**



$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{-x\mathbf{i} - (20\text{ in.})\mathbf{j} + z\mathbf{k}}{25\text{ in.}}$$

Collar A: $\quad \Sigma\mathbf{F} = 0\text{:} \quad P\mathbf{i} + N_y\mathbf{j} + N_z\mathbf{k} + T_{AB}\lambda_{AB} = 0$

Substitute for $\boldsymbol{\lambda}_{AB}$ and set coefficient of **i** equal to zero:

$$P - \frac{T_{AB}x}{25\text{ in.}} = 0 \tag{1}$$

Collar B: $\quad \Sigma\mathbf{F} = 0\text{:} \quad (60\text{ lb})\mathbf{k} + N_x'\mathbf{i} + N_y'\mathbf{j} - T_{AB}\lambda_{AB} = 0$

Substitute for $\boldsymbol{\lambda}_{AB}$ and set coefficient of **k** equal to zero:

$$60\text{ lb} - \frac{T_{AB}z}{25\text{ in.}} = 0 \tag{2}$$

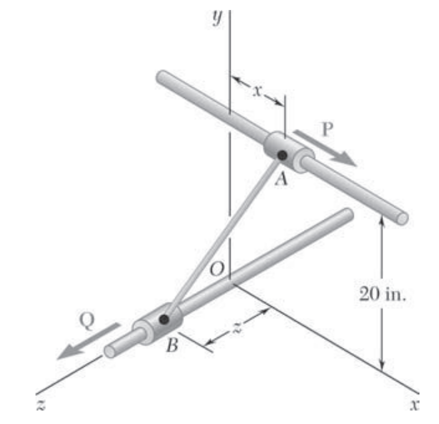
(*a*) $\quad x = 9$ in. $\quad (9\text{ in.})^2 + (20\text{ in.})^2 + z^2 = (25\text{ in.})^2$

$$z = 12\text{ in.}$$

From Eq. (2): $\quad \dfrac{60\text{ lb} - T_{AB}(12\text{ in.})}{25\text{ in.}} \qquad T_{AB} = 125.0\text{ lb}$ ◀

(*b*) From Eq. (1): $\quad P = \dfrac{(125.0\text{ lb})(9\text{ in.})}{25\text{ in.}} \qquad P = 45.0\text{ lb}$ ◀

PROBLEM 2.126



Collars A and B are connected by a 25-in.-long wire and can slide freely on frictionless rods. Determine the distances x and z for which the equilibrium of the system is maintained when $P = 120$ lb and $Q = 60$ lb.

SOLUTION

See Problem 2.125 for the diagrams and analysis leading to Equations (1) and (2) below:

$$P = \frac{T_{AB}x}{25\text{ in.}} = 0 \tag{1}$$

$$60\text{ lb} - \frac{T_{AB}z}{25\text{ in.}} = 0 \tag{2}$$

For $P = 120$ lb, Eq. (1) yields $\quad T_{AB}x = (25\text{ in.})(20\text{ lb}) \quad (1')$

From Eq. (2) $\quad T_{AB}z = (25\text{ in.})(60\text{ lb}) \quad (2')$

Dividing Eq. (1′) by (2′): $\quad \dfrac{x}{z} = 2 \quad (3)$

Now write $\quad x^2 + z^2 + (20\text{ in.})^2 = (25\text{ in.})^2 \quad (4)$

Solving (3) and (4) simultaneously

$$4z^2 + z^2 + 400 = 625$$

$$z^2 = 45$$

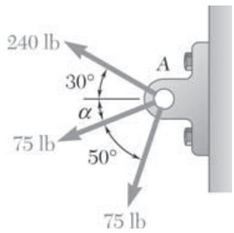
$$z = 6.708\text{ in.}$$

From Eq. (3) $\quad x = 2z = 2(6.708\text{ in.})$

$$= 13.416\text{ in.}$$

$x = 13.42$ in., $\quad z = 6.71$ in. ◀

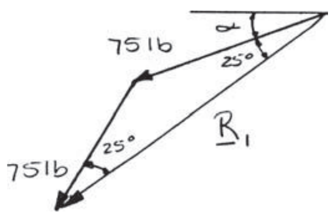
PROBLEM 2.127



The direction of the 75-lb forces may vary, but the angle between the forces is always 50°. Determine the value of α for which the resultant of the forces acting at A is directed horizontally to the left.

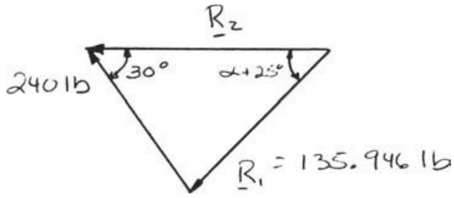
SOLUTION

We must first replace the two 75-lb forces by their resultant $\mathbf{R}_1$ using the triangle rule.



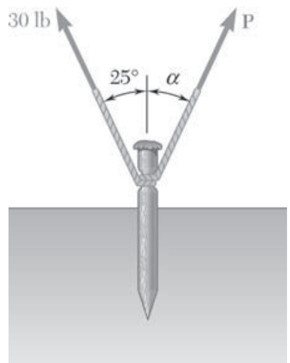
$$\mathbf{R}_1 = 2(75 \text{ lb})\cos 25°$$
$$= 135.946 \text{ lb}$$
$$\mathbf{R}_1 = 135.946 \text{ lb} \nearrow \alpha + 25°$$

Next we consider the resultant $\mathbf{R}_2$ of $\mathbf{R}_1$ and the 240-lb force where $\mathbf{R}_2$ must be horizontal and directed to the left. Using the triangle rule and law of sines,



$$\frac{\sin(\alpha + 25°)}{240 \text{ lb}} = \frac{\sin(30°)}{135.946}$$
$$\sin(\alpha + 25°) = 0.88270$$
$$\alpha + 25° = 61.970°$$
$$\alpha = 36.970°$$

$\alpha = 37.0°$ ◀

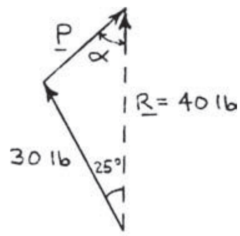


PROBLEM 2.128

A stake is being pulled out of the ground by means of two ropes as shown. Knowing the magnitude and direction of the force exerted on one rope, determine the magnitude and direction of the force **P** that should be exerted on the other rope if the resultant of these two forces is to be a 40-lb vertical force.

SOLUTION

Triangle rule:



Law of cosines: $P^2 = (30)^2 + (40)^2 - 2(30)(40)\cos 25°$

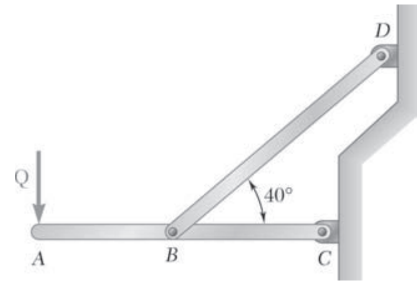
$P = 18.0239 \text{ lb}$

Law of sines: $\dfrac{\sin\alpha}{30 \text{ lb}} = \dfrac{\sin 25°}{18.0239 \text{ lb}}$

$\alpha = 44.703°$

$90° - \alpha = 45.297°$

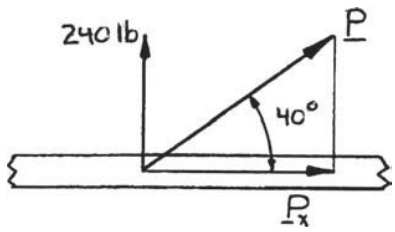
P $= 18.02 \text{ lb} \measuredangle 45.3°$ ◀



PROBLEM 2.129

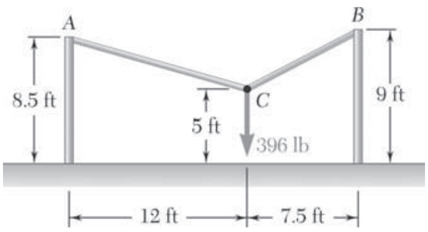
Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 240-lb vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION



(*a*) $$P = \frac{P_y}{\sin 35°} = \frac{240 \text{ lb}}{\sin 40°}$$ or $P = 373$ lb ◀

(*b*) $$P_x = \frac{P_y}{\tan 40°} = \frac{240 \text{ lb}}{\tan 40°}$$ or $P_x = 286$ lb ◀

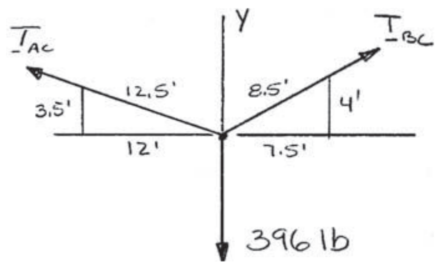


PROBLEM 2.130

Two cables are tied together at C and loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free Body Diagram at C:



$$\Sigma \mathbf{F}_x = 0: \quad -\frac{12 \text{ ft}}{12.5 \text{ ft}} T_{AC} + \frac{7.5 \text{ ft}}{8.5 \text{ ft}} T_{BC} = 0$$

$$T_{BC} = 1.08800 T_{AC}$$

$$\Sigma \mathbf{F}_y = 0: \quad \frac{3.5 \text{ ft}}{12 \text{ ft}} T_{AC} + \frac{4 \text{ ft}}{8.5 \text{ ft}} T_{BC} - 396 \text{ lb} = 0$$

(a)

$$\frac{3.5 \text{ ft}}{12.5 \text{ ft}} T_{AC} + \frac{4 \text{ ft}}{8.5 \text{ ft}} (1.08800 T_{AC}) - 396 \text{ lb} = 0$$

$$(0.28000 + 0.51200) T_{AC} = 396 \text{ lb}$$

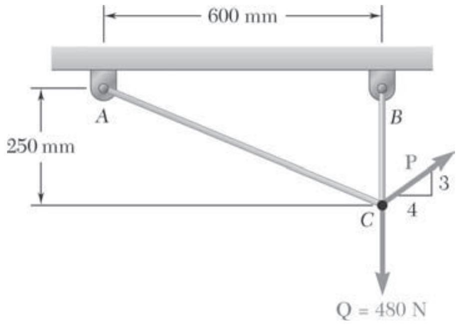
$$T_{AC} = 500.0 \text{ lb}$$

$T_{AC} = 500$ lb ◀

(b)

$$T_{BC} = (1.08800)(500.0 \text{ lb})$$

$T_{BC} = 544$ lb ◀

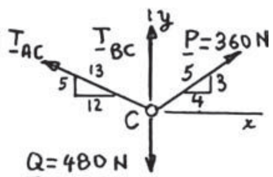


PROBLEM 2.131

Two cables are tied together at C and loaded as shown. Knowing that $P = 360$ N, determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

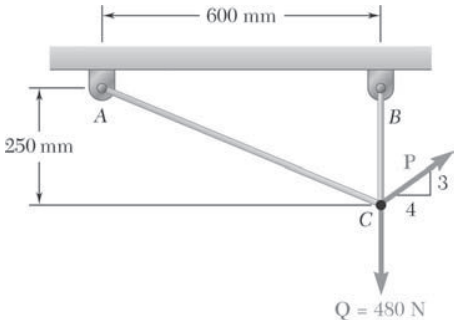
Free Body: *C*



(a) $\Sigma \mathbf{F}_x = 0$: $-\dfrac{12}{13}T_{AC} + \dfrac{4}{5}(360\text{ N}) = 0$ $T_{AC} = 312\text{ N}$ ◀

(b) $\Sigma \mathbf{F}_y = 0$: $\dfrac{5}{13}(312\text{ N}) + T_{BC} + \dfrac{3}{5}(360\text{ N}) - 480\text{ N} = 0$

$T_{BC} = 480\text{ N} - 120\text{ N} - 216\text{ N}$ $T_{BC} = 144\text{ N}$ ◀

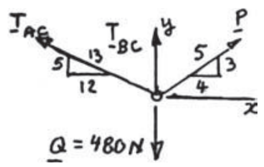


PROBLEM 2.132

Two cables are tied together at C and loaded as shown. Determine the range of values of P for which both cables remain taut.

SOLUTION

Free Body: *C*



$$\Sigma \mathbf{F}_x = 0: \quad -\frac{12}{13}T_{AC} + \frac{4}{5}\mathbf{P} = 0$$

$$T_{AC} = \frac{13}{15}P \qquad (1)$$

$$\Sigma \mathbf{F}_y = 0: \quad \frac{5}{13}T_{AC} + T_{BC} + \frac{3}{5}P - 480 \text{ N} = 0$$

Substitute for T_{AC} from (1):

$$\left(\frac{5}{13}\right)\left(\frac{13}{15}\right)P + T_{BC} + \frac{3}{5}P - 480 \text{ N} = 0$$

$$T_{BC} = 480 \text{ N} - \frac{14}{15}P \qquad (2)$$

From (1), $T_{AC} > 0$ requires $P > 0$.

From (2), $T_{BC} > 0$ requires $\frac{14}{15}P < 480$ N, $\quad P < 514.29$ N

Allowable range: $0 < P < 514$ N ◀

PROBLEM 2.133

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 69.3°$ and $\theta_z = 57.9°$. Knowing that the *y* component of the force is –174.0 lb, determine (*a*) the angle θ_y, (*b*) the other components and the magnitude of the force.

SOLUTION

(*a*) To determine θ_y, use the relation

$$\cos^2\theta_y + \cos^2\theta_y + \cos^2\theta_z = 1$$

or
$$\cos^2\theta_y = 1 - \cos^2\theta_x - \cos^2\theta_y$$

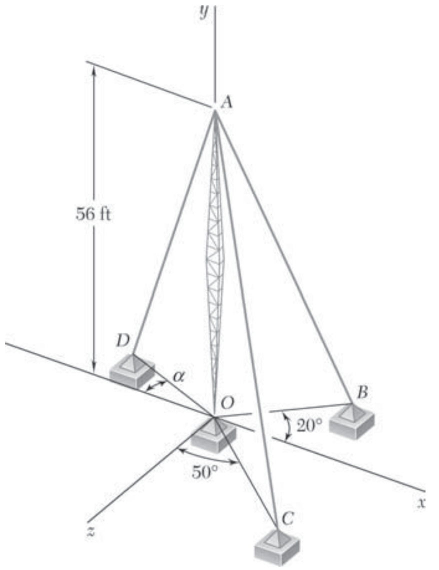
Since $F_y < 0$, we must have $\cos\theta_y < 0$

$$\cos\theta_y = -\sqrt{1 - \cos^2 69.3° - \cos^2 57.9°}$$
$$= -0.76985 \qquad \theta_y = 140.3° \blacktriangleleft$$

(*b*)
$$F = \frac{F_y}{\cos\theta_y} = \frac{-174.0 \text{ lb}}{-0.76985} = 226.02 \text{ lb} \qquad F = 226 \text{ lb} \blacktriangleleft$$

$$F_x = F\cos\theta_x = (226.02 \text{ lb})\cos 69.3° \qquad F_x = 79.9 \text{ lb} \blacktriangleleft$$

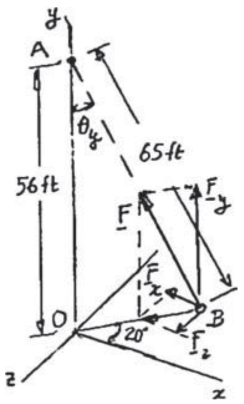
$$F_z = F\cos\theta_z = (226.02 \text{ lb})\cos 57.9° \qquad F_z = 120.1 \text{ lb} \blacktriangleleft$$



PROBLEM 2.134

Cable AB is 65 ft long, and the tension in that cable is 3900 lb. Determine (*a*) the *x*, *y*, and *z* components of the force exerted by the cable on the anchor *B*, (*b*) the angles θ_x, θ_y, and θ_z defining the direction of that force.

SOLUTION



From triangle AOB:

$$\cos\theta_y = \frac{56 \text{ ft}}{65 \text{ ft}} = 0.86154$$

$$\theta_y = 30.51°$$

(*a*)

$$F_x = -F\sin\theta_y\cos 20° = -(3900 \text{ lb})\sin 30.51°\cos 20°$$

$$F_x = -1861 \text{ lb} \blacktriangleleft$$

$$F_y = +F\cos\theta_y = (3900 \text{ lb})(0.86154) \qquad F_y = +3360 \text{ lb} \blacktriangleleft$$

$$F_z = +(3900 \text{ lb})\sin 30.51°\sin 20° \qquad F_z = +677 \text{ lb} \blacktriangleleft$$

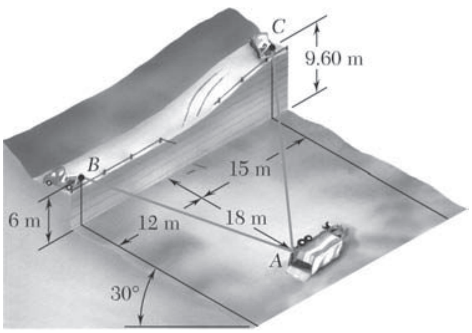
(*b*)

$$\cos\theta_x = \frac{F_x}{F} = -\frac{1861 \text{ lb}}{3900 \text{ lb}} = -0.4771 \qquad \theta_x = 118.5° \blacktriangleleft$$

From above:

$$\theta_y = 30.51° \qquad \theta_y = 30.5° \blacktriangleleft$$

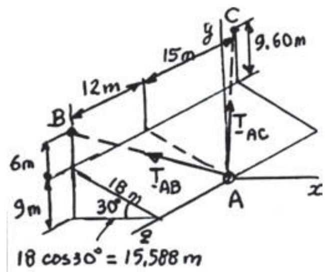
$$\cos\theta_z = \frac{F_z}{F} = +\frac{677 \text{ lb}}{3900 \text{ lb}} = +0.1736 \qquad \theta_z = 80.0° \blacktriangleleft$$



PROBLEM 2.135

In order to move a wrecked truck, two cables are attached at *A* and pulled by winches *B* and *C* as shown. Knowing that the tension is 10 kN in cable *AB* and 7.5 kN in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION



$$\overrightarrow{AB} = -15.588\mathbf{i} + 15\mathbf{j} + 12\mathbf{k}$$

$$AB = 24.739 \text{ m}$$

$$\overrightarrow{AC} = -15.588\mathbf{i} + 18.60\mathbf{j} - 15\mathbf{k}$$

$$AC = 28.530 \text{ m}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$\mathbf{T}_{AB} = (10 \text{ kN})\frac{-15.588\mathbf{i} + 15\mathbf{j} + 12\mathbf{k}}{24.739}$$

$$\mathbf{T}_{AB} = (6.301 \text{ kN})\mathbf{i} + (6.063 \text{ kN})\mathbf{j} + (4.851 \text{ kN})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}(7.5 \text{ kN})\frac{-15.588\mathbf{i} + 18.60\mathbf{j} - 15\mathbf{k}}{28.530}$$

$$\mathbf{T}_{AC} = -(4.098 \text{ kN})\mathbf{i} + (4.890 \text{ kN})\mathbf{j} - (3.943 \text{ kN})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = -(10.399 \text{ kN})\mathbf{i} + (10.953 \text{ kN})\mathbf{j} + (0.908 \text{ kN})\mathbf{k}$$

$$R = \sqrt{(10.399)^2 + (10.953)^2 + (0.908)^2}$$

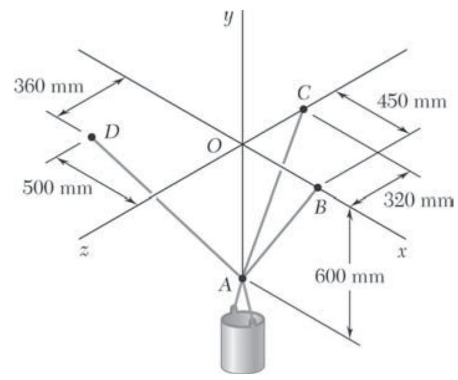
$$= 15.130 \text{ kN} \qquad R = 15.13 \text{ kN} \blacktriangleleft$$

$$\cos\theta_x = \frac{R_x}{R} = \frac{-10.399 \text{ kN}}{15.130 \text{ kN}} = -0.6873 \qquad \theta_z = 133.4° \blacktriangleleft$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{10.953 \text{ kN}}{15.130 \text{ kN}} = 0.7239 \qquad \theta_y = 43.6° \blacktriangleleft$$

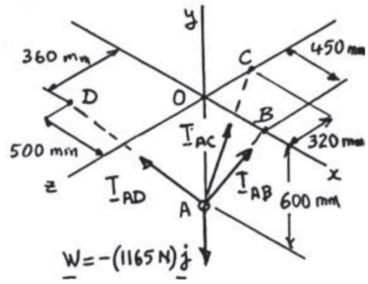
$$\cos\theta_z = \frac{R_z}{R} = \frac{0.908 \text{ kN}}{15.130 \text{ kN}} = 0.0600 \qquad \theta_z = 86.6° \blacktriangleleft$$

PROBLEM 2.136



A container of weight $W = 1165$ N is supported by three cables as shown. Determine the tension in each cable.

SOLUTION



Free Body: ***A*** $\qquad \Sigma\mathbf{F} = 0$: $\quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = 0$

$$\overrightarrow{AB} = (450\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j}$$

$$AB = 750\text{ mm}$$

$$\overrightarrow{AC} = (600\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k}$$

$$AC = 680\text{ mm}$$

$$\overrightarrow{AD} = (500\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}$$

$$AD = 860\text{ mm}$$

We have:

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(\frac{450}{750}\mathbf{i} + \frac{600}{750}\mathbf{j}\right)T_{AB}$$

$$= (0.6\mathbf{i} + 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(\frac{600}{680}\mathbf{j} - \frac{320}{680}\mathbf{k}\right)T_{AC} = \left(\frac{15}{17}\mathbf{j} - \frac{8}{17}\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \left(-\frac{500}{860}\mathbf{i} + \frac{600}{860}\mathbf{j} + \frac{360}{860}\mathbf{k}\right)T_{AD}$$

$$\mathbf{T}_{AD} = \left(-\frac{25}{43}\mathbf{i} + \frac{30}{43}\mathbf{j} + \frac{18}{43}\mathbf{k}\right)T_{AD}$$

Substitute into $\Sigma\mathbf{F} = 0$, factor **i**, **j**, **k**, and set their coefficient equal to zero:

$$0.6T_{AB} - \frac{25}{43}T_{AD} = 0 \qquad T_{AB} = 0.96899T_{AD} \tag{1}$$

$$0.8T_{AB} + \frac{15}{17}T_{AC} + \frac{30}{43}T_{AD} - 1165\text{ N} = 0 \tag{2}$$

$$-\frac{8}{17}T_{AC} + \frac{18}{43}T_{AD} = 0 \qquad T_{AC} = 0.88953T_{AD} \tag{3}$$

PROBLEM 2.136 (Continued)

Substitute for T_{AB} and T_{AC} from (1) and (3) into (2):

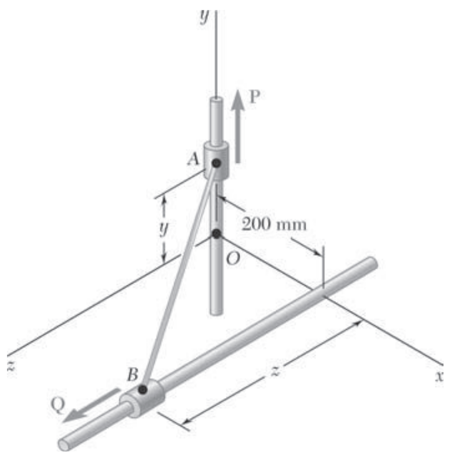
$$\left(0.8\times 0.96899+\frac{15}{17}\times 0.88953+\frac{30}{53}\right)T_{AD}-1165\text{ N}=0$$

$$2.2578T_{AD}-1165\text{ N}=0 \qquad T_{AD}=516\text{ N} \blacktriangleleft$$

From (1): $T_{AB}=0.96899(516\text{ N})$ $\qquad T_{AB}=500\text{ N} \blacktriangleleft$

From (3): $T_{AC}=0.88953(516\text{ N})$ $\qquad T_{AC}=459\text{ N} \blacktriangleleft$

PROBLEM 2.137

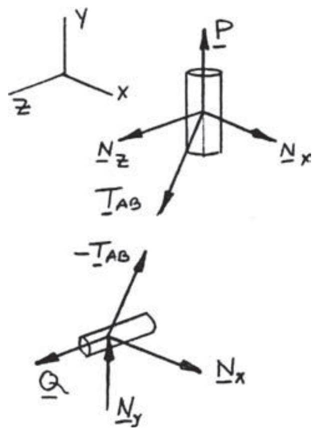


Collars A and B are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341\text{ N})\mathbf{j}$ is applied to collar A, determine (a) the tension in the wire when $y = 155$ mm, (b) the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the system.

SOLUTION

For both Problems 2.137 and 2.138:

Free Body Diagrams of Collars:



$$(AB)^2 = x^2 + y^2 + z^2$$

Here $$(0.525\text{ m})^2 = (.20\text{ m})^2 + y^2 + z^2$$

or $$y^2 + z^2 = 0.23563\text{ m}^2$$

Thus, why y given, z is determined,

Now
$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB}$$
$$= \frac{1}{0.525\text{ m}}(0.20\mathbf{i} - y\mathbf{j} + z\mathbf{k})\text{m}$$
$$= 0.38095\mathbf{i} - 1.90476y\mathbf{j} + 1.90476z\mathbf{k}$$

Where y and z are in units of meters, m.

From the F.B. Diagram of collar A: $\quad \Sigma\mathbf{F} = 0: \quad N_x\mathbf{i} + N_z\mathbf{k} + P\mathbf{j} + T_{AB}\lambda_{AB} = 0$

Setting the $\mathbf{j}$ coefficient to zero gives: $\quad P - (1.90476y)T_{AB} = 0$

With $$P = 341\text{ N}$$
$$T_{AB} = \frac{341\text{ N}}{1.90476y}$$

Now, from the free body diagram of collar B: $\quad \Sigma\mathbf{F} = 0: \quad N_x\mathbf{i} + N_y\mathbf{j} + Q\mathbf{k} - T_{AB}\boldsymbol{\lambda}_{AB} = 0$

Setting the $\mathbf{k}$ coefficient to zero gives: $\quad Q - T_{AB}(1.90476z) = 0$

And using the above result for T_{AB} we have $\quad Q = T_{AB}z = \dfrac{341\text{ N}}{(1.90476)y}(1.90476z) = \dfrac{(341\text{ N})(z)}{y}$

PROBLEM 2.137 (Continued)

Then, from the specifications of the problem, $y = 155 \text{ mm} = 0.155 \text{ m}$

$$z^2 = 0.23563 \text{ m}^2 - (0.155 \text{ m})^2$$
$$z = 0.46 \text{ m}$$

and

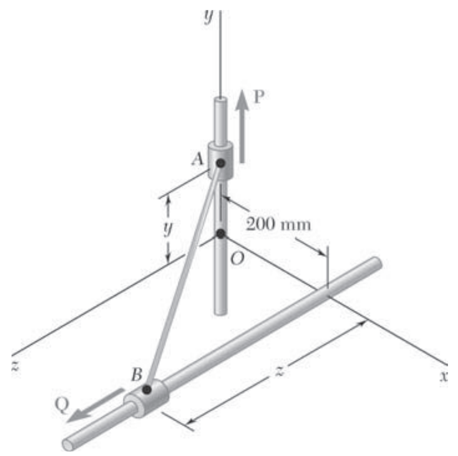
(*a*)
$$T_{AB} = \frac{341 \text{ N}}{0.155(1.90476)}$$
$$= 1155.00 \text{ N}$$

or $T_{AB} = 1.155 \text{ kN}$ ◀

and

(*b*)
$$Q = \frac{341 \text{ N}(0.46 \text{ m})(0.866)}{(0.155 \text{ m})}$$
$$= (1012.00 \text{ N})$$

or $Q = 1.012 \text{ kN}$ ◀



PROBLEM 2.138

Solve Problem 2.137 assuming that $y = 275$ mm.

PROBLEM 2.137 Collars A and B are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341\text{ N})\mathbf{j}$ is applied to collar A, determine (a) the tension in the wire when $y = 155$ mm, (b) the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the system.

SOLUTION

From the analysis of Problem 2.137, particularly the results:

$$y^2 + z^2 = 0.23563\text{ m}^2$$

$$T_{AB} = \frac{341\text{ N}}{1.90476y}$$

$$Q = \frac{341\text{ N}}{y}z$$

With $y = 275\text{ mm} = 0.275\text{ m}$, we obtain:

$$z^2 = 0.23563\text{ m}^2 - (0.275\text{ m})^2$$

$$z = 0.40\text{ m}$$

and

(a) $$T_{AB} = \frac{341\text{ N}}{(1.90476)(0.275\text{ m})} = 651.00$$

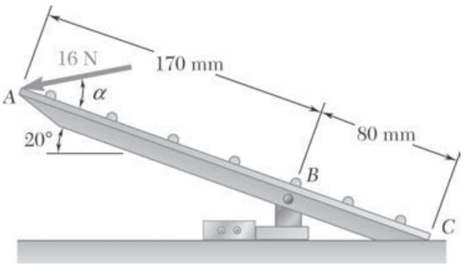
or $$T_{AB} = 651\text{ N} \blacktriangleleft$$

and

(b) $$Q = \frac{341\text{ N}(0.40\text{ m})}{(0.275\text{ m})}$$

or $$Q = 496\text{ N} \blacktriangleleft$$

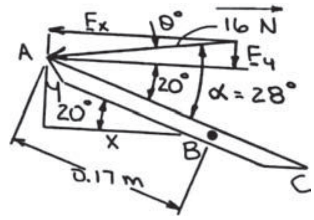
CHAPTER 3



PROBLEM 3.1

A foot valve for a pneumatic system is hinged at B. Knowing that $a = 28°$, determine the moment of the 16-N force about Point B by resolving the force into horizontal and vertical components.

SOLUTION



Note that $\theta = \alpha - 20° = 28° - 20° = 8°$

and $F_x = (16\text{ N})\cos 8° = 15.8443\text{ N}$

$F_y = (16\text{ N})\sin 8° = 2.2268\text{ N}$

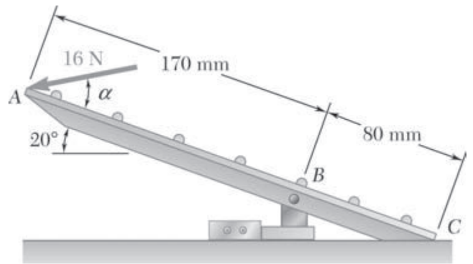
Also $x = (0.17\text{ m})\cos 20° = 0.159748\text{ m}$

$y = (0.17\text{ m})\sin 20° = 0.058143\text{ m}$

Noting that the direction of the moment of each force component about B is counterclockwise,

$$\begin{aligned} M_B &= xF_y + yF_x \\ &= (0.159748\text{ m})(2.2268\text{ N}) \\ &\quad + (0.058143\text{ m})(15.8443\text{ N}) \\ &= 1.277\text{ N}\cdot\text{m} \end{aligned}$$

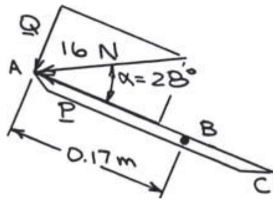
or $\mathbf{M}_B = 1.277\text{ N}\cdot\text{m}$ ↺ ◀



PROBLEM 3.2

A foot valve for a pneumatic system is hinged at B. Knowing that $a = 28°$, determine the moment of the 16-N force about Point B by resolving the force into components along ABC and in a direction perpendicular to ABC.

SOLUTION



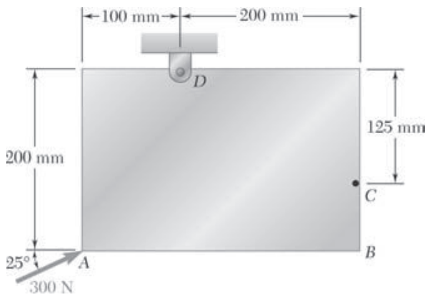
First resolve the 4-lb force into components **P** and **Q**, where

$$Q = (16\text{ N})\sin 28°$$
$$= 7.5115\text{ N}$$

Then

$$M_B = r_{A/B}Q$$
$$= (0.17\text{ m})(7.5115\text{ N})$$
$$= 1.277\text{ N}\cdot\text{m}$$

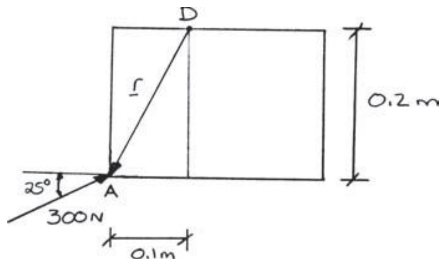
or $\mathbf{M}_B = 1.277\text{ N}\cdot\text{m}$ ↻ ◀



PROBLEM 3.3

A 300-N force is applied at A as shown. Determine (a) the moment of the 300-N force about D, (b) the smallest force applied at B that creates the same moment about D.

SOLUTION



(a)

$$F_x = (300\text{ N})\cos 25° = 271.89\text{ N}$$

$$F_y = (300\text{ N})\sin 25° = 126.785\text{ N}$$

$$\mathbf{F} = (271.89\text{ N})\mathbf{i} + (126.785\text{ N})\mathbf{j}$$

$$\mathbf{r} = \overrightarrow{DA} = -(0.1\text{ m})\mathbf{i} - (0.2\text{ m})\mathbf{j}$$

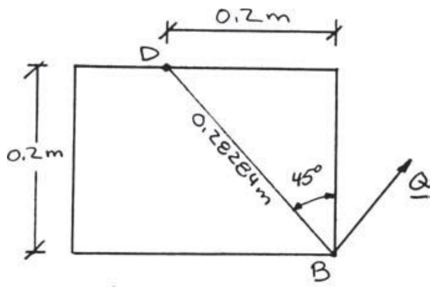
$$\mathbf{M}_D = \mathbf{r} \times \mathbf{F}$$

$$\mathbf{M}_D = [-(0.1\text{ m})\mathbf{i} - (0.2\text{ m})\mathbf{j}] \times [(271.89\text{ N})\mathbf{i} + (126.785\text{ N})\mathbf{j}]$$

$$= -(12.6785\text{ N}\cdot\text{m})\mathbf{k} + (54.378\text{ N}\cdot\text{m})\mathbf{k}$$

$$= (41.700\text{ N}\cdot\text{m})\mathbf{k}$$

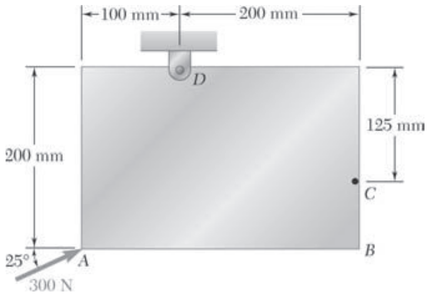
$$\mathbf{M}_D = 41.7\text{ N}\cdot\text{m} ◀$$



(b) The smallest force Q at B must be perpendicular to $\overrightarrow{DB}$ at 45°

$$\mathbf{M}_D = Q(\overrightarrow{DB})$$

$$41.700\text{ N}\cdot\text{m} = Q(0.28284\text{ m}) \qquad Q = 147.4\text{ N} \;\; 45° ◀$$



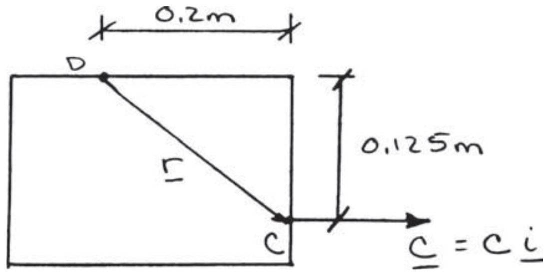
PROBLEM 3.4

A 300-N force is applied at A as shown. Determine (a) the moment of the 300-N force about D, (b) the magnitude and sense of the horizontal force applied at C that creates the same moment about D, (c) the smallest force applied at C that creates the same moment about D.

SOLUTION

(a) See Problem 3.3 for the figure and analysis leading to the determination of $\mathbf{M}_D$

$$\mathbf{M}_D = 41.7 \text{ N}\cdot\text{m} ◀$$



(b) Since $\mathbf{C}$ is horizontal $\mathbf{C} = C\,\mathbf{i}$

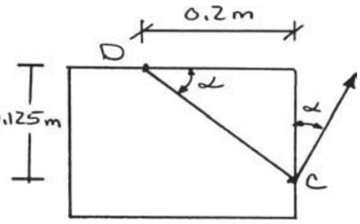
$$\mathbf{r} = \overrightarrow{DC} = (0.2 \text{ m})\mathbf{i} - (0.125 \text{ m})\mathbf{j}$$

$$\mathbf{M}_D = \mathbf{r} \times C\,\mathbf{i} = C(0.125 \text{ m})\mathbf{k}$$

$$41.7 \text{ N}\cdot\text{m} = (0.125 \text{ m})(C)$$

$$C = 333.60 \text{ N} \qquad C = 334 \text{ N} ◀$$

(c) The smallest force C must be perpendicular to DC; thus, it forms α with the vertical



$$\tan\alpha = \frac{0.125 \text{ m}}{0.2 \text{ m}}$$

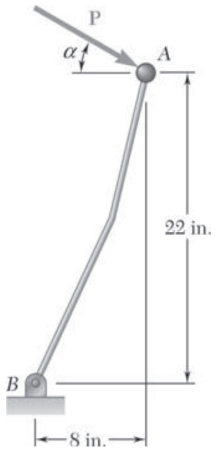
$$\alpha = 32.0°$$

$$\mathbf{M}_D = C(DC); \quad DC = \sqrt{(0.2 \text{ m})^2 + (0.125 \text{ m})^2}$$

$$= 0.23585 \text{ m}$$

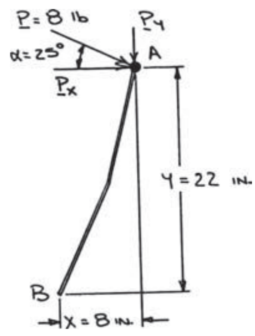
$$41.70 \text{ N}\cdot\text{m} = C(0.23585 \text{ m}) \qquad \mathbf{C} = 176.8 \text{ N} \angle 58.0° ◀$$

PROBLEM 3.5



An 8-lb force **P** is applied to a shift lever. Determine the moment of **P** about *B* when *a* is equal to 25°.

SOLUTION



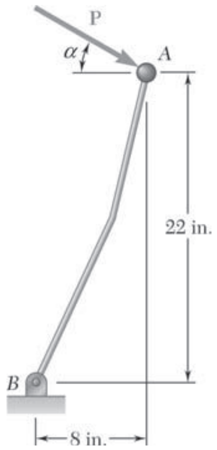
First note

$$P_x = (8 \text{ lb})\cos 25°$$
$$= 7.2505 \text{ lb}$$
$$P_y = (8 \text{ lb})\sin 25°$$
$$= 3.3809 \text{ lb}$$

Noting that the direction of the moment of each force component about *B* is clockwise, have

$$M_B = -xP_y - yP_x$$
$$= -(8 \text{ in.})(3.3809 \text{ lb})$$
$$- (22 \text{ in.})(7.2505 \text{ lb})$$
$$= -186.6 \text{ lb} \cdot \text{in.}$$

or $\mathbf{M}_B = 186.6 \text{ lb} \cdot \text{in.}$ ◀

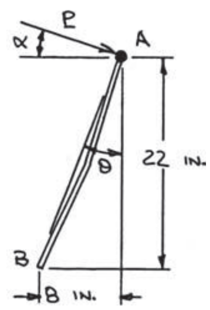


PROBLEM 3.6

For the shift lever shown, determine the magnitude and the direction of the smallest force **P** that has a 210-lb · in. clockwise moment about *B*.

SOLUTION

For **P** to be minimum, it must be perpendicular to the line joining Points *A* and *B*. Thus,



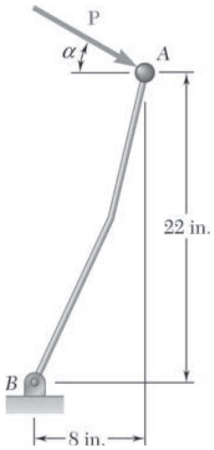
$$\alpha = \theta$$
$$= \tan^{-1}\frac{8\text{ in.}}{22\text{ in.}}$$
$$= 19.98^\circ$$

and $$M_B = dP_{\min}$$

Where $$d = r_{A/B}$$
$$= \sqrt{(8\text{ in.})^2 + (22\text{ in.})^2}$$
$$= 23.409\text{ in.}$$

Then $$P_{\min} = \frac{210\text{ lb}\cdot\text{in.}}{23.409\text{ in.}}$$
$$= 8.97\text{ lb}$$

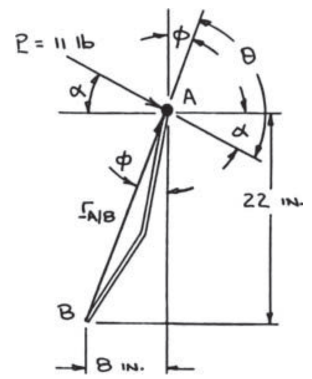
$\mathbf{P}_{\min} = 8.97$ lb ⦨ 19.98° ◀



PROBLEM 3.7

An 11-lb force **P** is applied to a shift lever. The moment of **P** about *B* is clockwise and has a magnitude of 250 lb · in. Determine the value of *a*.

SOLUTION



By definition $\quad M_B = r_{A/B} P \sin\theta$

where $\quad \theta = \alpha + (90° - \phi)$

and $\quad \phi = \tan^{-1}\dfrac{8 \text{ in.}}{22 \text{ in.}} = 19.9831°$

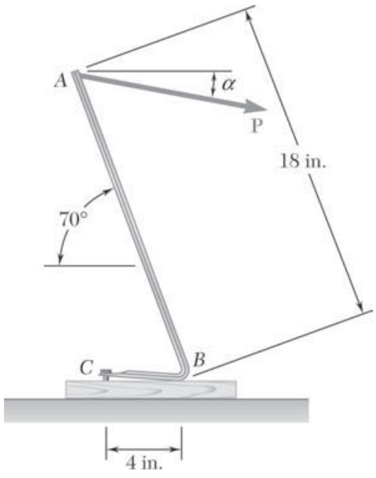
also
$$r_{A/B} = \sqrt{(8 \text{ in.})^2 + (22 \text{ in.})^2} = 23.409 \text{ in.}$$

Then
$$250 \text{ lb}\cdot\text{in} = (23.409 \text{ in.})(11 \text{ lb}) \times \sin(\alpha + 90° - 19.9831°)$$

or $\quad \sin(\alpha + 70.0169°) = 0.97088$

or $\quad \alpha + 70.0169° = 76.1391°$

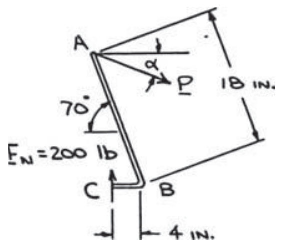
and $\quad \alpha + 70.0169° = 103.861°$ $\qquad \alpha = 6.12° \quad 33.8°$ ◀



PROBLEM 3.8

It is known that a vertical force of 200 lb is required to remove the nail at C from the board. As the nail first starts moving, determine (*a*) the moment about B of the force exerted on the nail, (*b*) the magnitude of the force **P** that creates the same moment about B if $a = 10°$, (*c*) the smallest force **P** that creates the same moment about B.

SOLUTION

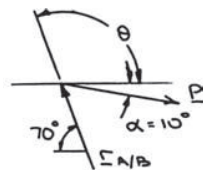


(*a*) We have $\quad M_B = r_{C/B}F_N$

$$= (4 \text{ in.})(200 \text{ lb})$$

$$= 800 \text{ lb} \cdot \text{in.}$$

or $\quad M_B = 800 \text{ lb} \cdot \text{in.}$ ◀



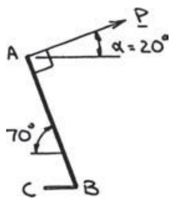
(*b*) By definition $\quad M_B = r_{A/B}P\sin\theta$

$$\theta = 10° + (180° - 70°)$$

$$= 120°$$

Then $\quad 800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) \times P\sin 120°$

or $\quad P = 51.3$ lb ◀



(*c*) For **P** to be minimum, it must be perpendicular to the line joining Points A and B. Thus, **P** must be directed as shown.

Thus $\quad M_B = dP_{\min}$

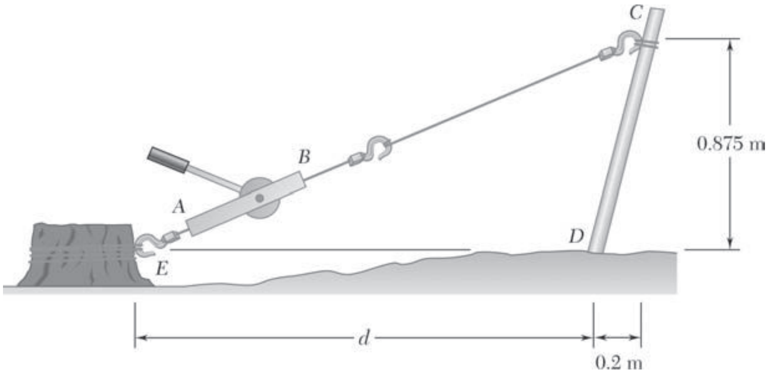
$$d = r_{A/B}$$

or $\quad 800 \text{ lb} \cdot \text{in.} = (18 \text{ in.})P_{\min}$

or $\quad P_{\min} = 44.4$ lb $\qquad \mathbf{P}_{\min} = 44.4 \text{ lb} \measuredangle 20°$ ◀

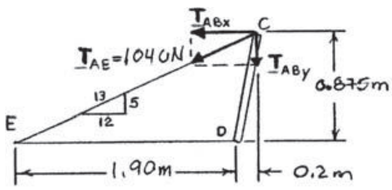
PROBLEM 3.9

A winch puller AB is used to straighten a fence post. Knowing that the tension in cable BC is 1040 N and length d is 1.90 m, determine the moment about D of the force exerted by the cable at C by resolving that force into horizontal and vertical components applied (a) at Point C, (b) at Point E.



SOLUTION

(a) Slope of line
$$EC = \frac{0.875\text{ m}}{1.90\text{ m} + 0.2\text{ m}} = \frac{5}{12}$$



(a)

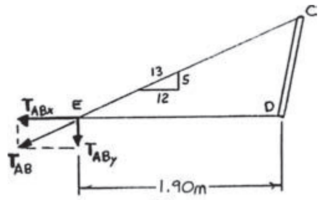
Then
$$\begin{aligned} T_{ABx} &= \frac{12}{13}(T_{AB}) \\ &= \frac{12}{13}(1040\text{ N}) \\ &= 960\text{ N} \end{aligned}$$

and
$$\begin{aligned} T_{ABy} &= \frac{5}{13}(1040\text{ N}) \\ &= 400\text{ N} \end{aligned}$$

Then
$$\begin{aligned} M_D &= T_{ABx}(0.875\text{ m}) - T_{ABy}(0.2\text{ m}) \\ &= (960\text{ N})(0.875\text{ m}) - (400\text{ N})(0.2\text{ m}) \\ &= 760\text{ N}\cdot\text{m} \end{aligned}$$

or $\mathbf{M}_D = 760\text{ N}\cdot\text{m}$ ↻ ◀

(b) We have
$$\begin{aligned} M_D &= T_{ABx}(y) + T_{ABx}(x) \\ &= (960\text{ N})(0) + (400\text{ N})(1.90\text{ m}) \\ &= 760\text{ N}\cdot\text{m} \end{aligned}$$

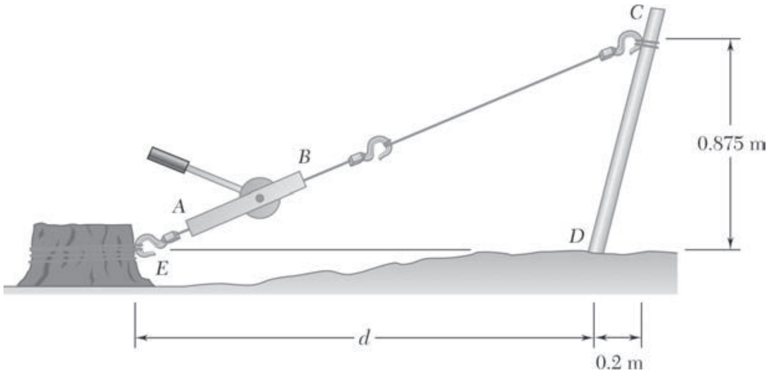


(b)

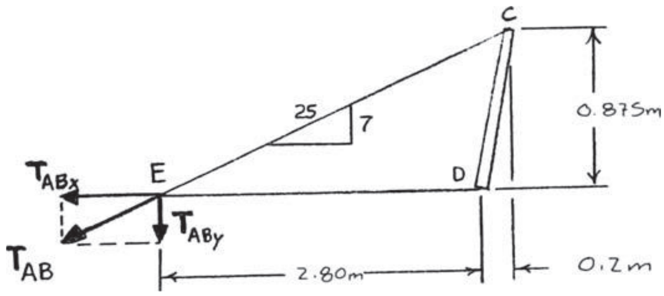
or $\mathbf{M}_D = 760\text{ N}\cdot\text{m}$ ↻ ◀

PROBLEM 3.10

It is known that a force with a moment of 960 N · m about D is required to straighten the fence post CD. If $d = 2.80$ m, determine the tension that must be developed in the cable of winch puller AB to create the required moment about Point D.



SOLUTION



Slope of line $\quad EC = \dfrac{0.875\text{ m}}{2.80\text{ m} + 0.2\text{ m}} = \dfrac{7}{24}$

Then $\quad T_{ABx} = \dfrac{24}{25} T_{AB}$

and $\quad T_{ABy} = \dfrac{7}{25} T_{AB}$

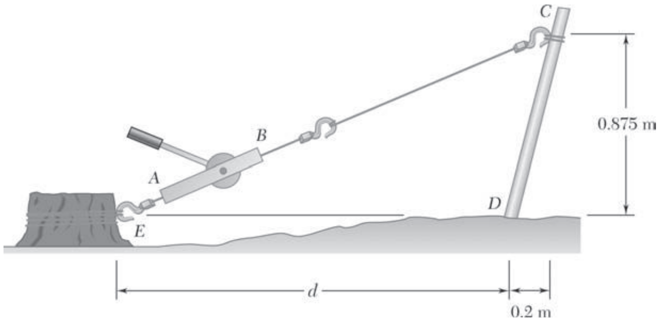
We have $\quad M_D = T_{ABx}(y) + T_{ABy}(x)$

$$960\text{ N}\cdot\text{m} = \frac{24}{25} T_{AB}(0) + \frac{7}{25} T_{AB}(2.80\text{ m})$$

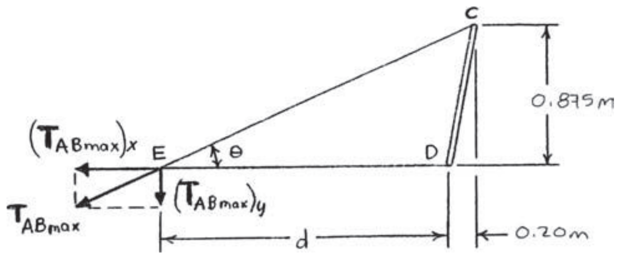
$T_{AB} = 1224$ N $\qquad$ or $T_{AB} = 1224$ N ◀

PROBLEM 3.11

It is known that a force with a moment of 960 N · m about D is required to straighten the fence post CD. If the capacity of winch puller AB is 2400 N, determine the minimum value of distance d to create the specified moment about Point D.



SOLUTION



The minimum value of d can be found based on the equation relating the moment of the force $\mathbf{T}_{AB}$ about D:

$$M_D = (T_{AB\max})_y(d)$$

where $$M_D = 960 \text{ N}\cdot\text{m}$$

$$(T_{AB\max})_y = T_{AB\max}\sin\theta = (2400 \text{ N})\sin\theta$$

Now $$\sin\theta = \frac{0.875m}{\sqrt{(d+0.20)^2+(0.875)^2}\text{ m}}$$

$$960 \text{ N}\cdot\text{m} = 2400 \text{ N}\left[\frac{0.875}{\sqrt{(d+0.20)^2+(0.875)^2}}\right](d)$$

or $$\sqrt{(d+0.20)^2+(0.875)^2} = 2.1875d$$

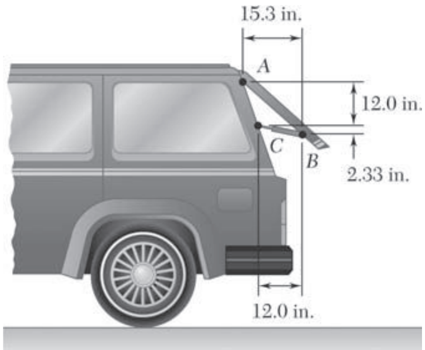
or $$(d+0.20)^2+(0.875)^2 = 4.7852d^2$$

or $$3.7852d^2 - 0.40d - .8056 = 0$$

Using the quadratic equation, the minimum values of d are 0.51719 m and $-.41151$ m.

Since only the positive value applies here, $d = 0.51719$ m

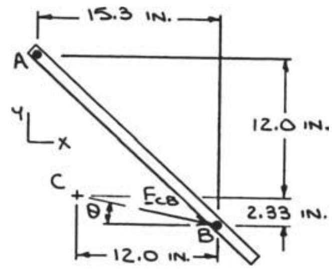
or $d = 517$ mm ◀



PROBLEM 3.12

The tailgate of a car is supported by the hydraulic lift BC. If the lift exerts a 125-lb force directed along its centerline on the ball and socket at B, determine the moment of the force about A.

SOLUTION



First note $$d_{CB} = \sqrt{(12.0 \text{ in.})^2 + (2.33 \text{ in.})^2}$$
$$= 12.2241 \text{ in.}$$

Then $$\cos\theta = \frac{12.0 \text{ in.}}{12.2241 \text{ in.}}$$
$$\sin\theta = \frac{2.33 \text{ in.}}{12.2241 \text{ in.}}$$

and $$\mathbf{F}_{CB} = F_{CB}\cos\theta\,\mathbf{i} - F_{CB}\sin\theta\,\mathbf{j}$$
$$= \frac{125 \text{ lb}}{12.2241 \text{ in.}}[(12.0 \text{ in.})\,\mathbf{i} - (2.33 \text{ in.})\,\mathbf{j}]$$

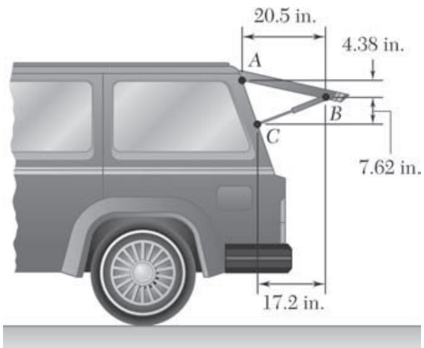
Now $$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

where $$\mathbf{r}_{B/A} = (15.3 \text{ in.})\,\mathbf{i} - (12.0 \text{ in.} + 2.33 \text{ in.})\,\mathbf{j}$$
$$= (15.3 \text{ in.})\,\mathbf{i} - (14.33 \text{ in.})\,\mathbf{j}$$

Then $$\mathbf{M}_A = [(15.3 \text{ in.})\mathbf{i} - (14.33 \text{ in.})\mathbf{j}] \times \frac{125 \text{ lb}}{12.2241 \text{ in.}}(12.0\mathbf{i} - 2.33\mathbf{j})$$
$$= (1393.87 \text{ lb}\cdot\text{in.})\mathbf{k}$$
$$= (116.156 \text{ lb}\cdot\text{ft})\mathbf{k}$$

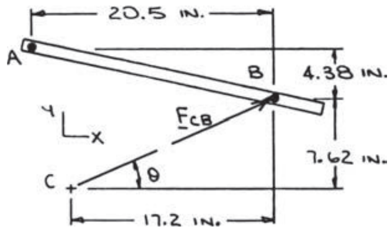
or $\mathbf{M}_A = 116.2 \text{ lb}\cdot\text{ft}$ ◀

PROBLEM 3.13



The tailgate of a car is supported by the hydraulic lift BC. If the lift exerts a 125-lb force directed along its centerline on the ball and socket at B, determine the moment of the force about A.

SOLUTION



First note
$$d_{CB} = \sqrt{(17.2\text{ in.})^2 + (7.62\text{ in.})^2}$$
$$= 18.8123\text{ in.}$$

Then
$$\cos\theta = \frac{17.2\text{ in.}}{18.8123\text{ in.}}$$
$$\sin\theta = \frac{7.62\text{ in.}}{18.8123\text{ in.}}$$

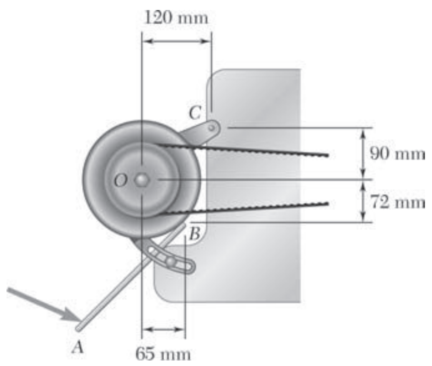
and
$$\mathbf{F}_{CB} = (F_{CB}\cos\theta)\mathbf{i} - (F_{CB}\sin\theta)\mathbf{j}$$
$$= \frac{125\text{ lb}}{18.8123\text{ in.}}[(17.2\text{ in.})\mathbf{i} + (7.62\text{ in.})\mathbf{j}]$$

Now
$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

where
$$\mathbf{r}_{B/A} = (20.5\text{ in.})\mathbf{i} - (4.38\text{ in.})\mathbf{j}$$

Then
$$\mathbf{M}_A = [(20.5\text{ in.})\mathbf{i} - (4.38\text{ in.})\mathbf{j}] \times \frac{125\text{ lb}}{18.8123\text{ in.}}(17.2\mathbf{i} - 7.62\mathbf{j})$$
$$= (1538.53\text{ lb}\cdot\text{in.})\mathbf{k}$$
$$= (128.2\text{ lb}\cdot\text{ft})\mathbf{k}$$

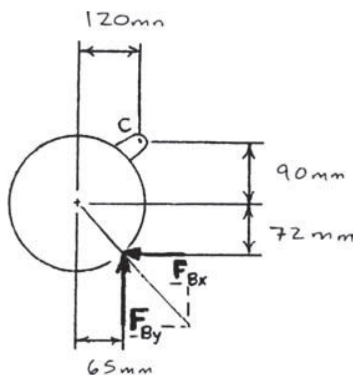
or $\mathbf{M}_A = 128.2\text{ lb}\cdot\text{ft}$ ◀



PROBLEM 3.14

A mechanic uses a piece of pipe AB as a lever when tightening an alternator belt. When he pushes down at A, a force of 485 N is exerted on the alternator at B. Determine the moment of that force about bolt C if its line of action passes through O.

SOLUTION



We have $\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_B$

Noting the direction of the moment of each force component about C is clockwise.

$$M_C = xF_{By} + yF_{Bx}$$

Where

$$x = 120 \text{ mm} - 65 \text{ mm} = 55 \text{ mm}$$

$$y = 72 \text{ mm} + 90 \text{ mm} = 162 \text{ mm}$$

and

$$F_{Bx} = \frac{65}{\sqrt{(65)^2 + (72)^2}}(485 \text{ N}) = 325 \text{ N}$$

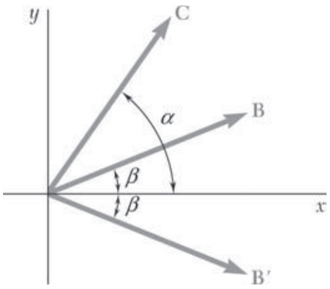
$$F_{By} = \frac{72}{\sqrt{(65)^2 + (72)^2}}(485 \text{ N}) = 360 \text{ N}$$

$$M_C = (55 \text{ mm})(360 \text{ N}) + (162)(325 \text{ N})$$

$$= 72450 \text{ N} \cdot \text{mm}$$

$$= 72.450 \text{ N} \cdot \text{m}$$

or $\mathbf{M}_C = 72.5 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.15

Form the vector products **B** × **C** and **B′** × **C**, where $B = B'$, and use the results obtained to prove the identity

$$\sin a \cos\beta = \frac{1}{2}\sin(a+\beta) + \frac{1}{2}\sin(a-\beta).$$

SOLUTION

Note:
$$\mathbf{B} = B(\cos\beta\,\mathbf{i} + \sin\beta\,\mathbf{j})$$
$$\mathbf{B}' = B(\cos\beta\,\mathbf{i} - \sin\beta\,\mathbf{j})$$
$$\mathbf{C} = C(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})$$

By definition:
$$|\mathbf{B}\times\mathbf{C}| = BC\sin(\alpha-\beta) \tag{1}$$
$$|\mathbf{B}'\times\mathbf{C}| = BC\sin(\alpha+\beta) \tag{2}$$

Now
$$\mathbf{B}\times\mathbf{C} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j})\times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$
$$= BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)\mathbf{k} \tag{3}$$

and
$$\mathbf{B}'\times\mathbf{C} = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j})\times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$
$$= BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)\mathbf{k} \tag{4}$$

Equating the magnitudes of $\mathbf{B}\times\mathbf{C}$ from Equations (1) and (3) yields:

$$BC\sin(\alpha-\beta) = BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha) \tag{5}$$

Similarly, equating the magnitudes of $\mathbf{B}'\times\mathbf{C}$ from Equations (2) and (4) yields:

$$BC\sin(\alpha+\beta) = BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \tag{6}$$

Adding Equations (5) and (6) gives:

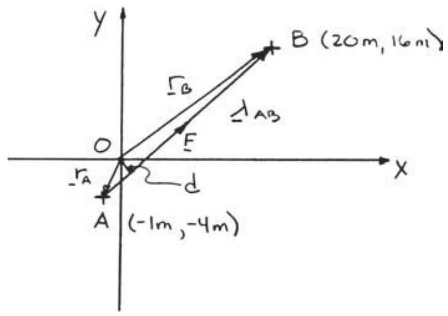
$$\sin(\alpha-\beta) + \sin(\alpha+\beta) = 2\cos\beta\sin\alpha$$

or $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$ ◀

PROBLEM 3.16

A line passes through the Points (20 m, 16 m) and (–1 m, –4 m). Determine the perpendicular distance d from the line to the origin O of the system of coordinates.

SOLUTION



$$d_{AB} = \sqrt{[20\text{ m} - (-1\text{ m})]^2 + [16\text{ m} - (-4\text{ m})]^2}$$
$$= 29\text{ m}$$

Assume that a force **F**, or magnitude F(N), acts at Point A and is directed from A to B.

Then, $$\mathbf{F} = F\boldsymbol{\lambda}_{AB}$$

Where $$\boldsymbol{\lambda}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{d_{AB}}$$
$$= \frac{1}{29}(21\mathbf{i} + 20\mathbf{j})$$

By definition $$\mathbf{M}_O = |\mathbf{r}_A \times \mathbf{F}| = dF$$

Where $$\mathbf{r}_A = -(1\text{ m})\mathbf{i} - (4\text{ m})\mathbf{j}$$

Then $$\mathbf{M}_O = [-(-1\text{ m})\mathbf{i} - (4\text{ m})\mathbf{j}] \times \frac{F}{29\text{ m}}[(21\text{ m})\mathbf{i} + (20\text{ m})\mathbf{j}]$$
$$= \frac{F}{29}[-(20)\mathbf{k} + (84)\mathbf{k}]$$
$$= \left(\frac{64}{29}F\right)\mathbf{k} \quad \text{N}\cdot\text{m}$$

Finally $$\left(\frac{64}{29}F\right) = d(F)$$
$$d = \frac{64}{29}\text{ m}$$

$d = 2.21\text{ m}$ ◀

PROBLEM 3.17

The vectors **P** and **Q** are two adjacent sides of a parallelogram. Determine the area of the parallelogram when (*a*) **P** = –7**i** + 3**j** – 3**k** and **Q** = 2**i** + 2**j** + 5**k**, (*b*) **P** = 6**i** – 5**j** – 2**k** and **Q** = –2**i** + 5**j** – **k**.

SOLUTION

(*a*) We have $A = |\mathbf{P} \times \mathbf{Q}|$

where $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$

$\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

Then
$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & 3 & -3 \\ 2 & 2 & 5 \end{vmatrix}$$

$$= [(15+6)\mathbf{i} + (-6+35)\mathbf{j} + (-14-6)\mathbf{k}]$$
$$= (21)\mathbf{i} + (29)\mathbf{j}(-20)\mathbf{k}$$

$$A = \sqrt{(20)^2 + (29)^2 + (-20)^2}$$

or $A = 41.0$ ◀

(*b*) We have $A = |\mathbf{P} \times \mathbf{Q}|$

where $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$

$\mathbf{Q} = -2\mathbf{i} + 5 \text{ in.} \mathbf{j} - 1\mathbf{k}$

Then
$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -5 & -2 \\ -2 & 5 & -1 \end{vmatrix}$$

$$= [(5+10)\mathbf{i} + (4+6)\mathbf{j} + (30-10)\mathbf{k}]$$
$$= (15)\mathbf{i} + (10)\mathbf{j} + (20)\mathbf{k}$$

$$A = \sqrt{(15)^2 + (10)^2 + (20)^2}$$

or $A = 26.9$ ◀

PROBLEM 3.18

A plane contains the vectors **A** and **B**. Determine the unit vector normal to the plane when **A** and **B** are equal to, respectively, (*a*) **i** + 2**j** – 5**k** and 4**i** – 7**j** – 5**k**, (*b*) 3**i** – 3**j** + 2**k** and –2**i** + 6**j** – 4**k**.

SOLUTION

(*a*) We have
$$\boldsymbol{\lambda} = \frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}$$

where
$$\mathbf{A} = 1\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$$
$$\mathbf{B} = 4\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$$

Then
$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & +2 & -5 \\ 4 & -7 & -5 \end{vmatrix}$$
$$= (-10-35)\mathbf{i} + (20+5)\mathbf{j} + (-7-8)\mathbf{k}$$
$$= 15(3\mathbf{i} - 1\mathbf{j} - 1\mathbf{k})$$

and
$$|\mathbf{A}\times\mathbf{B}| = 15\sqrt{(-3)^2 + (-1)^2 + (-1)^2} = 15\sqrt{11}$$

$$\boldsymbol{\lambda} = \frac{15(-3\mathbf{i} - 1\mathbf{j} - 1\mathbf{k})}{15\sqrt{11}}$$

or $\boldsymbol{\lambda} = \dfrac{1}{\sqrt{11}}(-3\mathbf{i} - \mathbf{j} - \mathbf{k})$ ◀

(*b*) We have
$$\boldsymbol{\lambda} = \frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}$$

where
$$\mathbf{A} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$
$$\mathbf{B} = -2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$$

Then
$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 2 \\ -2 & 6 & -4 \end{vmatrix}$$
$$= (12-12)\mathbf{i} + (-4+12)\mathbf{j} + (18-6)\mathbf{k}$$
$$= (8\mathbf{j} + 12\mathbf{k})$$

and
$$|\mathbf{A}\times\mathbf{B}| = 4\sqrt{(2)^2 + (3)^2} = 4\sqrt{13}$$

$$\boldsymbol{\lambda} = \frac{4(2\mathbf{j} + 3\mathbf{k})}{4\sqrt{13}}$$

or $\boldsymbol{\lambda} = \dfrac{1}{\sqrt{13}}(2\mathbf{j} + 3\mathbf{k})$ ◀

PROBLEM 3.19

Determine the moment about the origin O of the force $\mathbf{F} = 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ that acts at a Point A. Assume that the position vector of A is (a) $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, ($b$) $\mathbf{r} = 2\mathbf{i} + 2.5\mathbf{j} - 1.5\mathbf{k}$, (c) $\mathbf{r} = 2\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$.

SOLUTION

(a)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 4 \\ 4 & 5 & -3 \end{vmatrix}$$

$$= (9-20)\mathbf{i} + (16+6)\mathbf{j} + (10+12)\mathbf{k} \qquad \mathbf{M}_O = -11\mathbf{i} + 22\mathbf{j} + 22\mathbf{k} \blacktriangleleft$$

(b)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2.5 & -1.5 \\ 4 & 5 & -3 \end{vmatrix}$$

$$= (-7.5+7.5)\mathbf{i} + (-6+6)\mathbf{j} + (10-10)\mathbf{k} \qquad \mathbf{M}_O = 0 \blacktriangleleft$$

(c)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 5 & 6 \\ 4 & 5 & -3 \end{vmatrix}$$

$$= (-15-30)\mathbf{i} + (24+6)\mathbf{j} + (10-20)\mathbf{k} \qquad \mathbf{M}_O = -45\mathbf{i} + 30\mathbf{j} - 10\mathbf{k} \blacktriangleleft$$

Note: The answer to Part b could have been anticipated since the elements of the last two rows of the determinant are proportional.

PROBLEM 3.20

Determine the moment about the origin O of the force $\mathbf{F} = -2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ that acts at a Point A. Assume that the position vector of A is (*a*) $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, (*b*) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$, (c) $\mathbf{r} = -4\mathbf{i} + 6\mathbf{j} + 10\mathbf{k}$.

SOLUTION

(*a*)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ -2 & 3 & 5 \end{vmatrix}$$

$$= (5-3)\mathbf{i} + (-2-5)\mathbf{j} + (3+2)\mathbf{k}$$

$$\mathbf{M}_O = 2\mathbf{i} - 7\mathbf{j} + 5\mathbf{k} \blacktriangleleft$$

(*b*)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -5 \\ -2 & 3 & 5 \end{vmatrix}$$

$$= (15+15)\mathbf{i} + (10-10)\mathbf{j} + (6+6)\mathbf{k}$$

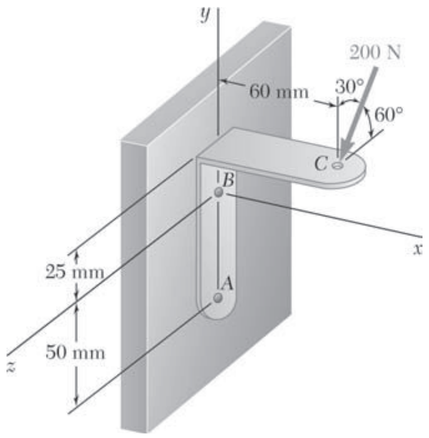
$$\mathbf{M}_O = 30\mathbf{i} + 12\mathbf{k} \blacktriangleleft$$

(*c*)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 6 & 10 \\ -2 & 3 & 5 \end{vmatrix}$$

$$= (30-30)\mathbf{i} + (-20+20)\mathbf{j} + (-12+12)\mathbf{k}$$

$$\mathbf{M}_O = 0 \blacktriangleleft$$

Note: The answer to Part *c* could have been anticipated since the elements of the last two rows of the determinant are proportional.



PROBLEM 3.21

A 200-N force is applied as shown to the bracket *ABC*. Determine the moment of the force about *A*.

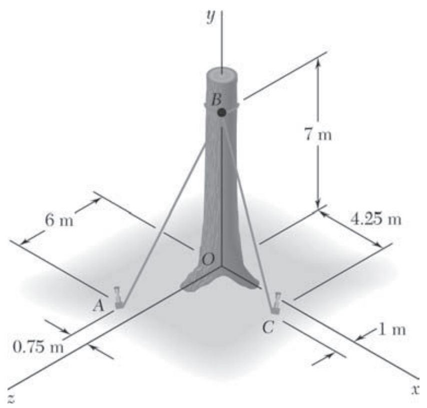
SOLUTION

We have
$$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$$

where
$$\mathbf{r}_{C/A} = (0.06 \text{ m})\mathbf{i} + (0.075 \text{ m})\mathbf{j}$$
$$\mathbf{F}_C = -(200 \text{ N})\cos 30°\mathbf{j} + (200 \text{ N})\sin 30°\mathbf{k}$$

Then
$$\mathbf{M}_A = 200\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06 & 0.075 & 0 \\ 0 & -\cos 30° & \sin 30° \end{vmatrix}$$
$$= 200[(0.075\sin 30°)\mathbf{i} - (0.06\sin 30°)\mathbf{j} - (0.06\cos 30°)\mathbf{k}]$$

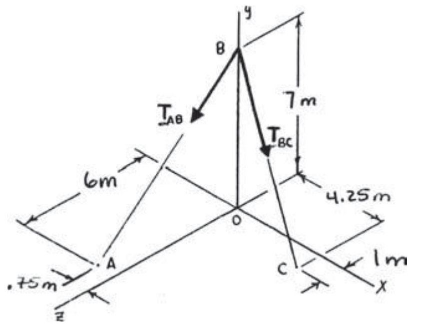
or $\mathbf{M}_A = (7.50 \text{ N}\cdot\text{m})\mathbf{i} - (6.00 \text{ N}\cdot\text{m})\mathbf{j} - (10.39 \text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.22

Before the trunk of a large tree is felled, cables *AB* and *BC* are attached as shown. Knowing that the tensions in cables *AB* and *BC* are 555 N and 660 N, respectively, determine the moment about *O* of the resultant force exerted on the tree by the cables at *B*.

SOLUTION



We have $\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F}_B$

where

$$\mathbf{r}_{B/O} = (7\text{ m})\mathbf{j}$$

$$\mathbf{F}_B = \mathbf{T}_{AB} + \mathbf{T}_{BC}$$

$$\mathbf{T}_{AB} = \boldsymbol{\lambda}_{BA} T_{AB}$$

$$= \frac{-(0.75\text{ m})\mathbf{i} - (7\text{ m})\mathbf{j} + (6\text{ m})\mathbf{k}}{\sqrt{(.75)^2 + (7)^2 + (6)^2}\text{ m}}(555\text{ N})$$

$$\mathbf{T}_{BC} = \boldsymbol{\lambda}_{BC} T_{BC}$$

$$= \frac{(4.25\text{ m})\mathbf{i} - (7\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}}{\sqrt{(4.25)^2 + (7)^2 + (1)^2}\text{ m}}(660\text{ N})$$

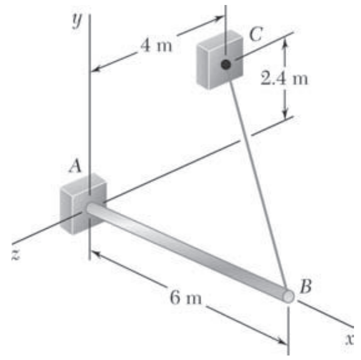
$$\mathbf{F}_B = [-(45.00\text{ N})\mathbf{i} - (420.0\text{ N})\mathbf{j} + (360.0\text{ N})\mathbf{k}]$$
$$+[(340.0\text{ N})\mathbf{i} - (560.0\text{ N})\mathbf{j} + (80.00\text{ N})\mathbf{k}]$$
$$= (295.0\text{ N})\mathbf{i} - (980.0\text{ N})\mathbf{j} + (440.0\text{ N})\mathbf{k}$$

and

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7 & 0 \\ 295 & 980 & 440 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (3080\text{ N}\cdot\text{m})\mathbf{i} - (2070\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_O = (3080\text{ N}\cdot\text{m})\mathbf{i} - (2070\text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.23

The 6-m boom *AB* has a fixed end *A*. A steel cable is stretched from the free end *B* of the boom to a Point *C* located on the vertical wall. If the tension in the cable is 2.5 kN, determine the moment about *A* of the force exerted by the cable at *B*.

SOLUTION

First note
$$d_{BC} = \sqrt{(-6)^2 + (2.4)^2 + (-4)^2}$$
$$= 7.6 \text{ m}$$

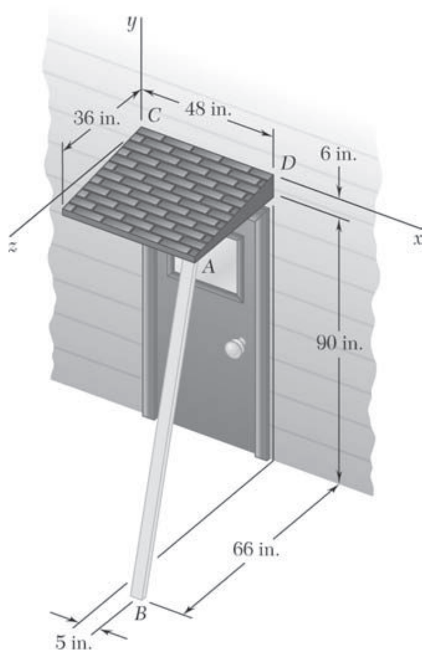
Then
$$\mathbf{T}_{BC} = \frac{2.5 \text{ kN}}{7.6}(-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

We have
$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$$

where
$$\mathbf{r}_{B/A} = (6 \text{ m})\mathbf{i}$$

Then
$$\mathbf{M}_A = (6 \text{ m})\mathbf{i} \times \frac{2.5 \text{ kN}}{7.6}(-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

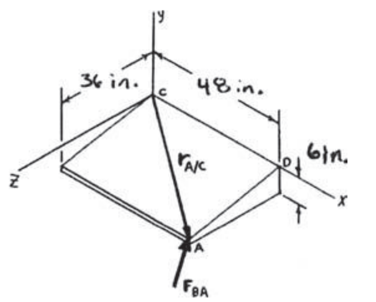
or $\mathbf{M}_A = (7.89 \text{ kN} \cdot \text{m})\mathbf{j} + (4.74 \text{ kN} \cdot \text{m})\mathbf{k}$ ◀



PROBLEM 3.24

A wooden board *AB*, which is used as a temporary prop to support a small roof, exerts at Point *A* of the roof a 57-lb force directed along *BA*. Determine the moment about *C* of that force.

SOLUTION



We have $\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}_{BA}$

where $\mathbf{r}_{A/C} = (48\text{ in.})\mathbf{i} - (6\text{ in.})\mathbf{j} + (36\text{ in.})\mathbf{k}$

and $\mathbf{F}_{BA} = \boldsymbol{\lambda}_{BA} F_{BA}$

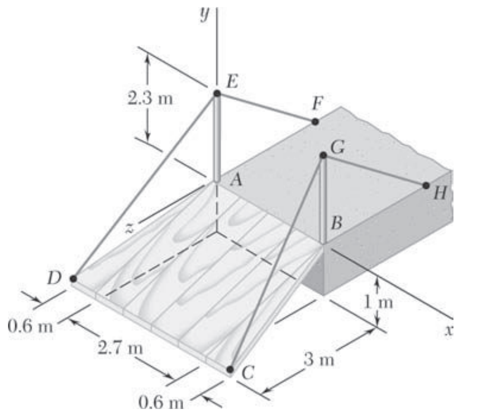
$$= \left[\frac{-(5\text{ in.})\mathbf{i} + (90\text{ in.})\mathbf{j} - (30\text{ in.})\mathbf{k}}{\sqrt{(5)^2 + (90)^2 + (30)^2}\text{ in.}}\right](57\text{ lb})$$

$$= -(3\text{ lb})\mathbf{i} + (54\text{ lb})\mathbf{j} - (18\text{ lb})\mathbf{k}$$

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 48 & 6 & 36 \\ 3 & 54 & 18 \end{vmatrix} \text{lb} \cdot \text{in.}$$

$$= -(1836\text{ lb} \cdot \text{in.})\mathbf{i} + (756\text{ lb} \cdot \text{in.})\mathbf{j} + (2574\text{ lb} \cdot \text{in.})$$

or $\mathbf{M}_C = -(153.0\text{ lb} \cdot \text{ft})\mathbf{i} + (63.0\text{ lb} \cdot \text{ft})\mathbf{j} + (215\text{ lb} \cdot \text{ft})\mathbf{k}$ ◀



PROBLEM 3.25

The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 810 N. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

SOLUTION

(*a*) We have $\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$

where

$$\mathbf{r}_{E/A} = (2.3\text{ m})\mathbf{j}$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE}$$

$$= \frac{(0.6\text{ m})\mathbf{i} + (3.3\text{ m})\mathbf{j} - (3\text{ m})\mathbf{k}}{\sqrt{(0.6)^2 + (3.3)^2 + (3)^2}\text{ m}}(810\text{ N})$$

$$= (108\text{ N})\mathbf{i} + (594\text{ N})\mathbf{j} - (540\text{ N})\mathbf{k}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.3 & 0 \\ 108 & 594 & -540 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(1242\text{ N}\cdot\text{m})\mathbf{i} - (248.4\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_A = -(1242\text{ N}\cdot\text{m})\mathbf{i} - (248\text{ N}\cdot\text{m})\mathbf{k}$ ◀

(*b*) We have $\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$

where

$$\mathbf{r}_{G/A} = (2.7\text{ m})\mathbf{i} + (2.3\text{ m})\mathbf{j}$$

$$\mathbf{T}_{CG} = \boldsymbol{\lambda}_{CG} T_{CG}$$

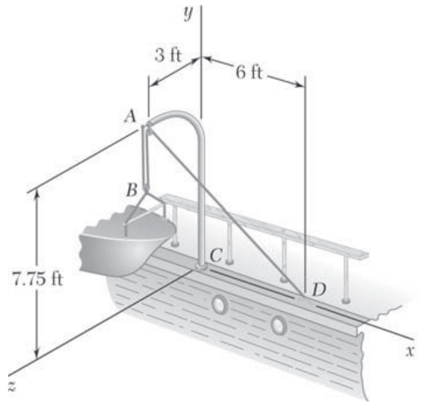
$$= \frac{-(.6\text{ m})\mathbf{i} + (3.3\text{ m})\mathbf{j} - (3\text{ m})\mathbf{k}}{\sqrt{(.6)^2 + (3.3)^2 + (3)^2}\text{ m}}(810\text{ N})$$

$$= -(108\text{ N})\mathbf{i} + (594\text{ N})\mathbf{j} - (540\text{ N})\mathbf{k}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 & 2.3 & 0 \\ -108 & 594 & -540 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(1242\text{ N}\cdot\text{m})\mathbf{i} + (1458\text{ N}\cdot\text{m})\mathbf{j} + (1852\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_A = -(1242\text{ N}\cdot\text{m})\mathbf{i} + (1458\text{ N}\cdot\text{m})\mathbf{j} + (1852\text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.26

A small boat hangs from two davits, one of which is shown in the figure. The tension in line *ABAD* is 82 lb. Determine the moment about *C* of the resultant force $\mathbf{R}_A$ exerted on the davit at *A*.

SOLUTION

We have $$\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD}$$

where $$\mathbf{F}_{AB} = -(82\text{ lb})\mathbf{j}$$

and $$\mathbf{F}_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD} = (82\text{ lb})\frac{6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k}}{10.25}$$

$$\mathbf{F}_{AD} = (48\text{ lb})\mathbf{i} - (62\text{ lb})\mathbf{j} - (24\text{ lb})\mathbf{k}$$

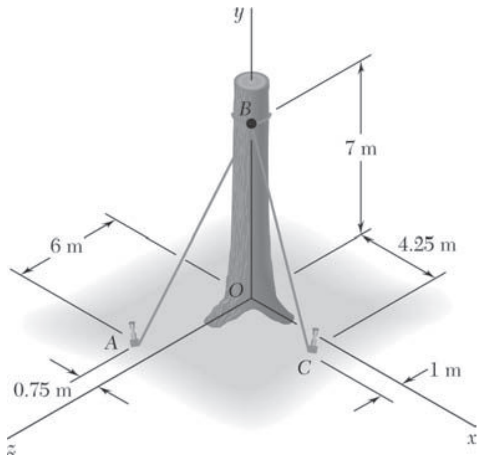
Thus $$\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD} = (48\text{ lb})\mathbf{i} - (226\text{ lb})\mathbf{j} - (24\text{ lb})\mathbf{k}$$

Also $$\mathbf{r}_{A/C} = (7.75\text{ ft})\mathbf{j} + (3\text{ ft})\mathbf{k}$$

Using Eq. (3.21): $$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7.75 & 3 \\ 48 & -226 & -24 \end{vmatrix}$$

$$= (492\text{ lb}\cdot\text{ft})\mathbf{i} + (144\text{ lb}\cdot\text{ft})\mathbf{j} - (372\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$\mathbf{M}_C = (492\text{ lb}\cdot\text{ft})\mathbf{i} + (144.0\text{ lb}\cdot\text{ft})\mathbf{j} - (372\text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

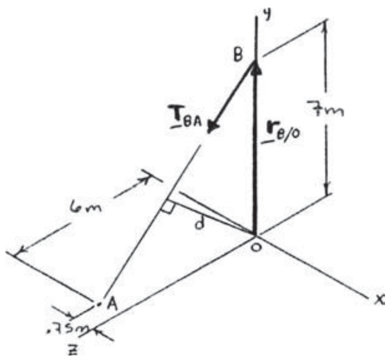


PROBLEM 3.27

In Problem 3.22, determine the perpendicular distance from Point O to cable AB.

PROBLEM 3.22 Before the trunk of a large tree is felled, cables AB and BC are attached as shown. Knowing that the tensions in cables AB and BC are 555 N and 660 N, respectively, determine the moment about O of the resultant force exerted on the tree by the cables at B.

SOLUTION



We have $\quad |\mathbf{M}_O| = T_{BA}d$

where $\quad d =$ perpendicular distance from O to line AB.

Now $\quad \mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BA}$

and $\quad \mathbf{r}_{B/O} = (7\text{ m})\mathbf{j}$

$$\mathbf{T}_{BA} = \boldsymbol{\lambda}_{BA}T_{AB}$$

$$= \frac{-(0.75\text{ m})\mathbf{i} - (7\text{ m})\mathbf{j} + (6\text{ m})\mathbf{k}}{\sqrt{(0.75)^2 + (7)^2 + (6)^2}\text{ m}}(555\text{ N})$$

$$= -(45.0\text{ N})\mathbf{i} - (420\text{ N})\mathbf{j} + (360\text{ N})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7 & 0 \\ -45 & -420 & 360 \end{vmatrix} \text{N} \cdot \text{m}$$

$$= (2520.0\text{ N} \cdot \text{m})\mathbf{i} + (315.00\text{ N} \cdot \text{m})\mathbf{k}$$

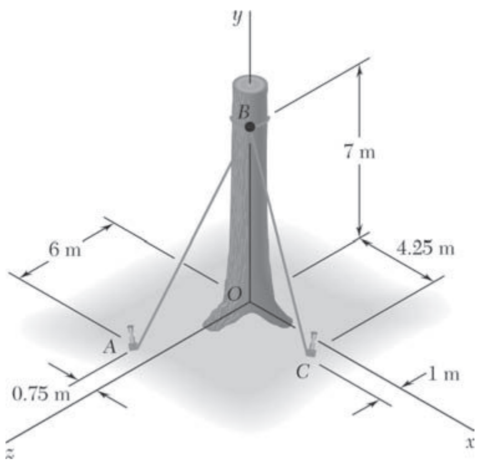
and

$$|\mathbf{M}_O| = \sqrt{(2520.0)^2 + (315.00)^2}$$

$$= 2539.6\text{ N} \cdot \text{m}$$

$$2539.6\text{ N} \cdot \text{m} = (555\text{ N})d$$

or $\quad d = 4.5759\text{ m} \qquad$ or $\quad d = 4.58\text{ m} \blacktriangleleft$



PROBLEM 3.28

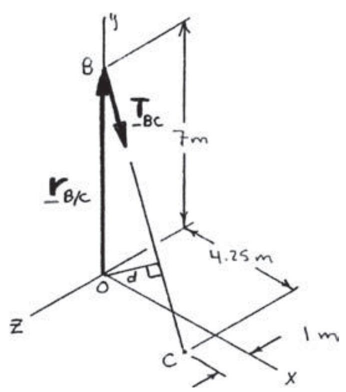
In Problem 3.22, determine the perpendicular distance from Point O to cable BC.

PROBLEM 3.22 Before the trunk of a large tree is felled, cables AB and BC are attached as shown. Knowing that the tensions in cables AB and BC are 555 N and 660 N, respectively, determine the moment about O of the resultant force exerted on the tree by the cables at B.

SOLUTION

We have $|\mathbf{M}_O| = T_{BC}d$

where d = perpendicular distance from O to line BC.



$$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BC}$$

$$\mathbf{r}_{B/O} = 7\ \text{m}\mathbf{j}$$

$$\mathbf{T}_{BC} = \boldsymbol{\lambda}_{BC} T_{BC}$$

$$= \frac{(4.25\ \text{m})\mathbf{i} - (7\ \text{m})\mathbf{j} + (1\ \text{m})\mathbf{k}}{\sqrt{(4.25)^2 + (7)^2 + (1)^2}\ \text{m}}(660\ \text{N})$$

$$= (340\ \text{N})\mathbf{i} - (560\ \text{N})\mathbf{j} + (80\ \text{N})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7 & 0 \\ 340 & -560 & 80 \end{vmatrix}$$

$$= (560\ \text{N}\cdot\text{m})\mathbf{i} - (2380\ \text{N}\cdot\text{m})\mathbf{k}$$

and

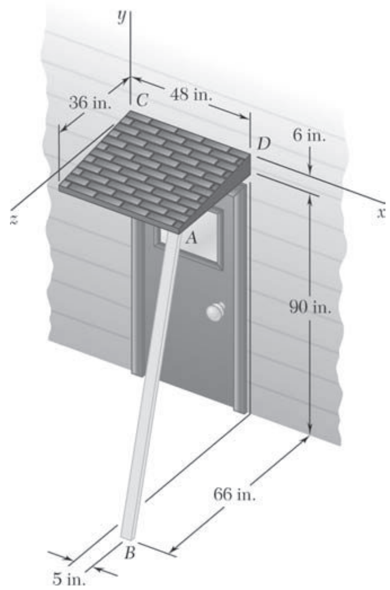
$$|\mathbf{M}_O| = \sqrt{(560)^2 + (2380)^2}$$

$$= 2445.0\ \text{N}\cdot\text{m}$$

$$2445.0\ \text{N}\cdot\text{m} = (660\ \text{N})d$$

$$d = 3.7045\ \text{m}$$

or $d = 3.70$ m ◀

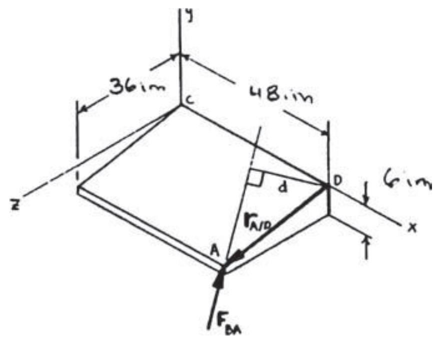


PROBLEM 3.29

In Problem 3.24, determine the perpendicular distance from Point D to a line drawn through Points A and B.

PROBLEM 3.24 A wooden board AB, which is used as a temporary prop to support a small roof, exerts at Point A of the roof a 57-lb force directed along BA. Determine the moment about C of that force.

SOLUTION



We have $|\mathbf{M}_D| = F_{BA}d$

where d = perpendicular distance from D to line AB.

$$\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{F}_{BA}$$

$$\mathbf{r}_{A/D} = -(6\text{ in.})\mathbf{j} + (36\text{ in.})\mathbf{k}$$

$$\mathbf{F}_{BA} = \boldsymbol{\lambda}_{BA} F_{BA}$$

$$= \frac{(-(5\text{ in.})\mathbf{i} + (90\text{ in.})\mathbf{j} - (30\text{ in.})\mathbf{k})}{\sqrt{(5)^2 + (90)^2 + (30)^2}\text{ in.}}(57\text{ lb})$$

$$= -(3\text{ lb})\mathbf{i} + (54\text{ lb})\mathbf{j} - (18\text{ lb})\mathbf{k}$$

$$\mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -6 & 36 \\ -3 & 54 & -18 \end{vmatrix} \text{lb} \cdot \text{in.}$$

$$= -(1836.00\text{ lb} \cdot \text{in.})\mathbf{i} - (108.000\text{ lb} \cdot \text{in.})\mathbf{j} - (18.0000\text{ lb} \cdot \text{in.})\mathbf{k}$$

and

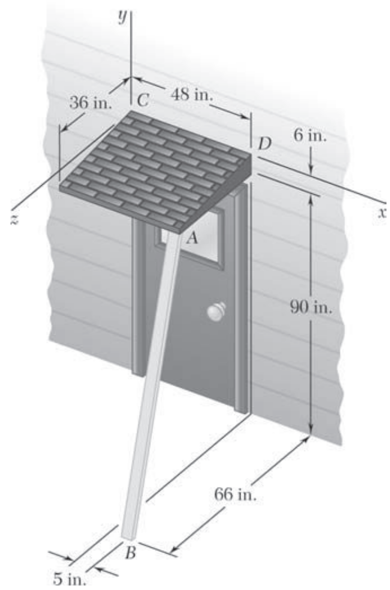
$$|\mathbf{M}_D| = \sqrt{(1836.00)^2 + (108.000)^2 + (18.0000)^2}$$

$$= 1839.26\text{ lb} \cdot \text{in.}$$

$$1839.26\text{ lb} \cdot \text{in} = (57\text{ lb})d$$

$$d = 32.268\text{ in.}$$

or $d = 32.3$ in. ◀

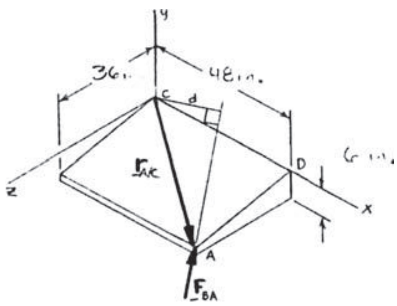


PROBLEM 3.30

In Problem 3.24, determine the perpendicular distance from Point C to a line drawn through Points A and B.

PROBLEM 3.24 A wooden board AB, which is used as a temporary prop to support a small roof, exerts at Point A of the roof a 57-lb force directed along BA. Determine the moment about C of that force.

SOLUTION



We have $\quad |\mathbf{M}_C| = F_{BA}d$

where $\quad d$ = perpendicular distance from C to line AB.

$$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}_{BA}$$

$$\mathbf{r}_{A/C} = (48 \text{ in.})\mathbf{i} - (6 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k}$$

$$\mathbf{F}_{BA} = \boldsymbol{\lambda}_{BA}F_{BA}$$

$$= \frac{(-(5 \text{ in.})\mathbf{i} + (90 \text{ in.})\mathbf{j} - (30 \text{ in.})\mathbf{k})}{\sqrt{(5)^2 + (90)^2 + (30)^2} \text{ in.}}(57 \text{ lb})$$

$$= -(3 \text{ lb})\mathbf{i} + (54 \text{ lb})\mathbf{j} - (18 \text{ lb})\mathbf{k}$$

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 48 & -6 & 36 \\ -3 & 54 & -18 \end{vmatrix} \text{lb} \cdot \text{in.}$$

$$= -(1836.00 \text{ lb} \cdot \text{in.})\mathbf{i} - (756.00 \text{ lb} \cdot \text{in.})\mathbf{j} + (2574.0 \text{ lb} \cdot \text{in.})\mathbf{k}$$

and

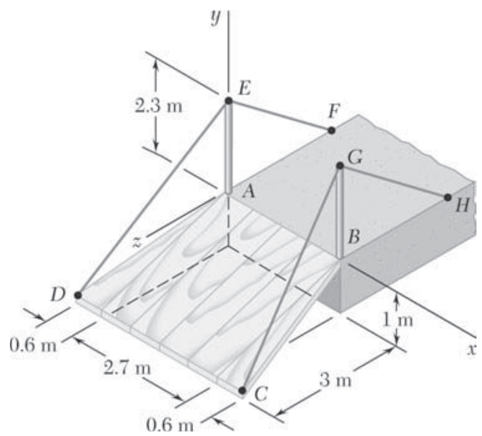
$$|\mathbf{M}_C| = \sqrt{(1836.00)^2 + (756.00)^2 + (2574.0)^2}$$

$$= 3250.8 \text{ lb} \cdot \text{in.}$$

$$3250.8 \text{ lb} \cdot \text{in.} = 57 \text{ lb}$$

$$d = 57.032 \text{ in.}$$

or $\quad d = 57.0$ in. ◀

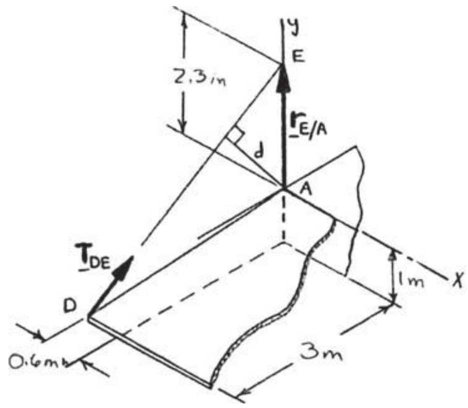


PROBLEM 3.31

In Problem 3.25, determine the perpendicular distance from Point A to portion DE of cable DEF.

PROBLEM 3.25 The ramp $ABCD$ is supported by cables at corners C and D. The tension in each of the cables is 810 N. Determine the moment about A of the force exerted by (*a*) the cable at D, (*b*) the cable at C.

SOLUTION



We have $\quad |\mathbf{M}_A| = T_{DE}d$

where $\quad d =$ perpendicular distance from A to line DE.

$$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$$

$$\mathbf{r}_{E/A} = (2.3\text{ m})\mathbf{j}$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE}\mathbf{T}_{DE}$$

$$= \frac{(0.6\text{ m})\mathbf{i} + (3.3\text{ m})\mathbf{j} - (3\text{ m})\mathbf{k}}{\sqrt{(0.6)^2 + (3.3)^2 + (3)^2}\text{ m}}(810\text{ N})$$

$$= (108\text{ N})\mathbf{i} + (594\text{ N})\mathbf{j} - (540\text{ N})\mathbf{k}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.3 & 0 \\ 108 & 594 & 540 \end{vmatrix}\text{N}\cdot\text{m}$$

$$= -(1242.00\text{ N}\cdot\text{m})\mathbf{i} - (248.00\text{ N}\cdot\text{m})\mathbf{k}$$

and $\quad |\mathbf{M}_A| = \sqrt{(1242.00)^2 + (248.00)^2}$

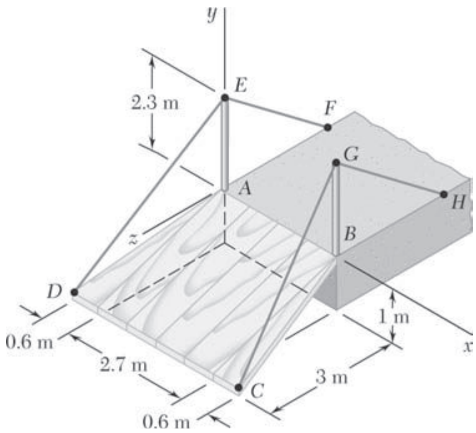
$$= 1266.52\text{ N}\cdot\text{m}$$

$$1266.52\text{ N}\cdot\text{m} = (810\text{ N})d$$

$$d = 1.56360\text{ m}$$

or $\quad d = 1.564\text{ m}$ ◀

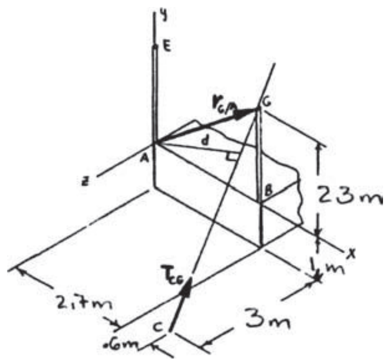
PROBLEM 3.32



In Problem 3.25, determine the perpendicular distance from Point A to a line drawn through Points C and G.

PROBLEM 3.25 The ramp $ABCD$ is supported by cables at corners C and D. The tension in each of the cables is 810 N. Determine the moment about A of the force exerted by (a) the cable at D, (b) the cable at C.

SOLUTION



We have $|\mathbf{M}_A| = T_{CG}d$

where $d =$ perpendicular distance from A to line CG.

$$\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$$

$$\mathbf{r}_{G/A} = (2.7\text{ m})\mathbf{i} + (2.3\text{ m})\mathbf{j}$$

$$\mathbf{T}_{CG} = \boldsymbol{\lambda}_{CG}T_{CG}$$

$$= \frac{-(0.6\text{ m})\mathbf{i} + (3.3\text{ m})\mathbf{j} - (3\text{ m})\mathbf{k}}{\sqrt{(0.6)^2 + (3.3)^2 + (3)^2}\text{ m}}(810\text{ N})$$

$$= -(108\text{ N})\mathbf{i} + (594\text{ N})\mathbf{j} - (540\text{ N})\mathbf{k}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 & 2.3 & 0 \\ -108 & 594 & -540 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(1242.00\text{ N}\cdot\text{m})\mathbf{i} + (1458.00\text{ N}\cdot\text{m})\mathbf{j} + (1852.00\text{ N}\cdot\text{m})\mathbf{k}$$

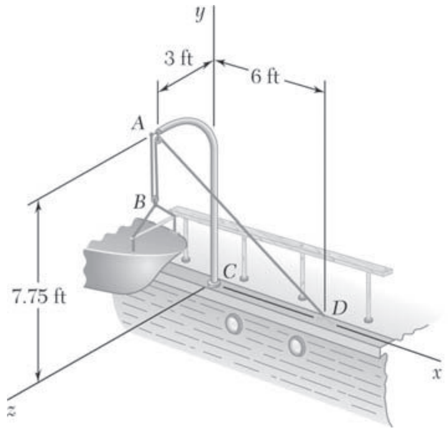
and $$|\mathbf{M}_A| = \sqrt{(1242.00)^2 + (1458.00)^2 + (1852.00)^2}$$

$$= 2664.3\text{ N}\cdot\text{m}$$

$$2664.3\text{ N}\cdot\text{m} = (810\text{ N})d$$

$$d = 3.2893\text{ m}$$

or $d = 3.29$ m ◀



PROBLEM 3.33

In Problem 3.26, determine the perpendicular distance from Point C to portion AD of the line $ABAD$.

PROBLEM 3.26 A small boat hangs from two davits, one of which is shown in the figure. The tension in line $ABAD$ is 82 lb. Determine the moment about C of the resultant force $\mathbf{R}_A$ exerted on the davit at A.

SOLUTION

First compute the moment about C of the force $\mathbf{F}_{DA}$ exerted by the line on D:

From Problem 3.26:

$$\mathbf{F}_{DA} = -\mathbf{F}_{AD}$$
$$= -(48 \text{ lb})\mathbf{i} + (62 \text{ lb})\mathbf{j} + (24 \text{ lb})\mathbf{k}$$
$$\mathbf{M}_C = \mathbf{r}_{D/C} \times \mathbf{F}_{DA}$$
$$= +(6 \text{ ft})\mathbf{i} \times [-(48 \text{ lb})\mathbf{i} + (62 \text{ lb})\mathbf{j} + (24 \text{ lb})\mathbf{k}]$$
$$= -(144 \text{ lb}\cdot\text{ft})\mathbf{j} + (372 \text{ lb}\cdot\text{ft})\mathbf{k}$$
$$\mathbf{M}_C = \sqrt{(144)^2 + (372)^2}$$
$$= 398.90 \text{ lb}\cdot\text{ft}$$

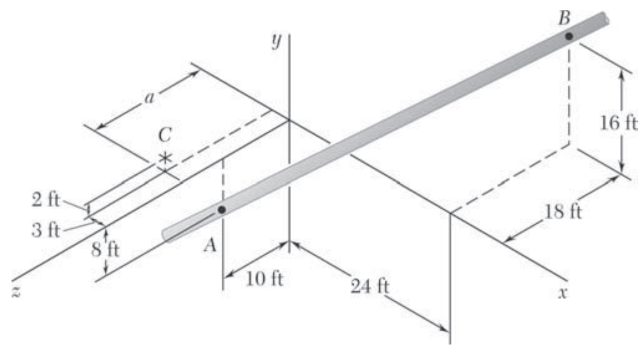
Then

$$\mathbf{M}_C = \mathbf{F}_{DA} d$$

Since

$$F_{DA} = 82 \text{ lb}$$
$$d = \frac{M_C}{F_{DA}}$$
$$= \frac{398.90 \text{ lb}\cdot\text{ft}}{82 \text{ lb}}$$

$d = 4.86$ ft ◀



PROBLEM 3.34

Determine the value of a that minimizes the perpendicular distance from Point C to a section of pipeline that passes through Points A and B.

SOLUTION

Assuming a force **F** acts along AB,

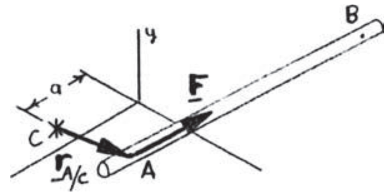
$$|\mathbf{M}_C| = |\mathbf{r}_{A/C} \times \mathbf{F}| = F(d)$$

Where $\quad d =$ perpendicular distance from C to line AB

$$\mathbf{F} = \boldsymbol{\lambda}_{AB} F$$

$$= \frac{(24\text{ ft})\,\mathbf{i} + (24\text{ ft})\,\mathbf{j} - (28)\,\mathbf{k}}{\sqrt{(24)^2 + (24)^2 + (18)^2}\text{ ft}} F$$

$$= \frac{F}{11}(6)\,\mathbf{i} + (6)\,\mathbf{j} - (7)\,\mathbf{k}$$



$$\mathbf{r}_{A/C} = (3\text{ ft})\mathbf{i} - (10\text{ ft})\mathbf{j} - (a - 10\text{ ft})\mathbf{k}$$

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -10 & 10a \\ 6 & 6 & -7 \end{vmatrix} \frac{F}{11}$$

$$= [(10 + 6a)\mathbf{i} + (81 - 6a)\mathbf{j} + 78\,\mathbf{k}]\frac{F}{11}$$

Since $\quad |\mathbf{M}_C| = \sqrt{|\mathbf{r}_{A/C} \times \mathbf{F}^2|} \quad$ or $\quad |\mathbf{r}_{A/C} \times \mathbf{F}^2| = (dF)^2$

$$\frac{1}{121}(10 + 6a)^2 + (81 - 6a)^2 + (78)^2 = d^2$$

Setting $\frac{d}{da}(d^2) = 0$ to find a to minimize d

$$\frac{1}{121}[2(6)(10 + 6a) + 2(-6)(81 - 6a)] = 0$$

Solving $\quad a = 5.92$ ft $\qquad$ or $\quad a = 5.92$ ft ◀

PROBLEM 3.35

Given the vectors $\mathbf{P} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{Q} = 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, and $\mathbf{S} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, compute the scalar products $\mathbf{P} \cdot \mathbf{Q}$, $\mathbf{P} \cdot \mathbf{S}$, and $\mathbf{Q} \cdot \mathbf{S}$.

SOLUTION

$$\begin{aligned}\mathbf{P} \cdot \mathbf{Q} &= (3\mathbf{i} - 1\mathbf{j} + 2\mathbf{k}) \cdot (4\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) \\ &= (3)(4) + (-1)(-5) + (2)(-3) \\ &= 1\end{aligned}$$

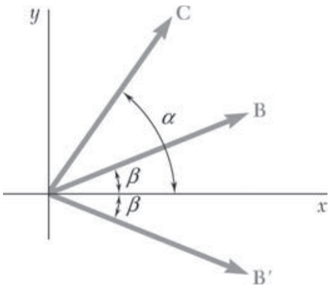
or $\mathbf{P} \cdot \mathbf{Q} = 1$ ◀

$$\begin{aligned}\mathbf{P} \cdot \mathbf{S} &= (3\mathbf{i} - 1\mathbf{j} + 2\mathbf{k}) \cdot (-2\mathbf{i} + 3\mathbf{j} - 1\mathbf{k}) \\ &= (3)(-2) + (-1)(3) + (2)(-1) \\ &= -11\end{aligned}$$

or $\mathbf{P} \cdot \mathbf{S} = -11$ ◀

$$\begin{aligned}\mathbf{Q} \cdot \mathbf{S} &= (4\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) \cdot (-2\mathbf{i} + 3\mathbf{j} - 1\mathbf{k}) \\ &= (4)(-2) + (5)(3) + (-3)(-1) \\ &= 10\end{aligned}$$

or $\mathbf{Q} \cdot \mathbf{S} = 10$ ◀

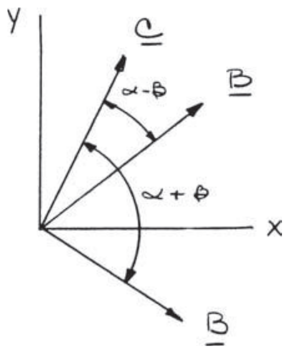


PROBLEM 3.36

Form the scalar products $\mathbf{B} \cdot \mathbf{C}$ and $\mathbf{B}' \cdot \mathbf{C}$, where $B = B'$, and use the results obtained to prove the identity

$$\cos a \cos \beta = \frac{1}{2}\cos(a+\beta) + \frac{1}{2}\cos(a-\beta).$$

SOLUTION



By definition $\quad \mathbf{B} \cdot \mathbf{C} = BC\cos(\alpha - \beta)$

where

$$\mathbf{B} = B[(\cos\beta)\mathbf{i} + (\sin\beta)\mathbf{j}]$$

$$\mathbf{C} = C[(\cos\alpha)\mathbf{i} + (\sin\alpha)\mathbf{j}]$$

$$(B\cos\beta)(C\cos\alpha) + (B\sin\beta)(C\sin\alpha) = BC\cos(\alpha-\beta)$$

or $$\cos\beta\cos\alpha + \sin\beta\sin\alpha = \cos(\alpha-\beta) \qquad (1)$$

By definition $\quad \mathbf{B}' \cdot \mathbf{C} = BC\cos(\alpha+\beta)$

where $$\mathbf{B}' = [(\cos\beta)\mathbf{i} - (\sin\beta)\mathbf{j}]$$

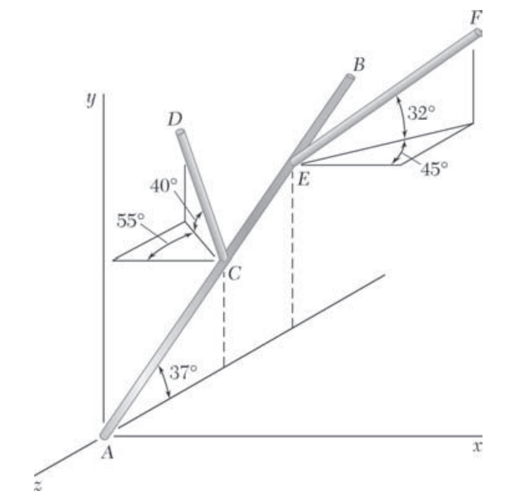
$$(B\cos\beta)(C\cos\alpha) + (-B\sin\beta)(C\sin\alpha) = BC\cos(\alpha+\beta)$$

or $$\cos\beta\cos\alpha - \sin\beta\sin\alpha = \cos(\alpha+\beta) \qquad (2)$$

Adding Equations (1) and (2),

$$2\cos\beta\cos\alpha = \cos(\alpha-\beta) + \cos(\alpha+\beta)$$

or $$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta) \blacktriangleleft$$



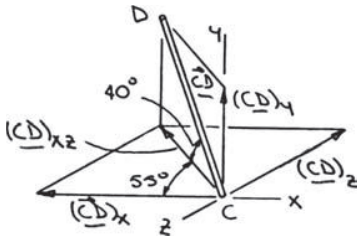
PROBLEM 3.37

Section *AB* of a pipeline lies in the *yz* plane and forms an angle of 37° with the *z* axis. Branch lines *CD* and *EF* join *AB* as shown. Determine the angle formed by pipes *AB* and *CD*.

SOLUTION

First note $\quad \overrightarrow{AB} = AB(\sin 37°\mathbf{j} - \cos 37°\mathbf{k})$

$$\mathbf{CD} = CD(-\cos 40° \cos 55°\mathbf{j} + \sin 40°\mathbf{j} - \cos 40° \sin 55°\mathbf{k})$$



Now $\quad \overrightarrow{AB} \cdot \overrightarrow{CD} = (AB)(CD)\cos\theta$

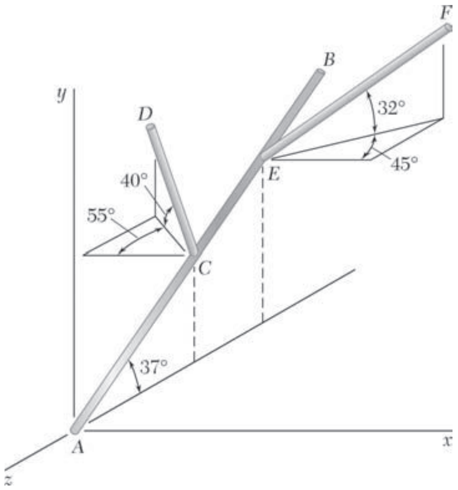
or $\quad AB(\sin 37°\mathbf{j} - \cos 37°\mathbf{k}) \cdot CD(-\cos 40° \cos 55°\mathbf{i} + \sin 40°\mathbf{j} - \cos 40° \sin 55°\mathbf{k})$

$$= (AB)(CD)\cos\theta$$

or $\quad \cos\theta = (\sin 37°)(\sin 40°) + (-\cos 37°)(-\cos 40° \sin 55°)$

$$= 0.88799$$

or $\theta = 27.4°$ ◀



PROBLEM 3.38

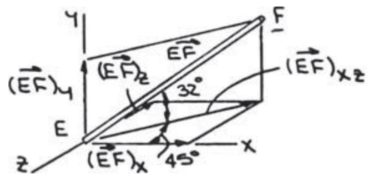
Section *AB* of a pipeline lies in the *yz* plane and forms an angle of 37° with the *z* axis. Branch lines *CD* and *EF* join *AB* as shown. Determine the angle formed by pipes *AB* and *EF*.

SOLUTION

First note

$$\overrightarrow{AB} = AB(\sin 37°\mathbf{j} - \cos 37°\mathbf{k})$$

$$\overrightarrow{EF} = EF(\cos 32°\cos 45°\mathbf{i} + \sin 32°\mathbf{j} - \cos 32°\sin 45°\mathbf{k})$$



Now

$$\overrightarrow{AB} \cdot \overrightarrow{EF} = (AB)(EF)\cos\theta$$

or

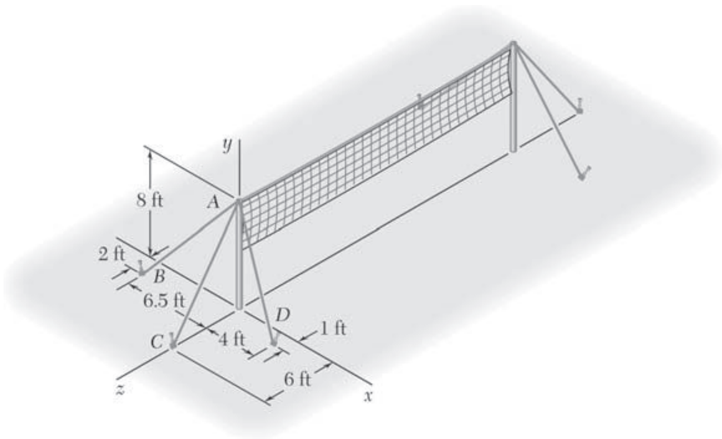
$$AB(\sin 37°\mathbf{j} - \cos 37°\mathbf{k}) \cdot EF(\cos 32°\cos 45°\mathbf{j} + \sin 32°\mathbf{j} - \cos 32°\sin 45°\mathbf{k})$$
$$= (AB)(EF)\cos\theta$$

or

$$\cos\theta = (\sin 37°)(\sin 32°) + (-\cos 37°)(-\cos 32°\sin 45°)$$
$$= 0.79782$$

or $\theta = 37.1°$ ◀

PROBLEM 3.39



Consider the volleyball net shown. Determine the angle formed by guy wires AB and AC.

SOLUTION

First note $\quad AB = \sqrt{(-6.5)^2 + (-8)^2 + (2)^2} = 10.5 \text{ ft}$

$$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2} = 10 \text{ ft}$$

and $\quad \overrightarrow{AB} = -(6.5 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$

$$\overrightarrow{AC} = -(8 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$

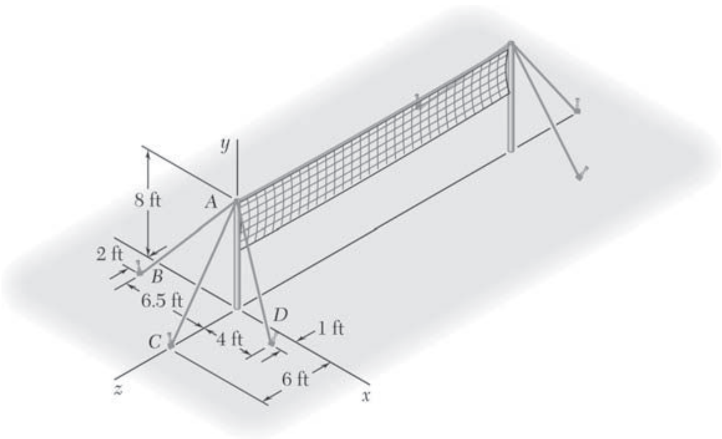
By definition $\quad \overrightarrow{AB} \cdot \overrightarrow{AC} = (AB)(AC)\cos\theta$

or $\quad (-6.5\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) \cdot (-8\mathbf{j} + 6\mathbf{k}) = (10.5)(10)\cos\theta$

$$(-6.5)(0) + (-8)(-8) + (2)(6) = 105\cos\theta$$

or $\quad \cos\theta = 0.72381 \qquad$ or $\theta = 43.6°$ ◀

PROBLEM 3.40



Consider the volleyball net shown. Determine the angle formed by guy wires AC and AD.

SOLUTION

First note

$$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2}$$
$$= 10 \text{ ft}$$
$$AD = \sqrt{(4)^2 + (-8)^2 + (1)^2}$$
$$= 9 \text{ ft}$$

and

$$\overrightarrow{AC} = -(8 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$
$$\overrightarrow{AD} = (4 \text{ ft})\mathbf{j} - (8 \text{ ft})\mathbf{j} + (1 \text{ ft})\mathbf{k}$$

By definition

$$\overrightarrow{AC} \cdot \overrightarrow{AD} = (AC)(AD)\cos\theta$$

or

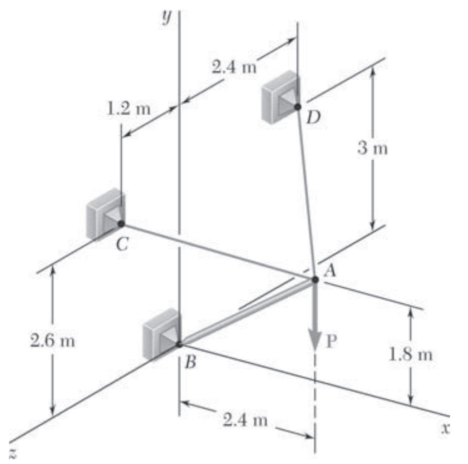
$$(-8\mathbf{j} + 6\mathbf{k}) \cdot (4\mathbf{i} - 8\mathbf{j} + \mathbf{k}) = (10)(9)\cos\theta$$
$$(0)(4) + (-8)(-8) + (6)(1) = 90\cos\theta$$

or

$$\cos\theta = 0.777\,78$$

or $\theta = 38.9°$ ◀

PROBLEM 3.41



Knowing that the tension in cable AC is 1260 N, determine (a) the angle between cable AC and the boom AB, (b) the projection on AB of the force exerted by cable AC at Point A.

SOLUTION

(a) First note

$$AC = \sqrt{(-2.4)^2 + (0.8)^2 + (1.2)^2}$$
$$= 2.8 \text{ m}$$
$$AB = \sqrt{(-2.4)^2 + (-1.8)^2 + (0)^2}$$
$$= 3.0 \text{ m}$$

and

$$\overrightarrow{AC} = -(2.4 \text{ m})\mathbf{i} + (0.8 \text{ m})\mathbf{j} + (1.2 \text{ m})\mathbf{k}$$
$$\overrightarrow{AB} = -(2.4 \text{ m})\mathbf{i} - (1.8 \text{ m})\mathbf{j}$$

By definition

$$\overrightarrow{AC} \cdot \overrightarrow{AB} = (AC)(AB)\cos\theta$$

or

$$(-2.4\mathbf{i} + 0.8\mathbf{j} + 1.2\mathbf{k}) \cdot (-2.4\mathbf{i} - 1.8\mathbf{j}) = (2.8)(30) \times \cos\theta$$

or

$$(-2.4)(-2.4) + (0.8)(-1.8) + (1.2)(0) = 8.4\cos\theta$$

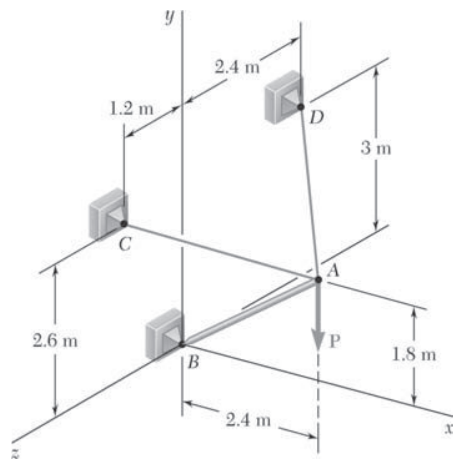
or

$$\cos\theta = 0.51429 \qquad \text{or} \quad \theta = 59.0° \blacktriangleleft$$

(b) We have

$$(T_{AC})_{AB} = \mathbf{T}_{AC} \cdot \boldsymbol{\lambda}_{AB}$$
$$= T_{AC}\cos\theta$$
$$= (1260 \text{ N})(0.51429)$$

$$\text{or} \quad (T_{AC})_{AB} = 648 \text{ N} \blacktriangleleft$$



PROBLEM 3.42

Knowing that the tension in cable AD is 405 N, determine (a) the angle between cable AD and the boom AB, (b) the projection on AB of the force exerted by cable AD at Point A.

SOLUTION

(a) First note

$$AD = \sqrt{(-2.4)^2 + (1.2)^2 + (-2.4)^2}$$
$$= 3.6 \text{ m}$$
$$AB = \sqrt{(-2.4)^2 + (-1.8)^2 + (0)^2}$$
$$= 3.0 \text{ m}$$

and

$$\overrightarrow{\mathbf{AD}} = -(2.4 \text{ m})\mathbf{i} + (1.2 \text{ m})\mathbf{j} - (2.4 \text{ m})\mathbf{k}$$
$$\mathbf{AB} = -(2.4 \text{ m})\mathbf{i} - (1.8 \text{ m})\mathbf{j}$$

By definition,

$$\overrightarrow{\mathbf{AD}} \cdot \mathbf{AB} = (AD)(AB)\cos\theta$$

$$(-2.4\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k}) \cdot (-2.4\mathbf{i} - 1.8\mathbf{j}) = (3.6)(3.0)\cos\theta$$
$$(-2.4)(-2.4) + (1.2)(-1.8) + (-2.4)(0) = 10.8\cos\theta$$

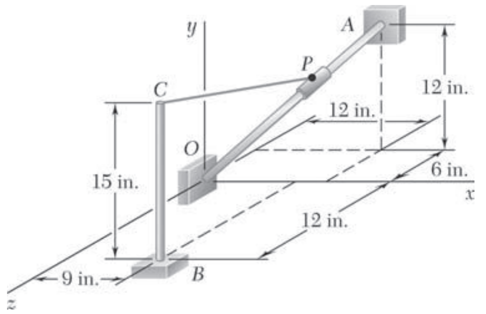
$$\cos\theta = \frac{1}{3}$$

$\theta = 70.5° \blacktriangleleft$

(b)

$$(T_{AD})_{AB} = \mathbf{T}_{AD} \cdot \boldsymbol{\lambda}_{AB}$$
$$= T_{AD}\cos\theta$$
$$= (405 \text{ N})\left(\frac{1}{3}\right)$$

$(T_{AD})_{AB} = 135.0 \text{ N} \blacktriangleleft$



PROBLEM 3.43

Slider P can move along rod OA. An elastic cord PC is attached to the slider and to the vertical member BC. Knowing that the distance from O to P is 6 in. and that the tension in the cord is 3 lb, determine (*a*) the angle between the elastic cord and the rod OA, (*b*) the projection on OA of the force exerted by cord PC at Point P.

SOLUTION

First note $$OA = \sqrt{(12)^2 + (12)^2 + (-6)^2} = 18 \text{ in.}$$

Then $$\boldsymbol{\lambda}_{OA} = \frac{\overrightarrow{OA}}{OA} = \frac{1}{18}(12\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$$
$$= \frac{1}{3}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

Now $$OP = 6 \text{ in.} \Rightarrow OP = \frac{1}{3}(OA)$$

The coordinates of Point P are (4 in., 4 in., −2 in.)

so that $$\overrightarrow{PC} = (5 \text{ in.})\mathbf{i} + (11 \text{ in.})\mathbf{j} + (14 \text{ in.})\mathbf{k}$$

and $$PC = \sqrt{(5)^2 + (11)^2 + (14)^2} = \sqrt{342} \text{ in.}$$

(*a*) We have $$\overrightarrow{PC} \cdot \boldsymbol{\lambda}_{OA} = (PC)\cos\theta$$

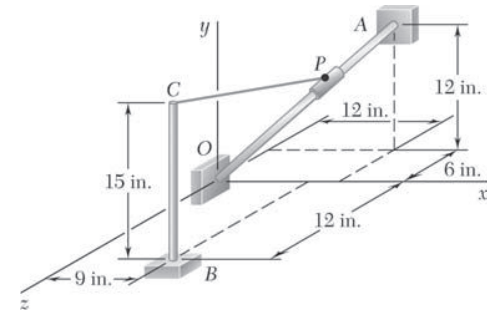
or $$(5\mathbf{i} + 11\mathbf{j} + 14\mathbf{k}) \cdot \frac{1}{3}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = \sqrt{342}\cos\theta$$

or $$\cos\theta = \frac{1}{3\sqrt{342}}[(5)(2) + (11)(2) + (14)(-1)]$$
$$= 0.324\,44$$

or $\theta = 71.1°$ ◀

(*b*) We have $$(T_{PC})_{OA} = \mathbf{T}_{PC} \cdot \boldsymbol{\lambda}_{OA}$$
$$= (T_{PC}\boldsymbol{\lambda}_{PC}) \cdot \boldsymbol{\lambda}_{OA}$$
$$= T_{PC}\frac{\overrightarrow{PC}}{PC} \cdot \boldsymbol{\lambda}_{OA}$$
$$= T_{PC}\cos\theta$$
$$= (3 \text{ lb})(0.324\,44)$$

or $(T_{PC})_{OA} = 0.973 \text{ lb}$ ◀



PROBLEM 3.44

Slider P can move along rod OA. An elastic cord PC is attached to the slider and to the vertical member BC. Determine the distance from O to P for which cord PC and rod OA are perpendicular.

SOLUTION

First note
$$OA = \sqrt{(12)^2 + (12)^2 + (-6)^2} = 18 \text{ in.}$$

Then
$$\boldsymbol{\lambda}_{OA} = \frac{\overrightarrow{OA}}{OA} = \frac{1}{18}(12\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$$
$$= \frac{1}{3}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

Let the coordinates of Point P be (x in., y in., z in.). Then
$$\overrightarrow{PC} = [(9 - x)\text{in.}]\mathbf{i} + (15 - y)\text{in.}]\mathbf{j} + [(12 - z)\text{in.}]\mathbf{k}$$

Also,
$$\overrightarrow{OP} = d_{OP}\boldsymbol{\lambda}_{OA} = \frac{d_{OP}}{3}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

and
$$\overrightarrow{OP} = (x \text{ in.})\mathbf{i} + (y \text{ in.})\mathbf{j} + (z \text{ in.})\mathbf{k}$$
$$x = \frac{2}{3}d_{OP} \quad y = \frac{2}{3}d_{OP} \quad z = \frac{1}{3}d_{OP}$$

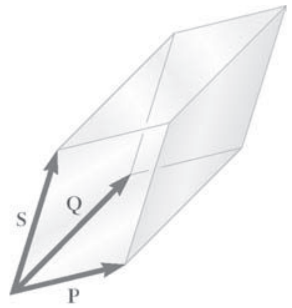
The requirement that OA and PC be perpendicular implies that
$$\boldsymbol{\lambda}_{OA} \cdot \overrightarrow{PC} = 0$$

or
$$\frac{1}{3}(2\mathbf{j} + 2\mathbf{j} - \mathbf{k}) \cdot [(9 - x)\mathbf{i} + (15 - y)\mathbf{j} + (12 - z)\mathbf{k}] = 0$$

or
$$(2)\left(9 - \frac{2}{3}d_{OP}\right) + (2)\left(15 - \frac{2}{3}d_{OP}\right) + (-1)\left[12 - \left(-\frac{1}{3}d_{OP}\right)\right] = 0$$

or $d_{OP} = 12.00$ in. ◀

PROBLEM 3.45



Determine the volume of the parallelepiped of Fig. 3.25 when (*a*) **P** = 4**i** − 3**j** + 2**k**, **Q** = −2**i** − 5**j** + **k**, and **S** = 7**i** + **j** − **k**, (*b*) **P** = 5**i** − **j** + 6**k**, **Q** = 2**i** + 3**j** + **k**, and **S** = −3**i** − 2**j** + 4**k**.

SOLUTION

Volume of a parallelepiped is found using the mixed triple product.

(*a*)

$$\text{Vol} = \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})$$
$$= \begin{vmatrix} 4 & -3 & 2 \\ -2 & -5 & 1 \\ 7 & 1 & -1 \end{vmatrix} \text{ in.}^3$$
$$= (20 - 21 - 4 + 70 + 6 - 4)$$
$$= 67$$

or $\text{Volume} = 67.0$ ◀

(*b*)

$$\text{Vol} = \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})$$
$$= \begin{vmatrix} 5 & -1 & 6 \\ 2 & 3 & 1 \\ -3 & -2 & 4 \end{vmatrix} \text{ in.}^3$$
$$= (60 + 3 - 24 + 54 + 8 + 10)$$
$$= 111$$

or $\text{Volume} = 111.0$ ◀

PROBLEM 3.46

Given the vectors $\mathbf{P} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}, \mathbf{Q} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k},$ and $\mathbf{S} = S_x\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, determine the value of S_x for which the three vectors are coplanar.

SOLUTION

If **P**, **Q**, and **S** are coplanar, then **P** must be perpendicular to $(\mathbf{Q} \times \mathbf{S})$.

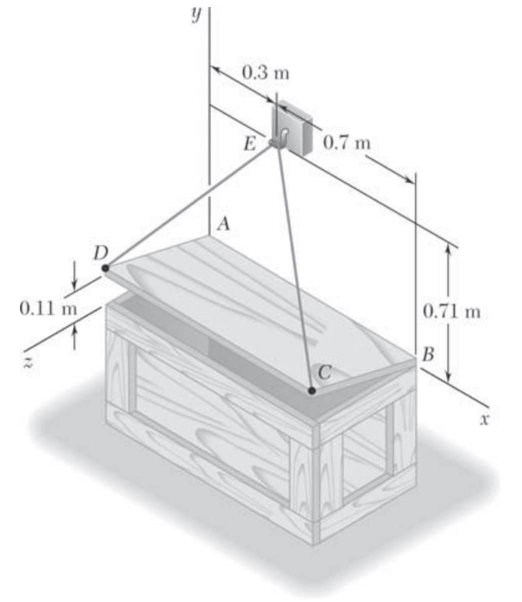
$$\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S}) = 0$$

(or, the volume of a parallelepiped defined by **P**, **Q**, and **S** is zero).

Then
$$\begin{vmatrix} 4 & -2 & 3 \\ 2 & 4 & -5 \\ S_x & -1 & 2 \end{vmatrix} = 0$$

or
$$32 + 10S_x - 6 - 20 + 8 - 12S_x = 0$$

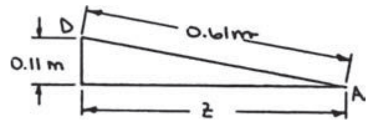
$S_x = 7$ ◀



PROBLEM 3.47

The 0.61×1.00-m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 66 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *D*.

SOLUTION



First note $$z = \sqrt{(0.61)^2 - (0.11)^2}$$
$$= 0.60 \text{ m}$$

Then $$d_{DE} = \sqrt{(0.3)^2 + (0.6)^2 + (-0.6)^2}$$
$$= 0.9 \text{ m}$$

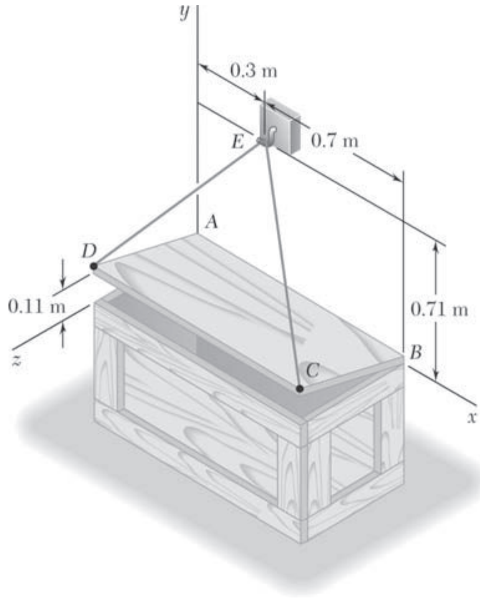
and $$\mathbf{T}_{DE} = \frac{66 \text{ N}}{0.9}(0.3\mathbf{i} + 0.6\mathbf{j} - 0.6\mathbf{k})$$
$$= 22[(1 \text{ N})\mathbf{i} + (2 \text{ N})\mathbf{j} - (2 \text{ N})\mathbf{k}]$$

Now $$\mathbf{M}_A = \mathbf{r}_{D/A} \times \mathbf{T}_{DE}$$

where $$\mathbf{r}_{D/A} = (0.11 \text{ m})\mathbf{j} + (0.60 \text{ m})\mathbf{k}$$

Then $$\mathbf{M}_A = 22\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.11 & 0.60 \\ 1 & 2 & -2 \end{vmatrix}$$
$$= 22[(-0.22 - 1.20)\mathbf{i} + 0.60\mathbf{j} - 0.11\mathbf{k}]$$
$$= -(31.24 \text{ N}\cdot\text{m})\mathbf{i} + (13.20 \text{ N}\cdot\text{m})\mathbf{j} - (2.42 \text{ N}\cdot\text{m})\mathbf{k}$$

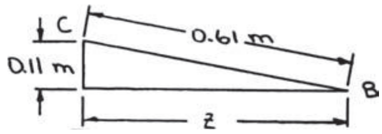
$$M_x = -31.2 \text{ N}\cdot\text{m}, \quad M_y = 13.20 \text{ N}\cdot\text{m}, \quad M_z = -2.42 \text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 3.48

The 0.61×1.00-m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 66 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *C*.

SOLUTION



First note $$z = \sqrt{(0.61)^2 - (0.11)^2}$$
$$= 0.60 \text{ m}$$

Then $$d_{CE} = \sqrt{(-0.7)^2 + (0.6)^2 + (-0.6)^2}$$
$$= 1.1 \text{ m}$$

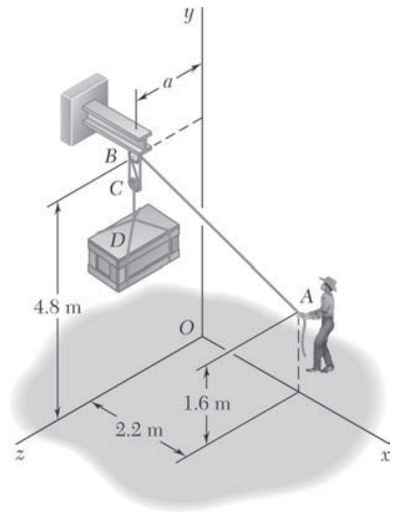
and $$\mathbf{T}_{CE} = \frac{66 \text{ N}}{1.1}(-0.7\mathbf{i} + 0.6\mathbf{j} - 0.6\mathbf{k})$$
$$= 6[-(7 \text{ N})\mathbf{i} + (6 \text{ N})\mathbf{j} - (6 \text{ N})\mathbf{k}]$$

Now $$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{CE}$$

where $$\mathbf{r}_{E/A} = (0.3 \text{ m})\mathbf{i} + (0.71 \text{ m})\mathbf{j}$$

Then $$\mathbf{M}_A = 6\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.71 & 0 \\ -7 & 6 & -6 \end{vmatrix}$$
$$= 6[-4.26\mathbf{i} + 1.8\mathbf{j} + (1.8 + 4.97)\mathbf{k}]$$
$$= -(25.56 \text{ N}\cdot\text{m})\mathbf{i} + (10.80 \text{ N}\cdot\text{m})\mathbf{j} + (40.62 \text{ N}\cdot\text{m})\mathbf{k}$$

$$M_x = -25.6 \text{ N}\cdot\text{m}, \quad M_y = 10.80 \text{ N}\cdot\text{m}, \quad M_z = 40.6 \text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 3.49

To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B. Knowing that the moments about the y and the z axes of the force exerted at B by portion AB of the rope are, respectively, 120 N · m and −460 N · m, determine the distance a.

SOLUTION

First note $$\overrightarrow{BA} = (2.2\text{ m})\mathbf{i} - (3.2\text{ m})\mathbf{j} - (a\text{ m})\mathbf{k}$$

Now $$\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{T}_{BA}$$

where $$\mathbf{r}_{A/D} = (2.2\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j}$$

$$\mathbf{T}_{BA} = \frac{T_{BA}}{d_{BA}}(2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k})\ (\text{N})$$

Then $$\mathbf{M}_D = \frac{T_{BA}}{d_{BA}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.2 & 1.6 & 0 \\ 2.2 & -3.2 & -a \end{vmatrix}$$

$$= \frac{T_{BA}}{d_{BA}}\{-1.6a\,\mathbf{i} + 2.2a\,\mathbf{j} + [(2.2)(-3.2) - (1.6)(2.2)]\mathbf{k}\}$$

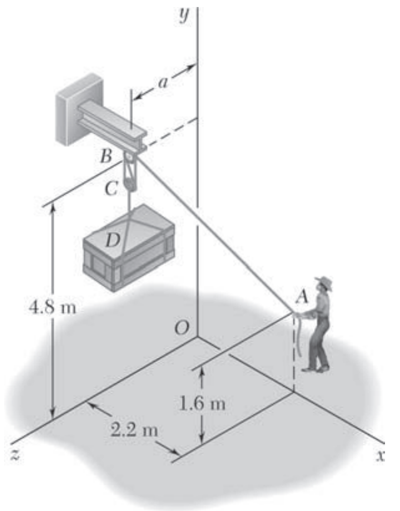
Thus $$M_y = 2.2\frac{T_{BA}}{d_{BA}}a \qquad (\text{N}\cdot\text{m})$$

$$M_z = -10.56\frac{T_{BA}}{d_{BA}} \qquad (\text{N}\cdot\text{m})$$

Then forming the ratio $$\frac{M_y}{M_z}$$

$$\frac{120\text{ N}\cdot\text{m}}{-460\text{ N}\cdot\text{m}} = \frac{2.2\frac{T_{BA}}{d_{BA}}\ (\text{N}\cdot\text{m})}{-10.56\frac{T_{BA}}{d_{BA}}\ (\text{N}\cdot\text{m})}$$

or $a = 1.252$ m ◀



PROBLEM 3.50

To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B. Knowing that the man applies a 195-N force to end A of the rope and that the moment of that force about the y axis is 132 N · m, determine the distance a.

SOLUTION

First note
$$d_{BA} = \sqrt{(2.2)^2 + (-3.2)^2 + (-a)^2}$$
$$= \sqrt{15.08 + a^2} \text{ m}$$

and
$$\mathbf{T}_{BA} = \frac{195 \text{ N}}{d_{BA}}(2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k})$$

Now
$$M_y = \mathbf{j} \cdot (\mathbf{r}_{A/D} \times \mathbf{T}_{BA})$$

where
$$\mathbf{r}_{A/0} = (2.2 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j}$$

Then
$$M_y = \frac{195}{d_{BA}}\begin{vmatrix} 0 & 1 & 0 \\ 2.2 & 1.6 & 0 \\ 2.2 & -3.2 & -a \end{vmatrix}$$
$$= \frac{195}{d_{BA}}(2.2a) \text{ (N} \cdot \text{m)}$$

Substituting for M_y and d_{BA}

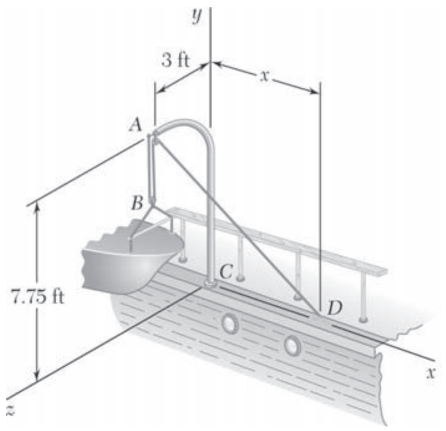
$$132 \text{ N} \cdot \text{m} = \frac{195}{\sqrt{15.08 + a^2}}(2.2a)$$

or
$$0.30769\sqrt{15.08 + a^2} = a$$

Squaring both sides of the equation

$$0.094675(15.08 + a^2) = a^2$$

or $a = 1.256$ m ◀



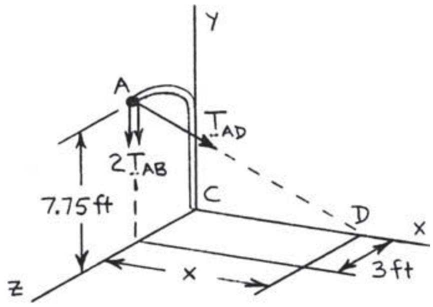
PROBLEM 3.51

A small boat hangs from two davits, one of which is shown in the figure. It is known that the moment about the z axis of the resultant force $\mathbf{R}_A$ exerted on the davit at A must not exceed 279 lb · ft in absolute value. Determine the largest allowable tension in line $ABAD$ when $x = 6$ ft.

SOLUTION

First note $\qquad \mathbf{R}_A = 2\mathbf{T}_{AB} + \mathbf{T}_{AD}$

Also note that only $\mathbf{T}_{AD}$ will contribute to the moment about the z axis.



Now $\qquad AD = \sqrt{(6)^2 + (-7.75)^2 + (-3)^2}$

$$= 10.25 \text{ ft}$$

Then, $\qquad \mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD}$

$$= \frac{T}{10.25}(6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})$$

Now $\qquad M_z = \mathbf{k}\cdot(\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$

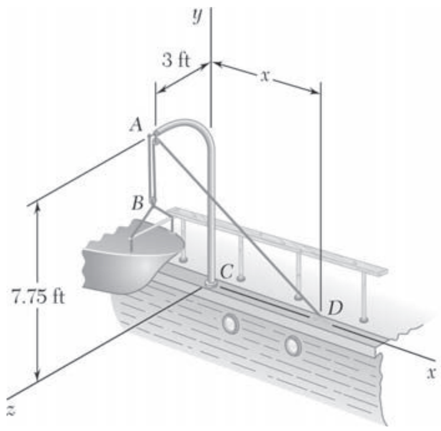
where $\qquad \mathbf{r}_{A/C} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$

Then for T_{max},

$$279 = \left| \frac{T_{max}}{10.25} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 7.75 & 3 \\ 6 & -7.75 & -3 \end{vmatrix} \right|$$

$$= \frac{T_{max}}{10.25}\left| -(1)(7.75)(6) \right|$$

or $T_{max} = 61.5$ lb ◀



PROBLEM 3.52

For the davit of Problem 3.51, determine the largest allowable distance x when the tension in line $ABAD$ is 60 lb.

SOLUTION

From the solution of Problem 3.51, $\mathbf{T}_{AD}$ is now

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD}$$

$$= \frac{60 \text{ lb}}{\sqrt{x^2 + (-7.75)^2 + (-3)^2}}(x\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})$$

Then $M_z = \mathbf{k} \cdot (\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$ becomes

$$279 = \left| \frac{60}{\sqrt{x^2 + (-7.75)^2 + (-3)^2}} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 7.75 & 3 \\ x & -7.75 & -3 \end{vmatrix} \right|$$

$$279 = \frac{60}{\sqrt{x^2 + 69.0625}} \left| -(1)(7.75)(x) \right|$$

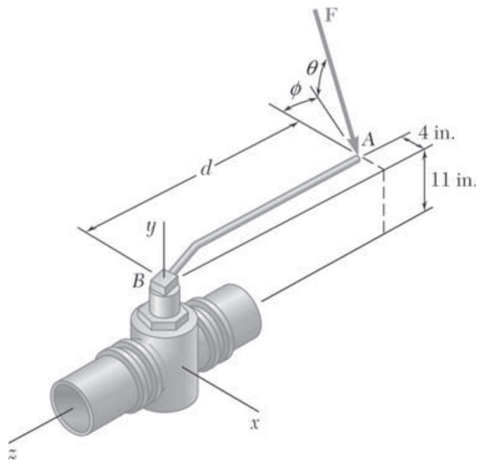
$$279\sqrt{x^2 + 69.0625} = 465x$$

$$0.6\sqrt{x^2 + 69.0625} = x$$

Squaring both sides: $0.36x^2 + 24.8625 = x^2$

$$x^2 = 38.848$$

$x = 6.23$ ft ◀



PROBLEM 3.53

To loosen a frozen valve, a force **F** of magnitude 70 lb is applied to the handle of the valve. Knowing that $\theta = 25°$, $M_x = -61 \text{ lb} \cdot \text{ft}$, and $M_z = -43 \text{ lb} \cdot \text{ft}$, determine ϕ and d.

SOLUTION

We have $\quad \Sigma \mathbf{M}_O: \quad \mathbf{r}_{A/O} \times \mathbf{F} = \mathbf{M}_O$

where $\quad \mathbf{r}_{A/O} = -(4 \text{ in.})\mathbf{i} + (11 \text{ in.})\mathbf{j} - (d)\mathbf{k}$

$$\mathbf{F} = F(\cos\theta\cos\phi\mathbf{i} - \sin\theta\mathbf{j} + \cos\theta\sin\phi\mathbf{k})$$

For $\quad F = 70 \text{ lb}, \quad \theta = 25°$

$$\mathbf{F} = (70 \text{ lb})[(0.90631\cos\phi)\mathbf{i} - 0.42262\mathbf{j} + (0.90631\sin\phi)\mathbf{k}]$$

$$\mathbf{M}_O = (70 \text{ lb})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 11 & -d \\ -0.90631\cos\phi & -0.42262 & 0.90631\sin\phi \end{vmatrix} \text{ in.}$$

$$= (70 \text{ lb})[(9.9694\sin\phi - 0.42262d)\mathbf{i} + (-0.90631d\cos\phi + 3.6252\sin\phi)\mathbf{j} + (1.69048 - 9.9694\cos\phi)\mathbf{k}] \text{ in.}$$

and

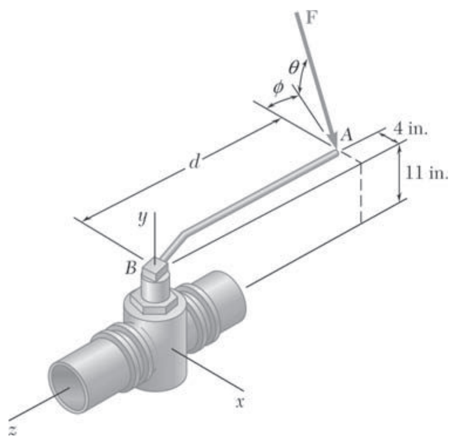
$$M_x = (70 \text{ lb})(9.9694\sin\phi - 0.42262d)\text{in.} = -(61 \text{ lb} \cdot \text{ft})(12 \text{ in./ft}) \qquad (1)$$

$$M_y = (70 \text{ lb})(-0.90631d\cos\phi + 3.6252\sin\phi) \text{ in.} \qquad (2)$$

$$M_z = (70 \text{ lb})(1.69048 - 9.9694\cos\phi) \text{ in.} = -43 \text{ lb} \cdot \text{ft}(12 \text{ in./ft}) \qquad (3)$$

From Equation (3) $\quad \phi = \cos^{-1}\left(\dfrac{634.33}{697.86}\right) = 24.636° \qquad$ or $\quad \phi = 24.6°$ ◀

From Equation (1) $\quad d = \left(\dfrac{1022.90}{29.583}\right) = 34.577 \text{ in.} \qquad$ or $\quad d = 34.6 \text{ in.}$ ◀



PROBLEM 3.54

When a force **F** is applied to the handle of the valve shown, its moments about the x and z axes are, respectively, $M_x = -77\text{ lb}\cdot\text{ft}$ and $M_z = -81\text{ lb}\cdot\text{ft}$. For $d = 27$ in., determine the moment M_y of **F** about the y axis.

SOLUTION

We have $\quad \Sigma\mathbf{M}_O: \quad \mathbf{r}_{A/O} \times \mathbf{F} = \mathbf{M}_O$

Where $\quad \mathbf{r}_{A/O} = -(4\text{ in.})\mathbf{i} + (11\text{ in.})\mathbf{j} - (27\text{ in.})\mathbf{k}$

$$\mathbf{F} = F(\cos\theta\cos\phi\mathbf{i} - \sin\theta\mathbf{j} + \cos\theta\sin\phi\mathbf{k})$$

$$\mathbf{M}_O = F\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 11 & -27 \\ \cos\theta\cos\phi & -\sin\theta & \cos\theta\sin\phi \end{vmatrix}\text{ lb}\cdot\text{in.}$$

$$= F[(11\cos\theta\sin\phi - 27\sin\theta)\mathbf{i} + (-27\cos\theta\cos\phi + 4\cos\theta\sin\phi)\mathbf{j} + (4\sin\theta - 11\cos\theta\cos\phi)\mathbf{k}](\text{lb}\cdot\text{in.})$$

and

$$M_x = F(11\cos\theta\sin\phi - 27\sin\theta)(\text{lb}\cdot\text{in.}) \tag{1}$$

$$M_y = F(-27\cos\theta\cos\phi + 4\cos\theta\sin\phi)\ (\text{lb}\cdot\text{in.}) \tag{2}$$

$$M_z = F(4\sin\theta - 11\cos\theta\cos\phi)\ (\text{lb}\cdot\text{in.}) \tag{3}$$

Now, Equation (1)

$$\cos\theta\sin\phi = \frac{1}{11}\left(\frac{M_x}{F} + 27\sin\theta\right) \tag{4}$$

and Equation (3)

$$\cos\theta\cos\phi = \frac{1}{11}\left(4\sin\theta - \frac{M_z}{F}\right) \tag{5}$$

Substituting Equations (4) and (5) into Equation (2),

$$M_y = F\left\{-27\left[\frac{1}{11}\left(4\sin\theta - \frac{M_z}{F}\right)\right] + 4\left[\frac{1}{11}\left(\frac{M_x}{F} + 27\sin\theta\right)\right]\right\}$$

or

$$M_y = \frac{1}{11}(27M_z + 4M_x)$$

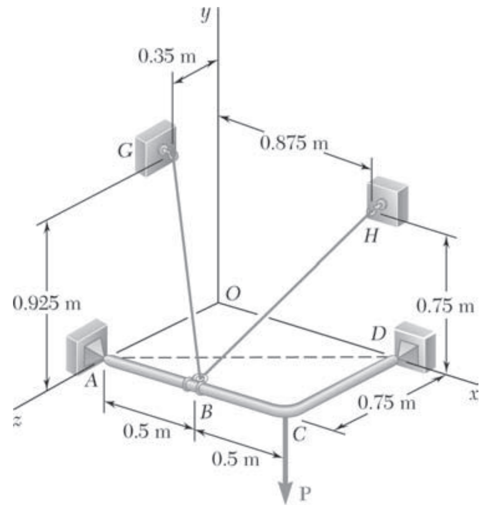
PROBLEM 3.54 (Continued)

Noting that the ratios $\frac{27}{11}$ and $\frac{4}{11}$ are the ratios of lengths, have

$$M_y = \frac{27}{11}(-81\text{ lb}\cdot\text{ft}) + \frac{4}{11}(-77\text{ lb}\cdot\text{ft})$$

$$= 226.82\text{ lb}\cdot\text{ft}$$

or $M_y = -227\text{ lb}\cdot\text{ft}$ ◀



PROBLEM 3.55

The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

SOLUTION

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH})$$

Where
$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

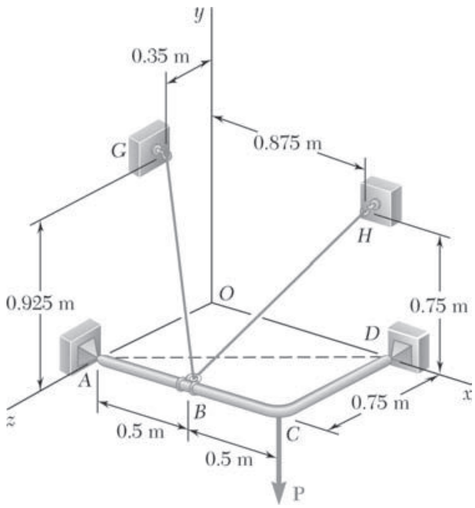
$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{i}$$

and
$$d_{BH} = \sqrt{(0.375)^2 + (0.75)^2 + (-0.75)^2}$$
$$= 1.125 \text{ m}$$

Then
$$\mathbf{T}_{BH} = \frac{450 \text{ N}}{1.125}(0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k})$$
$$= (150 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} - (300 \text{ N})\mathbf{k}$$

Finally
$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix}$$
$$= \frac{1}{5}[(-3)(0.5)(300)]$$

or $M_{AD} = -90.0 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.56

In Problem 3.55, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BG* of the cable.

SOLUTION

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG})$$

Where

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{j}$$

and

$$BG = \sqrt{(-0.5)^2 + (0.925)^2 + (-0.4)^2}$$

$$= 1.125 \text{ m}$$

Then

$$\overline{\mathbf{T}}_{BG} = \frac{450 \text{ N}}{1.125}(-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k})$$

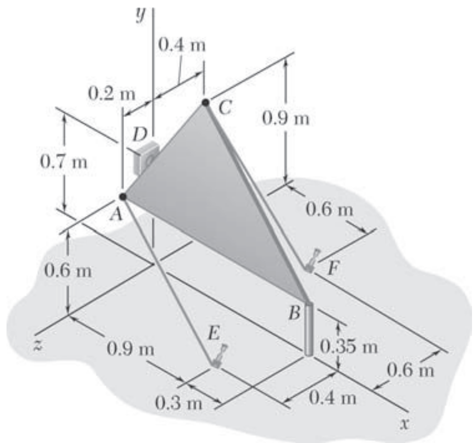
$$= -(200 \text{ N})\mathbf{i} + (370 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k}$$

Finally

$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ -200 & 370 & -160 \end{vmatrix}$$

$$= \frac{1}{5}[(-3)(0.5)(370)]$$

$M_{AD} = -111.0 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.57

The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 55 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

First note $$d_{AE} = \sqrt{(0.9)^2 + (-0.6)^2 + (0.2)^2} = 1.1 \text{ m}$$

Then $$\mathbf{T}_{AE} = \frac{55 \text{ N}}{1.1}(0.9\mathbf{i} - 0.6\mathbf{j} + 0.2\mathbf{k})$$
$$= 5[(9 \text{ N})\mathbf{i} - (6 \text{ N})\mathbf{j} + (2 \text{ N})\mathbf{k}]$$

Also $$DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2}$$
$$= 1.25 \text{ m}$$

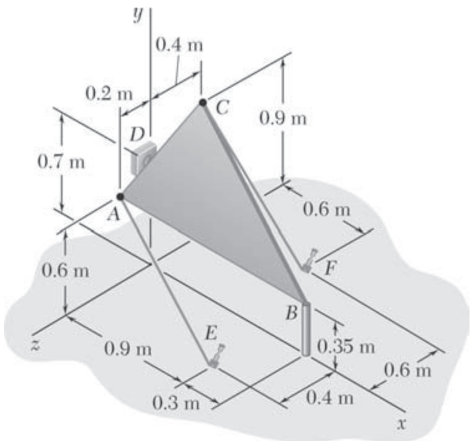
Then $$\boldsymbol{\lambda}_{DB} = \frac{\overrightarrow{DB}}{DB}$$
$$= \frac{1}{1.25}(1.2\mathbf{i} - 0.35\mathbf{j})$$
$$= \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Now $$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r}_{A/D} \times \mathbf{T}_{AE})$$

where $$\mathbf{T}_{DA} = -(0.1 \text{ m})\mathbf{j} + (0.2 \text{ m})\mathbf{k}$$

Then $$M_{DB} = \frac{1}{25}(5)\begin{vmatrix} 24 & -7 & 0 \\ 0 & -0.1 & 0.2 \\ 9 & -6 & 2 \end{vmatrix}$$
$$= \frac{1}{5}(-4.8 - 12.6 + 28.8)$$

or $M_{DB} = 2.28 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.58

The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 33 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

First note
$$d_{CF} = \sqrt{(0.6)^2 + (-0.9)^2 + (-0.2)^2} = 1.1 \text{ m}$$

Then
$$\mathbf{T}_{CF} = \frac{33 \text{ N}}{1.1}(0.6\mathbf{i} - 0.9\mathbf{j} + 0.2\mathbf{k})$$
$$= 3[(6 \text{ N})\mathbf{i} - (9 \text{ N})\mathbf{j} - (2 \text{ N})\mathbf{k}]$$

Also
$$DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2}$$
$$= 1.25 \text{ m}$$

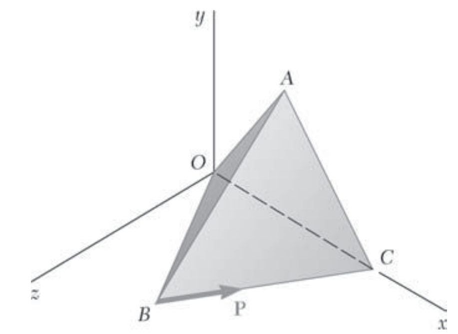
Then
$$\boldsymbol{\lambda}_{DB} = \frac{\overrightarrow{DB}}{DB}$$
$$= \frac{1}{1.25}(1.2\mathbf{i} - 0.35\mathbf{j})$$
$$= \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Now
$$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r}_{C/D} \times \mathbf{T}_{CF})$$

where
$$\mathbf{r}_{C/D} = (0.2 \text{ m})\mathbf{j} - (0.4 \text{ m})\mathbf{k}$$

Then
$$M_{DB} = \frac{1}{25}(3)\begin{vmatrix} 24 & -7 & 0 \\ 0 & 0.2 & -0.4 \\ 6 & -9 & -2 \end{vmatrix}$$
$$= \frac{3}{25}(-9.6 + 16.8 - 86.4)$$

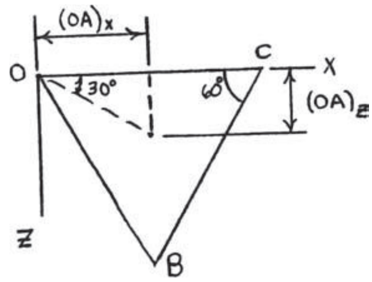
or $M_{DB} = -9.50 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.59

A regular tetrahedron has six edges of length a. A force **P** is directed as shown along edge BC. Determine the moment of **P** about edge OA.

SOLUTION



We have $$M_{OA} = \lambda_{OA} \cdot (\mathbf{r}_{C/O} \times \mathbf{P})$$

where

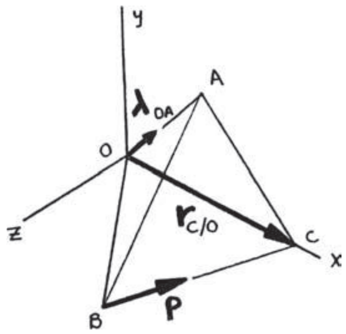
From triangle OBC $$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

Since $$(OA)^2 = (OA)_x^2 + (OA)_y^2 + (OA_z)^2$$

or $$a^2 = \left(\frac{a}{2}\right)^2 + (OA)_y^2 + \left(\frac{a}{2\sqrt{3}}\right)^2$$

$$(OA)_y = \sqrt{a^2 - \frac{a^2}{4} - \frac{a^2}{12}} = a\sqrt{\frac{2}{3}}$$



Then $$\mathbf{r}_{A/O} = \frac{a}{2}\mathbf{i} + a\sqrt{\frac{2}{3}}\,\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}$$

and $$\boldsymbol{\lambda}_{OA} = \frac{1}{2}\mathbf{i} + \sqrt{\frac{2}{3}}\mathbf{j} + \frac{1}{2\sqrt{3}}\mathbf{k}$$

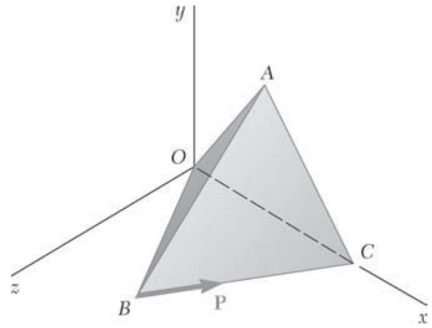
$$\mathbf{P} = \lambda_{BC} P = \frac{(a\sin 30°)\mathbf{i} - (a\cos 30°)\mathbf{k}}{a}(P) = \frac{P}{2}(\mathbf{i} - \sqrt{3}\mathbf{k})$$

$$\mathbf{r}_{C/O} = a\mathbf{i}$$

$$M_{OA} = \begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ 1 & 0 & -\sqrt{3} \end{vmatrix} (a)\left(\frac{P}{2}\right)$$

$$= \frac{aP}{2}\left(-\sqrt{\frac{2}{3}}\right)(1)(-\sqrt{3}) = \frac{aP}{\sqrt{2}}$$

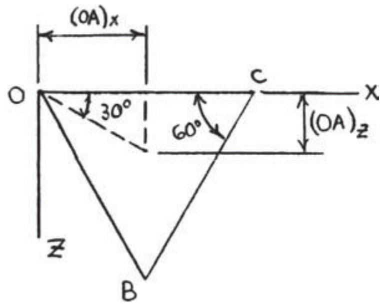
$$M_{OA} = \frac{aP}{\sqrt{2}} \blacktriangleleft$$



PROBLEM 3.60

A regular tetrahedron has six edges of length a. (a) Show that two opposite edges, such as OA and BC, are perpendicular to each other. (b) Use this property and the result obtained in Problem 3.59 to determine the perpendicular distance between edges OA and BC.

SOLUTION



(a) For edge OA to be perpendicular to edge BC,

$$\overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

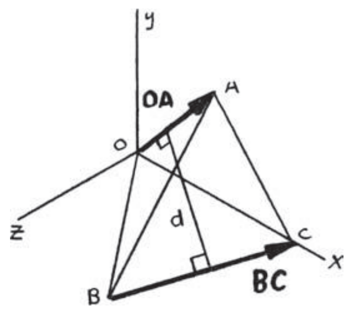
where

From triangle OBC

$$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

$$\overrightarrow{OA} = \left(\frac{a}{2}\right)\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}$$



and

$$\overrightarrow{BC} = (a\sin 30°)\mathbf{i} - (a\cos 30°)\mathbf{k}$$

$$= \frac{a}{2}\mathbf{i} - \frac{a\sqrt{3}}{2}\mathbf{k} = \frac{a}{2}(\mathbf{i} - \sqrt{3}\,\mathbf{k})$$

Then

$$\left[\frac{a}{2}\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}\right] \cdot (\mathbf{i} - \sqrt{3}\mathbf{k})\frac{a}{2} = 0$$

or

$$\frac{a^2}{4} + (OA)_y(0) - \frac{a^2}{4} = 0$$

$$\overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

so that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$. ◀

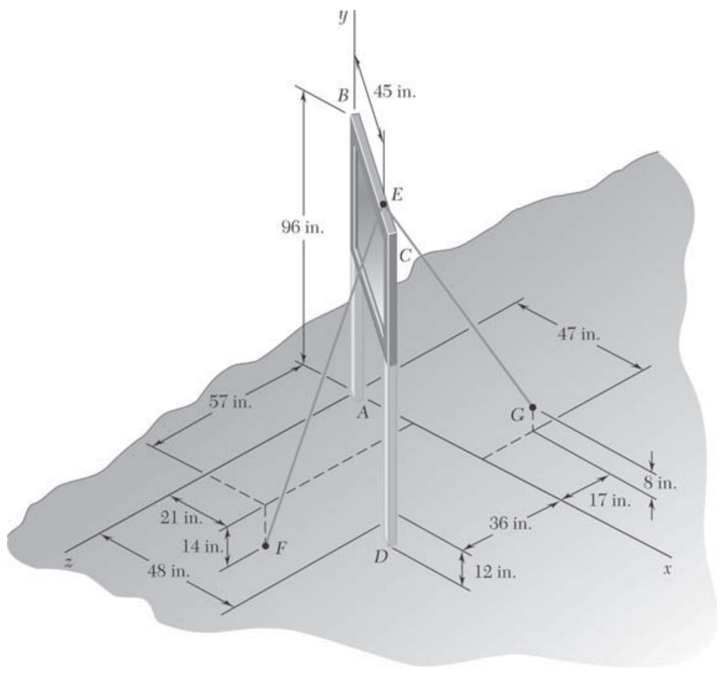
(b) Have $M_{OA} = Pd$, with P acting along BC and d the perpendicular distance from $\overrightarrow{OA}$ to $\overrightarrow{BC}$.

From the results of Problem 3.57

$$M_{OA} = \frac{Pa}{\sqrt{2}}$$

$$\frac{Pa}{\sqrt{2}} = Pd$$

or $d = \dfrac{a}{\sqrt{2}}$ ◀



PROBLEM 3.61

A sign erected on uneven ground is guyed by cables EF and EG. If the force exerted by cable EF at E is 46 lb, determine the moment of that force about the line joining Points A and D.

SOLUTION

First note that $BC = \sqrt{(48)^2 + (36)^2} = 60$ in. and that $\frac{BE}{BC} = \frac{45}{60} = \frac{3}{4}$. The coordinates of Point E are then $\left(\frac{3}{4} \times 48,\ 96,\ \frac{3}{4} \times 36\right)$ or (36 in., 96 in., 27 in.). Then

$$d_{EF} = \sqrt{(-15)^2 + (-110)^2 + (30)^2}$$
$$= 115 \text{ in.}$$

Then
$$\mathbf{T}_{EF} = \frac{46 \text{ lb}}{115}(-15\mathbf{i} - 110\mathbf{j} + 30\mathbf{k})$$
$$= 2[-(3 \text{ lb})\mathbf{i} - (22 \text{ lb})\mathbf{j} + (6 \text{ lb})\mathbf{k}]$$

Also
$$AD = \sqrt{(48)^2 + (-12)^2 + (36)^2}$$
$$= 12\sqrt{26} \text{ in.}$$

Then
$$\boldsymbol{\lambda}_{AD} = \frac{\overrightarrow{AD}}{AD}$$
$$= \frac{1}{12\sqrt{26}}(48\mathbf{i} - 12\mathbf{j} + 36\mathbf{k})$$
$$= \frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

Now
$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{E/A} \times \mathbf{T}_{EF})$$

PROBLEM 3.61 (Continued)

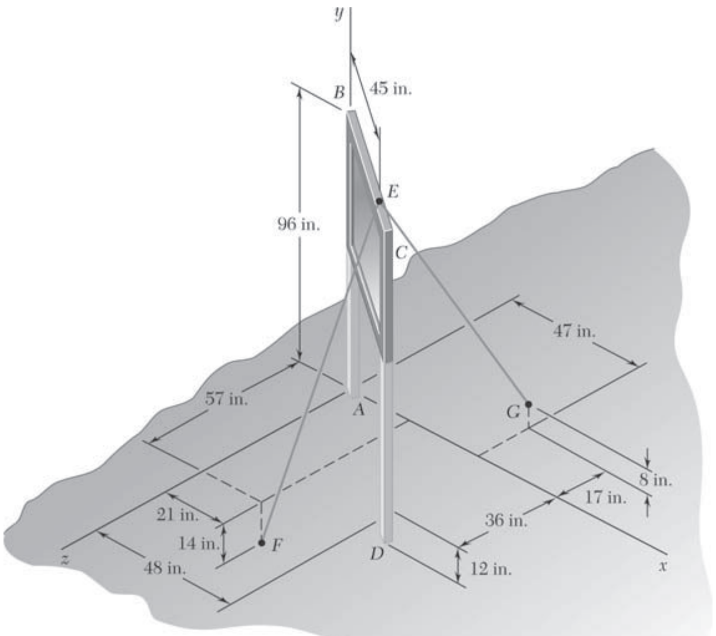
where $\quad \mathbf{r}_{E/A} = (36 \text{ in.})\mathbf{i} + (96 \text{ in.})\mathbf{j} + (27 \text{ in.})\mathbf{k}$

Then

$$M_{AD} = \frac{1}{\sqrt{26}}(2)\begin{vmatrix} 4 & -1 & 3 \\ 36 & 96 & 27 \\ -3 & -22 & 6 \end{vmatrix}$$

$$= \frac{2}{\sqrt{26}}(2304 + 81 - 2376 + 864 + 216 + 2376)$$

or $\quad M_{AD} = 1359 \text{ lb} \cdot \text{in.}$ ◀



PROBLEM 3.62

A sign erected on uneven ground is guyed by cables *EF* and *EG*. If the force exerted by cable *EG* at *E* is 54 lb, determine the moment of that force about the line joining Points *A* and *D*.

SOLUTION

First note that $BC = \sqrt{(48)^2 + (36)^2} = 60$ in. and that $\frac{BE}{BC} = \frac{45}{60} = \frac{3}{4}$. The coordinates of Point *E* are then $\left(\frac{3}{4}\times 48,\ 96,\ \frac{3}{4}\times 36\right)$ or (36 in., 96 in., 27 in.). Then

$$d_{EG} = \sqrt{(11)^2 + (-88)^2 + (-44)^2}$$
$$= 99 \text{ in.}$$

Then
$$\mathbf{T}_{EG} = \frac{54 \text{ lb}}{99}(11\mathbf{i} - 88\mathbf{j} - 44\mathbf{k})$$
$$= 6[(1 \text{ lb})\mathbf{i} - (8 \text{ lb})\mathbf{j} - (4 \text{ lb})\mathbf{k}]$$

Also
$$AD = \sqrt{(48)^2 + (-12)^2 + (36)^2}$$
$$= 12\sqrt{26} \text{ in.}$$

Then
$$\boldsymbol{\lambda}_{AD} = \frac{\overrightarrow{AD}}{AD}$$
$$= \frac{1}{12\sqrt{26}}(48\mathbf{i} - 12\mathbf{j} + 36\mathbf{k})$$
$$= \frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

Now
$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{E/A} \times \mathbf{T}_{EG})$$

PROBLEM 3.62 (Continued)

where $\mathbf{r}_{E/A} = (36\text{ in.})\mathbf{i} + (96\text{ in.})\mathbf{j} + (27\text{ in.})\mathbf{k}$

Then
$$M_{AD} = \frac{1}{\sqrt{26}}(6)\begin{vmatrix} 4 & -1 & 3 \\ 36 & 96 & 27 \\ 1 & -8 & -4 \end{vmatrix}$$

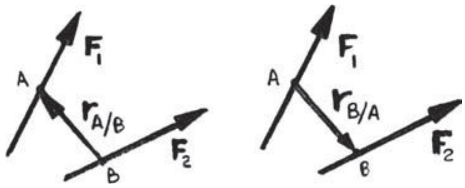
$$= \frac{6}{\sqrt{26}}(-1536 - 27 - 864 - 288 - 144 + 864)$$

or $M_{AD} = -2350\text{ lb}\cdot\text{in.}$ ◀

PROBLEM 3.63

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in space have the same magnitude F. Prove that the moment of $\mathbf{F}_1$ about the line of action of $\mathbf{F}_2$ is equal to the moment of $\mathbf{F}_2$ about the line of action of $\mathbf{F}_1$.

SOLUTION



First note that $\mathbf{F}_1 = F_1\lambda_1$ and $\mathbf{F}_2 = F_2\lambda_2$

Let $M_1 =$ moment of $\mathbf{F}_2$ about the line of action of $\mathbf{M}_1$ and $M_2 =$ moment of $\mathbf{F}_1$ about the line of action of $\mathbf{M}_2$

Now, by definition

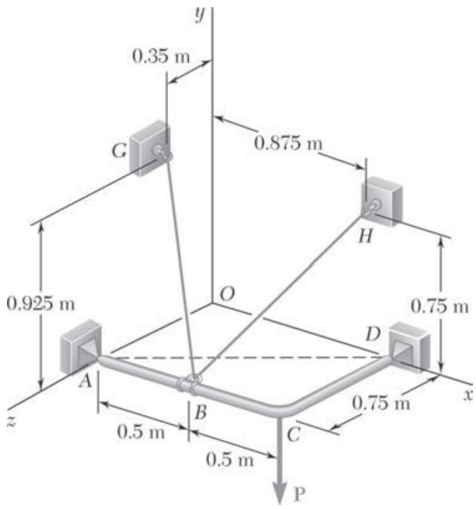
$$M_1 = \lambda_1 \cdot (\mathbf{r}_{B/A} \times \mathbf{F}_2)$$
$$= \lambda_1 \cdot (\mathbf{r}_{B/A} \times \lambda_2)F_2$$
$$M_2 = \lambda_2 \cdot (\mathbf{r}_{A/B} \times \mathbf{F}_1)$$
$$= \lambda_2 \cdot (\mathbf{r}_{A/B} \times \lambda_1)F_1$$

Since $F_1 = F_2 = F$ and $\mathbf{r}_{A/B} = -\mathbf{r}_{B/A}$

$$M_1 = \lambda_1 \cdot (\mathbf{r}_{B/A} \times \lambda_2)F$$
$$M_2 = \lambda_2 \cdot (-\mathbf{r}_{B/A} \times \lambda_1)F$$

Using Equation (3.39) $\lambda_1 \cdot (\mathbf{r}_{B/A} \times \lambda_2) = \lambda_2 \cdot (-\mathbf{r}_{B/A} \times \lambda_1)$

so that $M_2 = \lambda_1 \cdot (\mathbf{r}_{B/A} \times \lambda_2)F$ $\quad M_{12} = M_{21}$ ◀



PROBLEM 3.64

In Problem 3.55, determine the perpendicular distance between portion *BH* of the cable and the diagonal *AD*.

PROBLEM 3.55 The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

SOLUTION

From the solution to Problem 3.55: $T_{BH} = 450\text{ N}$

$$\mathbf{T}_{BH} = (150\text{ N})\mathbf{i} + (300\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}$$

$$|M_{AD}| = 90.0\text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{BH}$ will contribute to the moment of $\mathbf{T}_{BH}$ about line $\overrightarrow{AD}$.

Now
$$(T_{BH})_{\text{parallel}} = \mathbf{T}_{BH} \cdot \boldsymbol{\lambda}_{AD}$$
$$= (150\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}) \cdot \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$
$$= \frac{1}{5}[(150)(4) + (-300)(-3)]$$
$$= 300\text{ N}$$

Also
$$\mathbf{T}_{BH} = (\mathbf{T}_{BH})_{\text{parallel}} + (\mathbf{T}_{BH})_{\text{perpendicular}}$$

so that
$$(T_{BH})_{\text{perpendicular}} = \sqrt{(450)^2 - (300)^2} = 335.41\text{ N}$$

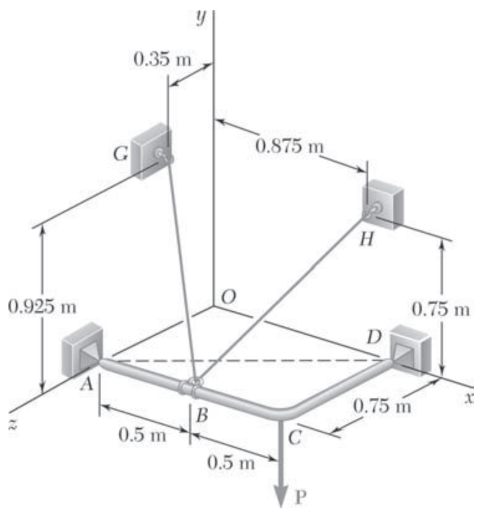
Since $\boldsymbol{\lambda}_{AD}$ and $(\mathbf{T}_{BH})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{AD} = d(T_{BH})_{\text{perpendicular}}$$

or
$$90.0\text{ N}\cdot\text{m} = d(335.41\text{ N})$$

$$d = 0.26833\text{ m}$$

$d = 0.268\text{ m}$ ◀



PROBLEM 3.65

In Problem 3.56, determine the perpendicular distance between portion *BG* of the cable and the diagonal *AD*.

PROBLEM 3.56 In Problem 3.55, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BG* of the cable.

SOLUTION

From the solution to Problem 3.56: $\mathbf{T}_{BG} = 450\text{ N}$

$$\mathbf{T}_{BG} = -(200\text{ N})\mathbf{i} + (370\text{ N})\mathbf{j} - (160\text{ N})\mathbf{k}$$

$$|M_{AD}| = 111\text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{BG}$ will contribute to the moment of $\mathbf{T}_{BG}$ about line $\overline{AD}$.

Now $$(T_{BG})_{\text{parallel}} = \mathbf{T}_{BG}\cdot\boldsymbol{\lambda}_{AD}$$

$$= (-200\mathbf{i} + 370\mathbf{j} - 160\mathbf{k})\cdot\frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$= \frac{1}{5}[(-200)(4) + (-160)(-3)]$$

$$= -64\text{ N}$$

Also $$\overline{\mathbf{T}}_{BG} = (\mathbf{T}_{BG})_{\text{parallel}} + (\mathbf{T}_{BG})_{\text{perpendicular}}$$

so that $$(\mathbf{T}_{BG})_{\text{perpendicular}} = \sqrt{(450)^2 - (-64)^2} = 445.43\text{ N}$$

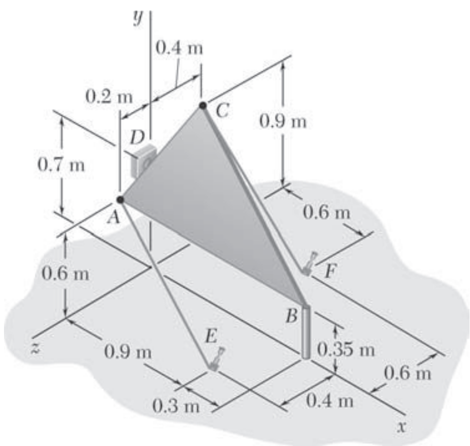
Since $\boldsymbol{\lambda}_{AD}$ and $(\mathbf{T}_{BG})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{AD} = d(T_{BG})_{\text{perpendicular}}$$

or $$111\text{ N}\cdot\text{m} = d(445.43\text{ N})$$

$$d = 0.24920\text{ m}$$

$d = 0.249\text{ m}$ ◀



PROBLEM 3.66

In Problem 3.57, determine the perpendicular distance between cable *AE* and the line joining Points *D* and *B*.

PROBLEM 3.57 The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 55 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

From the solution to Problem 3.57 $\quad \mathbf{T}_{AE} = 55 \text{ N}$

$$\mathbf{T}_{AE} = 5[(9 \text{ N})\mathbf{i} - (6 \text{ N})\mathbf{j} + (2 \text{ N})\mathbf{k}]$$

$$|M_{DB}| = 2.28 \text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{AE}$ will contribute to the moment of $\mathbf{T}_{AE}$ about line $\overrightarrow{DB}$.

Now
$$(T_{AE})_{\text{parallel}} = \mathbf{T}_{AE} \cdot \boldsymbol{\lambda}_{DB}$$
$$= 5(9\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$
$$= \frac{1}{5}[(9)(24) + (-6)(-7)]$$
$$= 51.6 \text{ N}$$

Also
$$\mathbf{T}_{AE} = (\mathbf{T}_{AE})_{\text{parallel}} + (\mathbf{T}_{AE})_{\text{perpendicular}}$$

so that
$$(\mathbf{T}_{AE})_{\text{perpendicular}} = \sqrt{(55)^2 + (51.6)^2} = 19.0379 \text{ N}$$

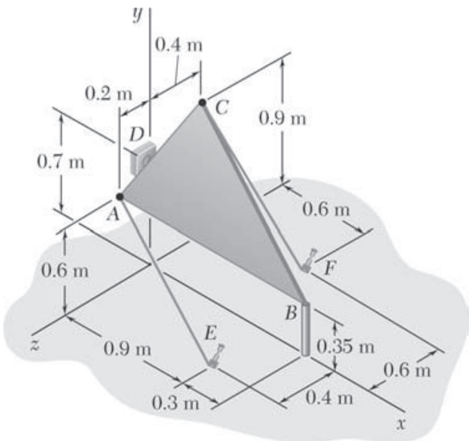
Since $\boldsymbol{\lambda}_{DB}$ and $(\mathbf{T}_{AE})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{DB} = d(T_{AE})_{\text{perpendicular}}$$

or
$$2.28 \text{ N}\cdot\text{m} = d(19.0379 \text{ N})$$

$$d = 0.119761 \qquad\qquad d = 0.1198 \text{ m} \blacktriangleleft$$

PROBLEM 3.67



In Problem 3.58, determine the perpendicular distance between cable *CF* and the line joining Points *D* and *B*.

PROBLEM 3.58 The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 33 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

From the solution to Problem 3.58 $\quad T_{CF} = 33 \text{ N}$

$$\mathbf{T}_{CF} = 3[(6 \text{ N})\mathbf{i} - (9 \text{ N})\mathbf{j} - (2 \text{ N})\mathbf{k}]$$

$$|M_{DB}| = 9.50 \text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{CF}$ will contribute to the moment of $\mathbf{T}_{CF}$ about line $\overrightarrow{DB}$.

Now
$$(\mathbf{T}_{CF})_{\text{parallel}} = \mathbf{T}_{CF} \cdot \boldsymbol{\lambda}_{DB}$$
$$= 3(6\mathbf{i} - 9\mathbf{j} - 2\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$
$$= \frac{3}{25}[(6)(24) + (-9)(-7)]$$
$$= 24.84 \text{ N}$$

Also
$$\mathbf{T}_{CF} = (\mathbf{T}_{CF})_{\text{parallel}} + (\mathbf{T}_{CF})_{\text{perpendicular}}$$

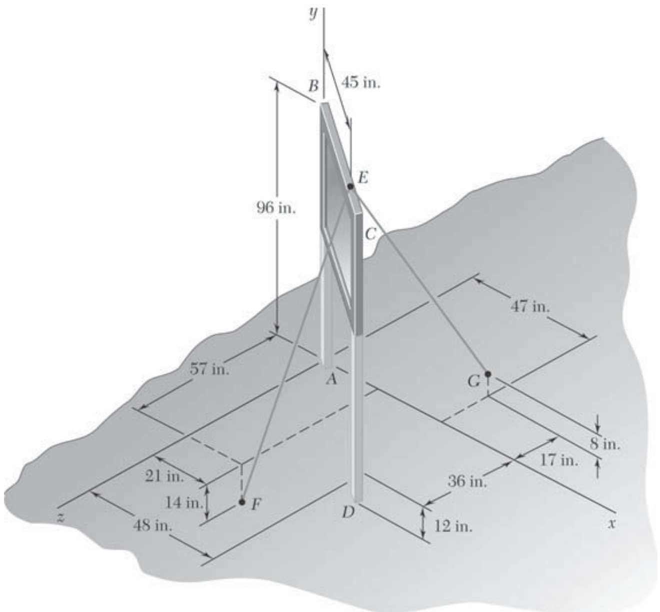
so that
$$(\mathbf{T}_{CF})_{\text{perpendicular}} = \sqrt{(33)^2 - (24.84)^2}$$
$$= 21.725 \text{ N}$$

Since $\boldsymbol{\lambda}_{DB}$ and $(\mathbf{T}_{CF})_{\text{perpendicular}}$ are perpendicular, it follows that

$$|M_{DB}| = d(T_{CF})_{\text{perpendicular}}$$

or
$$9.50 \text{ N}\cdot\text{m} = d \times 21.725 \text{ N}$$

or $d = 0.437$ m ◀



PROBLEM 3.68

In Problem 3.61, determine the perpendicular distance between cable *EF* and the line joining Points *A* and *D*.

PROBLEM 3.61 A sign erected on uneven ground is guyed by cables *EF* and *EG*. If the force exerted by cable *EF* at *E* is 46 lb, determine the moment of that force about the line joining Points *A* and *D*.

SOLUTION

From the solution to Problem 3.61

$$T_{EF} = 46 \text{ lb}$$

$$\mathbf{T}_{EF} = 2[-(3 \text{ lb})\mathbf{i} - (22 \text{ lb})\mathbf{j} + (6 \text{ lb})\mathbf{k}]$$

$$|M_{AD}| = 1359 \text{ lb} \cdot \text{in.}$$

$$\boldsymbol{\lambda}_{AD} = \frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{EF}$ will contribute to the moment of $\mathbf{T}_{EF}$ about line $\overrightarrow{AD}$.

Now
$$(T_{EF})_{\text{parallel}} = \mathbf{T}_{EF} \cdot \boldsymbol{\lambda}_{AD}$$

$$= 2(-3\mathbf{i} - 22\mathbf{j} + 6\mathbf{k}) \cdot \frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

$$= \frac{2}{\sqrt{26}}[(-3)(4) + (-22)(-1) + (6)(3)]$$

$$= 10.9825 \text{ lb}$$

Also
$$\mathbf{T}_{EF} = (\mathbf{T}_{EF})_{\text{parallel}} + (\mathbf{T}_{EF})_{\text{perpendicular}}$$

so that
$$(\mathbf{T}_{EF})_{\text{perpendicular}} = \sqrt{(46)^2 - (10.9825)^2} = 44.670 \text{ lb}$$

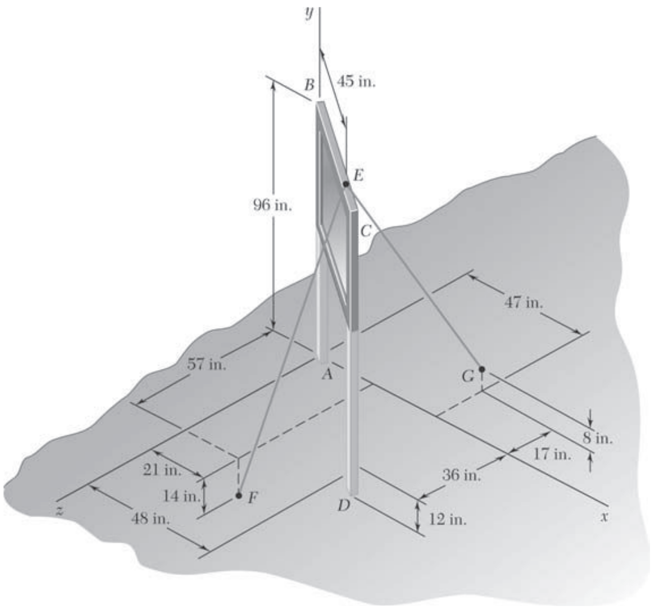
Since $\boldsymbol{\lambda}_{AD}$ and $(\mathbf{T}_{EF})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{AD} = d(T_{EF})_{\text{perpendicular}}$$

or
$$1359 \text{ lb} \cdot \text{in.} = d \times 44.670 \text{ lb}$$

or $d = 30.4$ in. ◀

PROBLEM 3.69



In Problem 3.62, determine the perpendicular distance between cable EG and the line joining Points A and D.

PROBLEM 3.62 A sign erected on uneven ground is guyed by cables EF and EG. If the force exerted by cable EG at E is 54 lb, determine the moment of that force about the line joining Points A and D.

SOLUTION

From the solution to Problem 3.62

$$T_{EG} = 54 \text{ lb}$$

$$\mathbf{T}_{EG} = 6[(1 \text{ lb})\mathbf{i} - (8 \text{ lb})\mathbf{j} - (4 \text{ lb})\mathbf{k}]$$

$$|M_{AD}| = 2350 \text{ lb} \cdot \text{in.}$$

$$\boldsymbol{\lambda}_{AD} = \frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{EG}$ will contribute to the moment of $\mathbf{T}_{EG}$ about line $\overrightarrow{AD}$.

Now

$$(T_{EG})_{\text{parallel}} = \mathbf{T}_{EG} \cdot \boldsymbol{\lambda}_{AD}$$

$$= 6(\mathbf{i} - 8\mathbf{j} - 4\mathbf{k}) \cdot \frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

$$= \frac{6}{\sqrt{26}}[(1)(4) + (-8)(-1) + (-4)(3)] = 0$$

Thus,

$$(\mathbf{T}_{EG})_{\text{perpendicular}} = \mathbf{T}_{EG} = 54 \text{ lb}$$

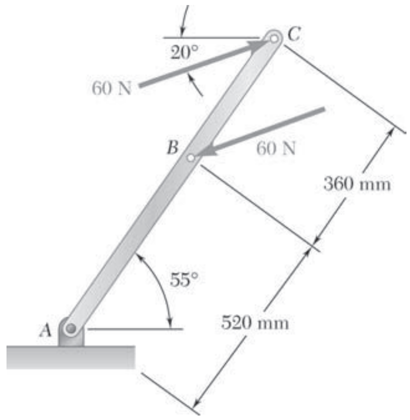
Since $\boldsymbol{\lambda}_{AD}$ and $(\mathbf{T}_{EG})_{\text{perpendicular}}$ are perpendicular, it follows that

$$|M_{AD}| = d(T_{EG})_{\text{perpendicular}}$$

or

$$2350 \text{ lb} \cdot \text{in.} = d \times 54 \text{ lb}$$

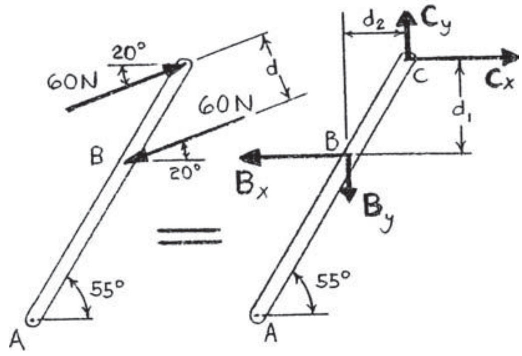
or $d = 43.5$ in. ◀



PROBLEM 3.70

Two parallel 60-N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces (*a*) by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples, (*b*) by using the perpendicular distance between the two forces, (*c*) by summing the moments of the two forces about Point *A*.

SOLUTION



(*a*) We have $\Sigma \mathbf{M}_B: \quad -d_1 C_x + d_2 C_y = \mathbf{M}$

where

$$d_1 = (0.360 \text{ m})\sin 55° = 0.29489 \text{ m}$$

$$d_2 = (0.360 \text{ m})\sin 55° = 0.20649 \text{ m}$$

$$C_x = (60 \text{ N})\cos 20° = 56.382 \text{ N}$$

$$C_y = (60 \text{ N})\sin 20° = 20.521 \text{ N}$$

$$\mathbf{M} = -(0.29489 \text{ m})(56.382 \text{ N})\mathbf{k} + (0.20649 \text{ m})(20.521 \text{ N})\mathbf{k}$$

$$= -(12.3893 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m} \curvearrowright$ ◀

(*b*) We have

$$\mathbf{M} = Fd(-\mathbf{k})$$

$$= 60 \text{ N}[(0.360 \text{ m})\sin(55° - 20°)](-\mathbf{k})$$

$$= -(12.3893 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m} \curvearrowright$ ◀

(*c*) We have $\Sigma \mathbf{M}_A: \quad \Sigma(\mathbf{r}_A \times \mathbf{F}) = \mathbf{r}_{B/A} \times \mathbf{F}_B + \mathbf{r}_{C/A} \times \mathbf{F}_C = \mathbf{M}$

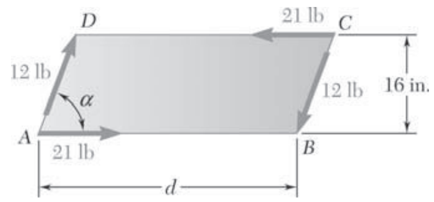
$$M = (0.520 \text{ m})(60 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55° & \sin 55° & 0 \\ -\cos 20° & -\sin 20° & 0 \end{vmatrix}$$

$$+ (0.800 \text{ m})(60 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55° & \sin 55° & 0 \\ \cos 20° & \sin 20° & 0 \end{vmatrix}$$

$$= (17.8956 \text{ N}\cdot\text{m} - 30.285 \text{ N}\cdot\text{m})\mathbf{k}$$

$$= -(12.3892 \text{ N}\cdot\text{m})\mathbf{k}$$

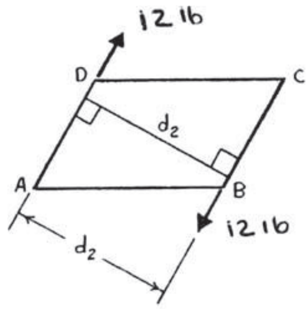
or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m} \curvearrowright$ ◀



PROBLEM 3.71

A plate in the shape of a parallelogram is acted upon by two couples. Determine (*a*) the moment of the couple formed by the two 21-lb forces, (*b*) the perpendicular distance between the 12-lb forces if the resultant of the two couples is zero, (*c*) the value of α if the resultant couple is $72\ \text{lb}\cdot\text{in.}$ clockwise and *d* is 42 in.

SOLUTION



(*a*) We have $M_1 = d_1 F_1$

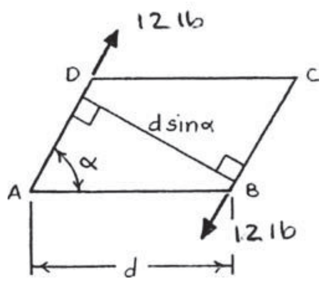
where $d_1 = 16$ in.

$F_1 = 21$ lb

$$M_1 = (16\text{ in.})(21\text{ lb})$$
$$= 336\ \text{lb}\cdot\text{in.} \qquad \text{or}\quad \mathbf{M}_1 = 336\ \text{lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$

(*b*) We have $\mathbf{M}_1 + \mathbf{M}_2 = 0$

or $336\ \text{lb}\cdot\text{in.} - d_2(12\text{ lb}) = 0 \qquad d_2 = 28.0\text{ in.} \blacktriangleleft$

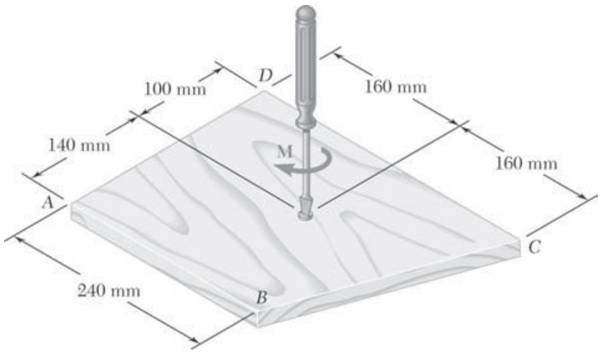


(*c*) We have $\mathbf{M}_{\text{total}} = \mathbf{M}_1 + \mathbf{M}_2$

or $-72\ \text{lb}\cdot\text{in.} = 336\ \text{lb}\cdot\text{in.} - (42\text{ in.})(\sin\alpha)(12\text{ lb})$

$$\sin\alpha = 0.80952$$

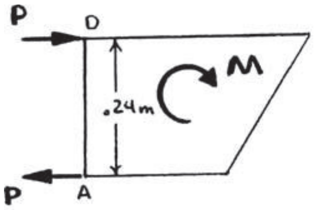
and $\alpha = 54.049°$ or $\alpha = 54.0° \blacktriangleleft$



PROBLEM 3.72

A couple **M** of magnitude $18\ \text{N}\cdot\text{m}$ is applied to the handle of a screwdriver to tighten a screw into a block of wood. Determine the magnitudes of the two smallest horizontal forces that are equivalent to **M** if they are applied (*a*) at corners *A* and *D*, (*b*) at corners *B* and *C*, (*c*) anywhere on the block.

SOLUTION



(*a*) We have $M = Pd$

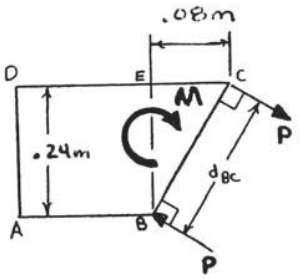
or $18\ \text{N}\cdot\text{m} = P(.24\ \text{m})$

$P = 75.0\ \text{N}$ or $P_{\min} = 75.0\ \text{N}$ ◀

(*b*) $d_{BC} = \sqrt{(BE)^2 + (EC)^2}$

$= \sqrt{(.24\ \text{m})^2 + (.08\ \text{m})^2}$

$= 0.25298\ \text{m}$



We have $M = Pd$

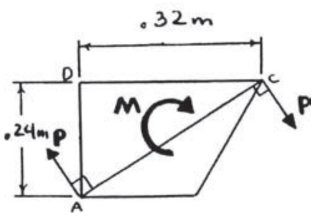
$18\ \text{N}\cdot\text{m} = P(0.25298\ \text{m})$

$P = 71.152\ \text{N}$ or $P = 71.2\ \text{N}$ ◀

(*c*) $d_{AC} = \sqrt{(AD)^2 + (DC)^2}$

$= \sqrt{(0.24\ \text{m})^2 + (0.32\ \text{m})^2}$

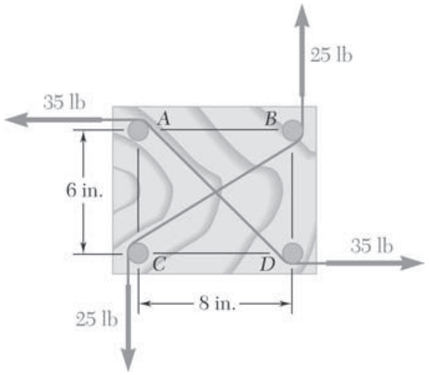
$= 0.4\ \text{m}$



We have $M = Pd_{AC}$

$18\ \text{N}\cdot\text{m} = P(0.4\ \text{m})$

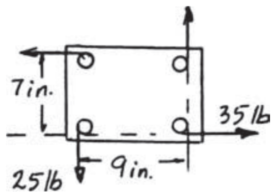
$P = 45.0\ \text{N}$ or $P = 45.0\ \text{N}$ ◀



PROBLEM 3.73

Four 1-in.-diameter pegs are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. (*a*) Determine the resultant couple acting on the board. (*b*) If only one string is used, around which pegs should it pass and in what directions should it be pulled to create the same couple with the minimum tension in the string? (*c*) What is the value of that minimum tension?

SOLUTION



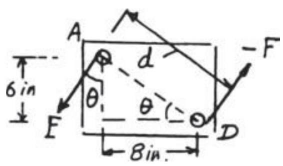
(*a*)

$$+\curvearrowright M = (35\text{ lb})(7\text{ in.}) + (25\text{ lb})(9\text{ in.})$$
$$= 245\text{ lb}\cdot\text{in.} + 225\text{ lb}\cdot\text{in.}$$

$$M = 470\text{ lb}\cdot\text{in.} \curvearrowright \blacktriangleleft$$

(*b*) With only one string, pegs *A* and *D*, or *B* and *C* should be used. We have

$$\tan\theta = \frac{6}{8} \qquad \theta = 36.9° \qquad 90° - \theta = 53.1°$$



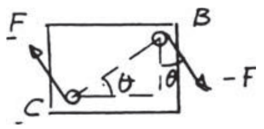
Direction of forces:

With pegs *A* and *D*: $\theta = 53.1° \blacktriangleleft$

With pegs *B* and *C*: $\theta = 53.1° \blacktriangleleft$

(*c*) The distance between the centers of the two pegs is

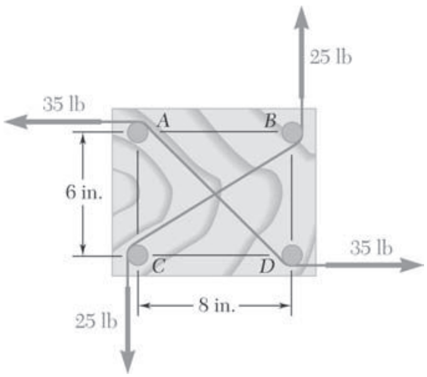
$$\sqrt{8^2 + 6^2} = 10\text{ in.}$$



Therefore, the perpendicular distance *d* between the forces is

$$d = 10\text{ in.} + 2\left(\frac{1}{2}\text{in.}\right)$$
$$= 11\text{ in.}$$

We must have $\quad M = Fd \quad 470\text{ lb}\cdot\text{in.} = F(11\text{ in.}) \qquad F = 42.7\text{ lb} \blacktriangleleft$



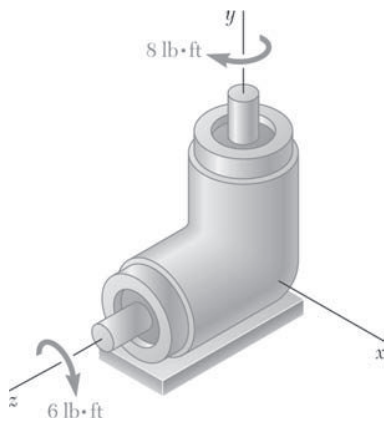
PROBLEM 3.74

Four pegs of the same diameter are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. Determine the diameter of the pegs knowing that the resultant couple applied to the board is $485 \text{ lb} \cdot \text{in.}$ counterclockwise.

SOLUTION

$$M = d_{AD}F_{AD} + d_{BC}F_{BC}$$

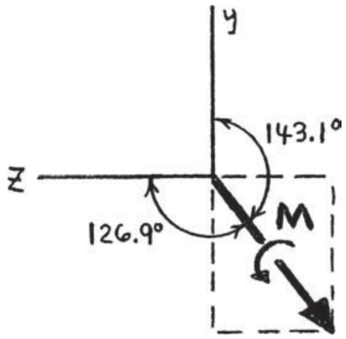
$$485 \text{ lb} \cdot \text{in.} = [(6+d)\text{in.}](35 \text{ lb}) + [(8+d)\text{in.}](25 \text{ lb}) \qquad d = 1.250 \text{ in.} \blacktriangleleft$$



PROBLEM 3.75

The shafts of an angle drive are acted upon by the two couples shown. Replace the two couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Based on $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

where $\mathbf{M}_1 = -(8 \text{ lb}\cdot\text{ft})\mathbf{j}$

$\mathbf{M}_2 = -(6 \text{ lb}\cdot\text{ft})\mathbf{k}$

$\mathbf{M} = -(8 \text{ lb}\cdot\text{ft})\mathbf{j} - (6 \text{ lb}\cdot\text{ft})\mathbf{k}$

and $|\mathbf{M}| = \sqrt{(8)^2 + (6)^2} = 10 \text{ lb}\cdot\text{ft}$ or $M = 10.00 \text{ lb}\cdot\text{ft}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|}$$

$$= \frac{-(8 \text{ lb}\cdot\text{ft})\mathbf{j} - (6 \text{ lb}\cdot\text{ft})\mathbf{k}}{10 \text{ lb}\cdot\text{ft}}$$

$$= -0.8\mathbf{j} - 0.6\mathbf{k}$$

or $\mathbf{M} = |\mathbf{M}|\boldsymbol{\lambda} = (10 \text{ lb}\cdot\text{ft})(-0.8\mathbf{j} - 0.6\mathbf{k})$

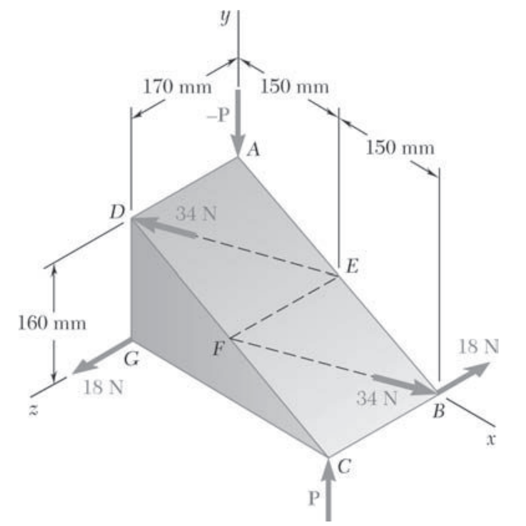
$\cos\theta_x = 0 \qquad \theta_x = 90°$

$\cos\theta_y = -0.8 \qquad \theta_y = 143.130°$

$\cos\theta_z = -0.6 \qquad \theta_z = 126.870°$

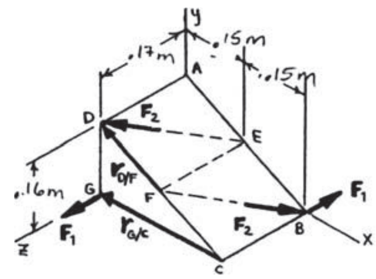
or $\theta_x = 90.0° \quad \theta_y = 143.1° \quad \theta_z = 126.9°$ ◀

PROBLEM 3.76



If $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



We have $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

where $\mathbf{M}_1 = \mathbf{r}_{G/C} \times \mathbf{F}_1$

$$\mathbf{r}_{G/C} = -(0.3\text{ m})\mathbf{i}$$

$$\mathbf{F}_1 = (18\text{ N})\mathbf{k}$$

$$\mathbf{M}_1 = -(0.3\text{ m})\mathbf{i} \times (18\text{ N})\mathbf{k}$$

$$= (5.4\text{ N}\cdot\text{m})\mathbf{j}$$

Also, $\mathbf{M}_2 = \mathbf{r}_{D/F} \times \mathbf{F}_2$

$$\mathbf{r}_{D/F} = -(.15\text{ m})\mathbf{i} + (.08\text{ m})\mathbf{j}$$

$$\mathbf{F}_2 = \lambda_{ED} F_2$$

$$= \frac{(.15\text{ m})\mathbf{i} + (.08\text{ m})\mathbf{j} + (.17\text{ m})\mathbf{k}}{\sqrt{(.15)^2 + (.08)^2 + (.17)^2}\text{ m}}(34\text{ N})$$

$$= 141.421\text{ N}\cdot\text{m}(.15\mathbf{i} + .08\mathbf{j} + .17\mathbf{k})$$

$$\mathbf{M}_2 = 141.421\text{ N}\cdot\text{m}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -.15 & .08 & 0 \\ -.15 & .08 & .17 \end{vmatrix}$$

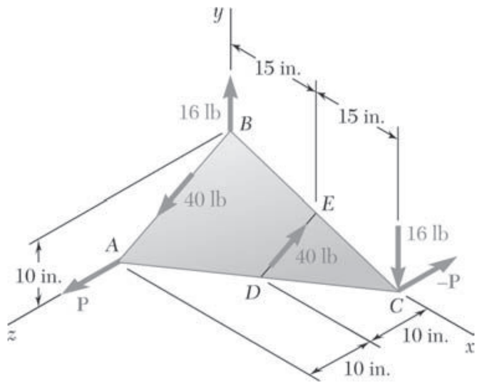
$$= 141.421(.0136\mathbf{i} + 0.0255\mathbf{j})\text{N}\cdot\text{m}$$

PROBLEM 3.76 (Continued)

and $$\mathbf{M} = [(5.4\ \text{N}\cdot\text{m})\mathbf{j}] + [141.421(.0136\mathbf{i} + .0255\mathbf{j})\ \text{N}\cdot\text{m}]$$
$$= (1.92333\ \text{N}\cdot\text{m})\mathbf{i} + (9.0062\ \text{N}\cdot\text{m})\mathbf{j}$$

$$|\mathbf{M}| = \sqrt{(M_x)^2 + (M_y)^2}$$
$$= \sqrt{(1.92333)^2 + (9.0062)^2}$$
$$= 9.2093\ \text{N}\cdot\text{m} \qquad \text{or} \quad M = 9.21\ \text{N}\cdot\text{m} \blacktriangleleft$$

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{(1.92333\ \text{N}\cdot\text{m})\mathbf{i} + (9.0062\ \text{N}\cdot\text{m})\mathbf{j}}{9.2093\ \text{N}\cdot\text{m}}$$
$$= 0.20885 + 0.97795$$
$$\cos\theta_x = 0.20885$$
$$\theta_x = 77.945° \qquad \text{or} \qquad \theta_x = 77.9° \blacktriangleleft$$
$$\cos\theta_y = 0.97795$$
$$\theta_y = 12.054° \qquad \text{or} \qquad \theta_y = 12.05° \blacktriangleleft$$
$$\cos\theta_z = 0.0$$
$$\theta_z = 90° \qquad \text{or} \qquad \theta_z = 90.0° \blacktriangleleft$$



PROBLEM 3.77

If $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2; \quad F_1 = 16 \text{ lb}, \quad F_2 = 40 \text{ lb}$$

$$\mathbf{M_1} = \mathbf{r}_C \times \mathbf{F}_1 = (30 \text{ in.})\mathbf{i} \times [-(16 \text{ lb})\mathbf{j}] = -(480 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\mathbf{M_2} = \mathbf{r}_{E/B} \times \mathbf{F}_2; \quad \mathbf{r}_{E/B} = (15 \text{ in.})\mathbf{i} - (5 \text{ in.})\mathbf{j}$$

$$d_{DE} = \sqrt{(0)^2 + (5)^2 + (10)^2} = 5\sqrt{5} \text{ in.}$$

$$F_2 = \frac{40 \text{ lb}}{5\sqrt{5}}(5\mathbf{j} - 10\mathbf{k})$$

$$= 8\sqrt{5}[(1 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}]$$

$$\mathbf{M}_2 = 8\sqrt{5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & -5 & 0 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= 8\sqrt{5}[(10 \text{ lb}\cdot\text{in.})\mathbf{i} + (30 \text{ lb}\cdot\text{in.})\mathbf{j} + (15 \text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$\mathbf{M} = -(480 \text{ lb}\cdot\text{in.})\mathbf{k} + 8\sqrt{5}[(10 \text{ lb}\cdot\text{in.})\mathbf{i} + (30 \text{ lb}\cdot\text{in.})\mathbf{j} + (15 \text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$= (178.885 \text{ lb}\cdot\text{in.})\mathbf{i} + (536.66 \text{ lb}\cdot\text{in.})\mathbf{j} - (211.67 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$M = \sqrt{(178.885)^2 + (536.66)^2 + (-211.67)^2}$$

$$= 603.99 \text{ lb}\cdot\text{in}$$

$M = 604 \text{ lb}\cdot\text{in.}$ ◀

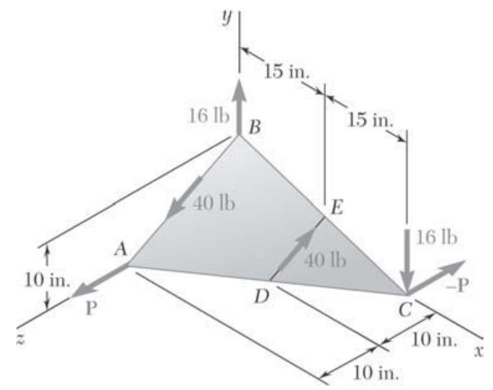
$$\boldsymbol{\lambda}_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.29617\mathbf{i} + 0.88852\mathbf{j} - 0.35045\mathbf{k}$$

$$\cos\theta_x = 0.29617$$

$$\cos\theta_y = 0.88852$$

$$\cos\theta_z = -0.35045$$

$\theta_x = 72.8° \quad \theta_y = 27.3° \quad \theta_z = 110.5°$ ◀



PROBLEM 3.78

If $P = 20$ lb, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

From the solution to Problem. 3.77

16 lb force: $M_1 = -(480 \text{ lb}\cdot\text{in.})\mathbf{k}$

40 lb force: $M_2 = 8\sqrt{5}[(10 \text{ lb}\cdot\text{in.})\mathbf{i} + (30 \text{ lb}\cdot\text{in.})\mathbf{j} + (15 \text{ lb}\cdot\text{in.})\mathbf{k}]$

$P = 20$ lb
$$\begin{aligned} M_3 &= \mathbf{r}_C \times P \\ &= (30 \text{ in.})\mathbf{i} \times (20 \text{ lb})\mathbf{k} \\ &= (600 \text{ lb}\cdot\text{in.})\mathbf{j} \end{aligned}$$

$$\begin{aligned} \mathbf{M} &= \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 \\ &= -(480)\mathbf{k} + 8\sqrt{5}\,(10\mathbf{i} + 30\mathbf{j} + 15\mathbf{k}) + 600\mathbf{j} \\ &= (178.885 \text{ lb}\cdot\text{in})\mathbf{i} + (1136.66 \text{ lb}\cdot\text{in.})\mathbf{j} - (211.67 \text{ lb}\cdot\text{in.})\mathbf{k} \end{aligned}$$

$$\begin{aligned} M &= \sqrt{(178.885)^2 + (113.66)^2 + (211.67)^2} \\ &= 1169.96 \text{ lb}\cdot\text{in} \end{aligned}$$

$M = 1170 \text{ lb}\cdot\text{in.}$ ◀

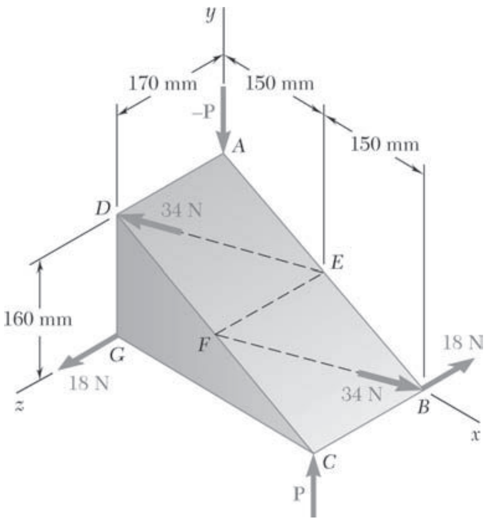
$$\lambda_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.152898\mathbf{i} + 0.97154\mathbf{j} - 0.180921\mathbf{k}$$

$\cos\theta_x = 0.152898$

$\cos\theta_y = 0.97154$

$\cos\theta_z = -0.180921$

$\theta_x = 81.2° \quad \theta_y = 13.70° \quad \theta_z = 100.4°$ ◀



PROBLEM 3.79

If $P = 20$ N, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

We have $\quad \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

where

$$\mathbf{M}_1 = \mathbf{r}_{G/C} \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0 & 0 \\ 0 & 0 & 18 \end{vmatrix} \mathrm{N \cdot m} = (5.4\ \mathrm{N \cdot m})\mathbf{j}$$

$$\mathbf{M}_2 = \mathbf{r}_{D/F} \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -.15 & .08 & 0 \\ -.15 & .08 & .17 \end{vmatrix} 141.421\ \mathrm{N \cdot m}$$

$$= 141.421(.0136\mathbf{i} + .0255\mathbf{j})\mathrm{N \cdot m}$$

(See Solution to Problem 3.76.)

$$\mathbf{M}_3 = \mathbf{r}_{C/A} \times \mathbf{F}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0 & 0.17 \\ 0 & 20 & 0 \end{vmatrix} \mathrm{N \cdot m}$$

$$= -(3.4\ \mathrm{N \cdot m})\mathbf{i} + (6\ \mathrm{N \cdot m})\mathbf{k}$$

$$\mathbf{M} = [(1.92333 - 3.4)\mathbf{i} + (5.4 + 3.6062)\mathbf{j} + (6)\mathbf{k}]\ \mathrm{N \cdot m}$$

$$= -(1.47667\ \mathrm{N \cdot m})\mathbf{i} + (9.0062\ \mathrm{N \cdot m})\mathbf{j} + (6\ \mathrm{N \cdot m})$$

$$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

$$= \sqrt{(1.47667) + (9.0062) + (6)^2}$$

$$= 10.9221\ \mathrm{N \cdot m} \qquad \text{or} \quad M = 10.92\ \mathrm{N \cdot m} \blacktriangleleft$$

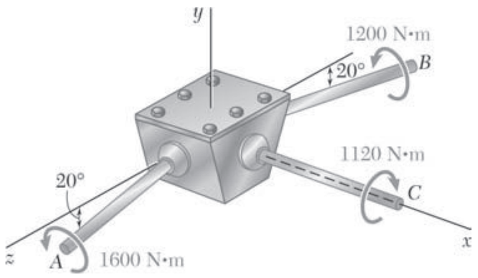
PROBLEM 3.79 (Continued)

$$\boldsymbol{\lambda} = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{-1.47667 + 9.0062 + 6}{10.9221}$$
$$= -0.135200\mathbf{i} + 0.82459\mathbf{j} + 0.54934\mathbf{k}$$

$\cos\theta_x = -0.135200 \quad \theta_x = 97.770$ or $\theta_x = 97.8°$ ◀

$\cos\theta_y = 0.82459 \quad \theta_y = 34.453$ or $\theta_y = 34.5°$ ◀

$\cos\theta_z = 0.54934 \quad \theta_z = 56.678$ or $\theta_z = 56.7°$ ◀



PROBLEM 3.80

Shafts A and B connect the gear box to the wheel assemblies of a tractor, and shaft C connects it to the engine. Shafts A and B lie in the vertical yz plane, while shaft C is directed along the x axis. Replace the couples applied to the shafts with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

Represent the given couples by the following couple vectors:

$$\begin{aligned}\mathbf{M}_A &= -1600\sin 20^\circ\mathbf{j} + 1600\cos 20^\circ\mathbf{k}\\ &= -(547.232\ \text{N}\cdot\text{m})\mathbf{j} + (1503.51\ \text{N}\cdot\text{m})\mathbf{k}\\ \mathbf{M}_B &= 1200\sin 20^\circ\mathbf{j} + 1200\cos 20^\circ\mathbf{k}\\ &= (410.424\ \text{N}\cdot\text{m})\mathbf{j} + (1127.63\ \text{N}\cdot\text{m})\mathbf{k}\\ \mathbf{M}_C &= -(1120\ \text{N}\cdot\text{m})\mathbf{i}\end{aligned}$$

The single equivalent couple is

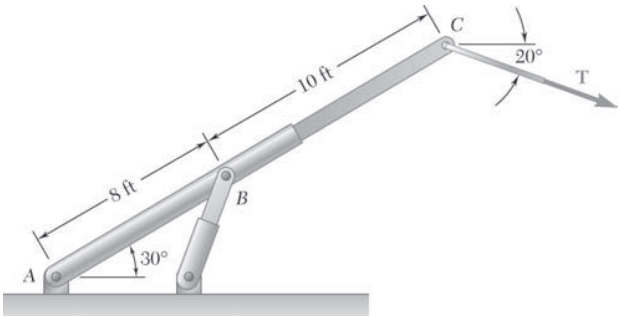
$$\begin{aligned}\mathbf{M} &= \mathbf{M}_A + \mathbf{M}_B + \mathbf{M}_C\\ &= -(1120\ \text{N}\cdot\text{m})\mathbf{i} - (136.808\ \text{N}\cdot\text{m})\mathbf{j} + (2631.1\ \text{N}\cdot\text{m})\mathbf{k}\\ M &= \sqrt{(1120)^2 + (136.808)^2 + (2631.1)^2}\\ &= 2862.8\ \text{N}\cdot\text{m}\end{aligned}$$

$$\cos\theta_x = \frac{-1120}{2862.8}$$

$$\cos\theta_y = \frac{-136.808}{2862.8}$$

$$\cos\theta_z = \frac{2631.1}{2862.8}$$

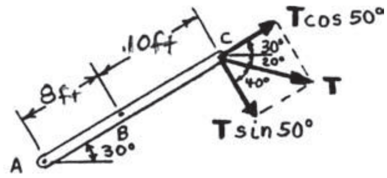
$$M = 2860\ \text{N}\cdot\text{m} \quad \theta_x = 113.0^\circ \quad \theta_y = 92.7^\circ \quad \theta_z = 23.2^\circ \blacktriangleleft$$

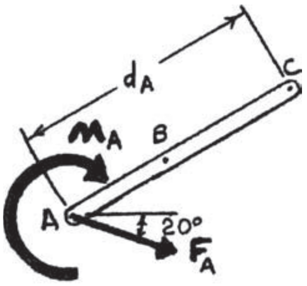


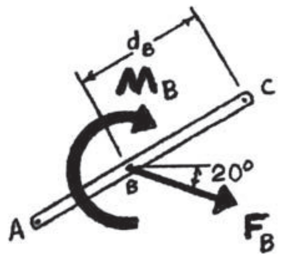
PROBLEM 3.81

The tension in the cable attached to the end C of an adjustable boom ABC is 560 lb. Replace the force exerted by the cable at C with an equivalent force-couple system (*a*) at A, (*b*) at B.

SOLUTION







(*a*) Based on $\Sigma F: \quad F_A = T = 560 \text{ lb}$

or $\mathbf{F}_A = 560 \text{ lb} \; \measuredangle \; 20°$ ◀

$\Sigma M_A: \quad M_A = (T \sin 50°)(d_A)$

$= (560 \text{ lb})\sin 50°(18 \text{ ft})$

$= 7721.7 \text{ lb} \cdot \text{ft}$

or $\mathbf{M}_A = 7720 \text{ lb} \cdot \text{ft}$ ↻ ◀

(*b*) Based on $\Sigma F: \quad F_B = T = 560 \text{ lb}$

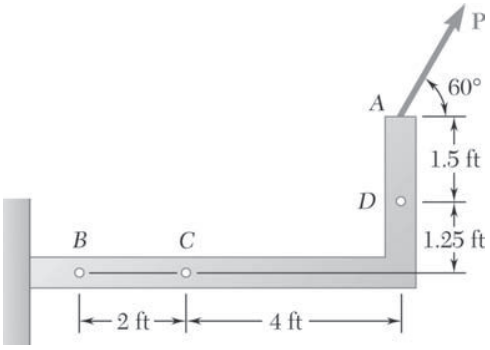
or $\mathbf{F}_B = 560 \text{ lb} \; \measuredangle \; 20°$ ◀

$\Sigma M_B: \quad M_B = (T \sin 50°)(d_B)$

$= (560 \text{ lb})\sin 50°(10 \text{ ft})$

$= 4289.8 \text{ lb} \cdot \text{ft}$

or $\mathbf{M}_B = 4290 \text{ lb} \cdot \text{ft}$ ↻ ◀



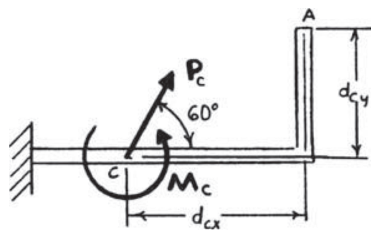
PROBLEM 3.82

A 160-lb force **P** is applied at Point A of a structural member. Replace **P** with (a) an equivalent force-couple system at C, (b) an equivalent system consisting of a vertical force at B and a second force at D.

SOLUTION

(a) Based on $\quad \Sigma F: \quad P_C = P = 160 \text{ lb}$ $\qquad$ or $\mathbf{P}_C = 160 \text{ lb} \measuredangle 60°$ ◀

$$\Sigma M_C: \quad M_C = -P_x d_{cy} + P_y d_{Cx}$$



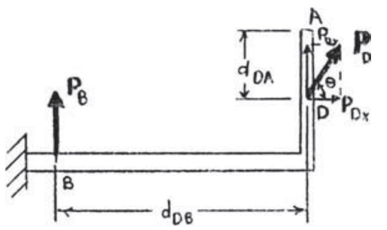
where

$$P_x = (160 \text{ lb})\cos 60°$$
$$= 80 \text{ lb}$$
$$P_y = (160 \text{ lb})\sin 60°$$
$$= 138.564 \text{ lb}$$
$$d_{Cx} = 4 \text{ ft}$$
$$d_{Cy} = 2.75 \text{ ft}$$

$$M_C = (80 \text{ lb})(2.75 \text{ ft}) + (138.564 \text{ lb})(4 \text{ ft})$$
$$= 220 \text{ lb}\cdot\text{ft} + 554.26 \text{ lb}\cdot\text{ft}$$
$$= 334.26 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_C = 334 \text{ lb}\cdot\text{ft} ↻$ ◀

(b) Based on $\quad \Sigma F_x: \quad P_{Dx} = P\cos 60°$



$$= (160 \text{ lb})\cos 60°$$
$$= 80 \text{ lb}$$

$$\Sigma M_D: \quad (P\cos 60°)(d_{DA}) = P_B(d_{DB})$$

$$[(160 \text{ lb})\cos 60°](1.5 \text{ ft}) = P_B(6 \text{ ft})$$
$$P_B = 20.0 \text{ lb}$$

or $\mathbf{P}_B = 20.0 \text{ lb} \uparrow$ ◀

PROBLEM 3.82 (Continued)

$$\Sigma F_y: \quad P\sin 60° = P_B + P_{Dy}$$

$$(160 \text{ lb})\sin 60° = 20.0 \text{ lb} + P_{Dy}$$

$$P_{Dy} = 118.564 \text{ lb}$$

$$P_D = \sqrt{(P_{Dx})^2 + (P_{Dy})^2}$$

$$= \sqrt{(80)^2 + (118.564)^2}$$

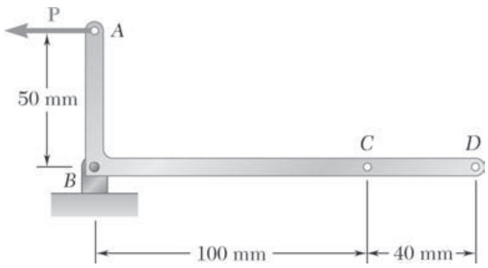
$$= 143.029 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{P_{Dy}}{P_{Dx}}\right)$$

$$= \tan^{-1}\left(\frac{118.564}{80}\right)$$

$$= 55.991°$$

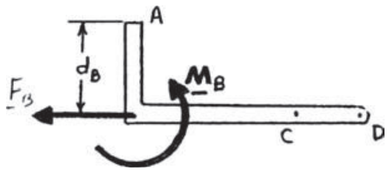
or $P_D = 143.0 \text{ lb} \measuredangle 56.0°$ ◀



PROBLEM 3.83

The 80-N horizontal force **P** acts on a bell crank as shown. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two vertical forces at *C* and *D* that are equivalent to the couple found in Part *a*.

SOLUTION



(*a*) Based on $\Sigma F: \quad F_B = F = 80 \text{ N}$ or $\mathbf{F}_B = 80.0 \text{ N} \longleftarrow$ ◀

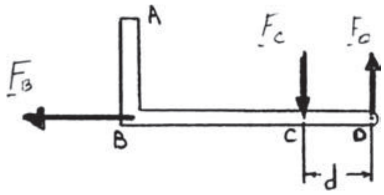
$$\Sigma M: \quad M_B = Fd_B$$

$$= 80 \text{ N } (.05 \text{ m})$$

$$= 4.0000 \text{ N} \cdot \text{m}$$

or $\mathbf{M}_B = 4.00 \text{ N} \cdot \text{m}$ ↻ ◀

(*b*) If the two vertical forces are to be equivalent to $\mathbf{M}_B$, they must be a couple. Further, the sense of the moment of this couple must be counterclockwise.



Then, with F_C and F_D acting as shown,

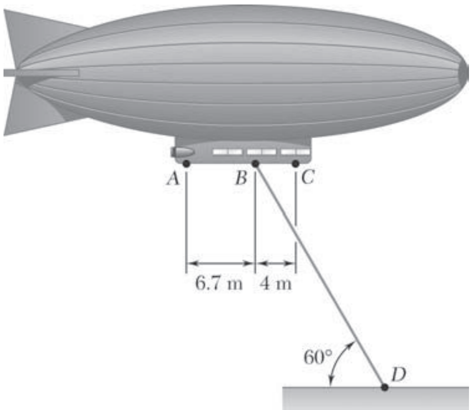
$$\Sigma M: \quad M_D = F_C d$$

$$4.0000 \text{ N} \cdot \text{m} = F_C (.04 \text{ m})$$

$F_C = 100.000 \text{ N}$ or $\mathbf{F}_C = 100.0 \text{ N} \downarrow$ ◀

$$\Sigma F_y: \quad 0 = F_D - F_C$$

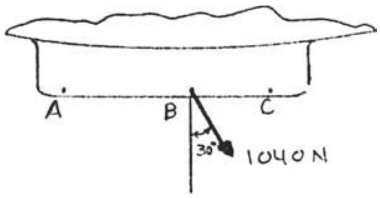
$F_D = 100.000 \text{ N}$ or $\mathbf{F}_D = 100.0 \text{ N} \uparrow$ ◀

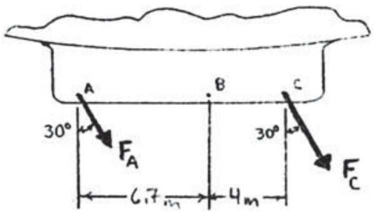


PROBLEM 3.84

A dirigible is tethered by a cable attached to its cabin at B. If the tension in the cable is 1040 N, replace the force exerted by the cable at B with an equivalent system formed by two parallel forces applied at A and C.

SOLUTION





Require the equivalent forces acting at A and C be parallel and at an angle of α with the vertical.

Then for equivalence,

$$\Sigma F_x: \quad (1040 \text{ N})\sin 30° = F_A \sin\alpha + F_B \sin\alpha \tag{1}$$

$$\Sigma F_y: \quad -(1040 \text{ N})\cos 30° = -F_A \cos\alpha - F_B \cos\alpha \tag{2}$$

Dividing Equation (1) by Equation (2),

$$\frac{(1040 \text{ N})\sin 30°}{-(1040 \text{ N})\cos 30°} = \frac{(F_A + F_B)\sin\alpha}{-(F_A + F_B)\cos\alpha}$$

Simplifying yields $\alpha = 30°$

Based on

$$\Sigma M_C: \quad [(1040 \text{ N})\cos 30°](4 \text{ m}) = (F_A \cos 30°)(10.7 \text{ m})$$

$$F_A = 388.79 \text{ N}$$

or $\mathbf{F}_A = 389 \text{ N} \measuredangle 60°$ ◀

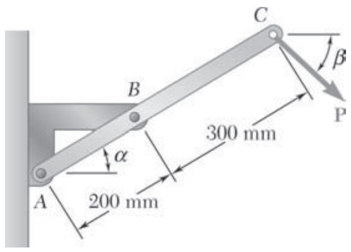
Based on

$$\Sigma M_A: \quad -[(1040 \text{ N})\cos 30°](6.7 \text{ m}) = (F_C \cos 30°)(10.7 \text{ m})$$

$$F_C = 651.21 \text{ N}$$

or $\mathbf{F}_C = 651 \text{ N} \measuredangle 60°$ ◀

PROBLEM 3.85



The force **P** has a magnitude of 250 N and is applied at the end C of a 500-mm rod AC attached to a bracket at A and B. Assuming $\alpha = 30°$ and $\beta = 60°$, replace **P** with (a) an equivalent force-couple system at B, (b) an equivalent system formed by two parallel forces applied at A and B.

SOLUTION

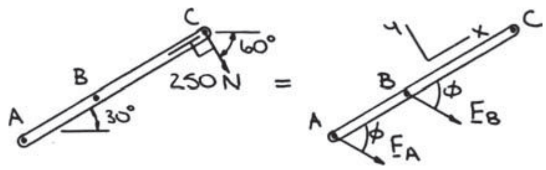
(a) Equivalence requires $\Sigma \mathbf{F}$: $\mathbf{F} = \mathbf{P}$ or $\mathbf{F} = 250 \text{ N} \; 60°$

$$\Sigma \mathbf{M}_B: \quad M = -(0.3 \text{ m})(250 \text{ N}) = -75 \text{ N} \cdot \text{m}$$

The equivalent force-couple system at B is

$\mathbf{F} = 250 \text{ N} \; 60°$ $\qquad$ $\mathbf{M} = 75.0 \text{ N} \cdot \text{m}$ ◀

(b) Require



Equivalence then requires

$$\Sigma F_x: \quad 0 = F_A \cos\phi + F_B \cos\phi$$

$$F_A = -F_B \quad \text{or} \quad \cos\phi = 0$$

$$\Sigma F_y: \quad -250 = -F_A \sin\phi - F_B \sin\phi$$

Now if $\qquad F_A = -F_B \Rightarrow -250 = 0$ reject

$$\cos\phi = 0$$

or $\qquad \phi = 90°$

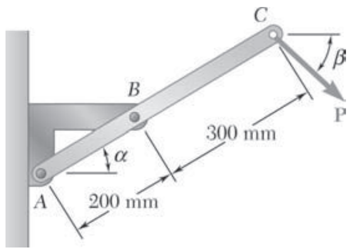
and $\qquad F_A + F_B = 250$

Also $\qquad \Sigma M_B: \quad -(0.3 \text{ m})(250 \text{ N}) = (0.2\text{m})F_A$

or $\qquad F_A = -375 \text{ N}$

and $\qquad F_B = 625 \text{ N}$

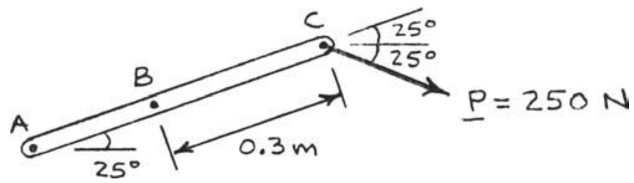
$\mathbf{F}_A = 375 \text{ N} \; 60°$ $\qquad$ $\mathbf{F}_B = 625 \text{ N} \; 60°$ ◀



PROBLEM 3.86

Solve Problem 3.85, assuming $\alpha = \beta = 25°$.

SOLUTION



(*a*) Equivalence requires

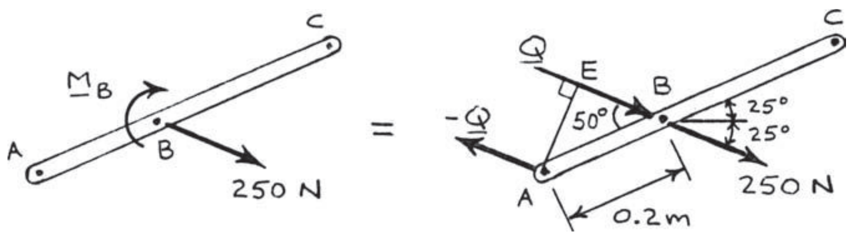
$$\Sigma \mathbf{F}: \quad \mathbf{F}_B = \mathbf{P} \quad \text{or} \quad \mathbf{F}_B = 250 \text{ N} \searrow 25.0°$$

$$\Sigma \mathbf{M}_B: \quad M_B = -(0.3 \text{ m})[(250 \text{ N})\sin 50°] = -57.453 \text{ N}\cdot\text{m}$$

The equivalent force-couple system at *B* is

$$\mathbf{F}_B = 250 \text{ N} \searrow 25.0° \qquad \mathbf{M}_B = 57.5 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) Require

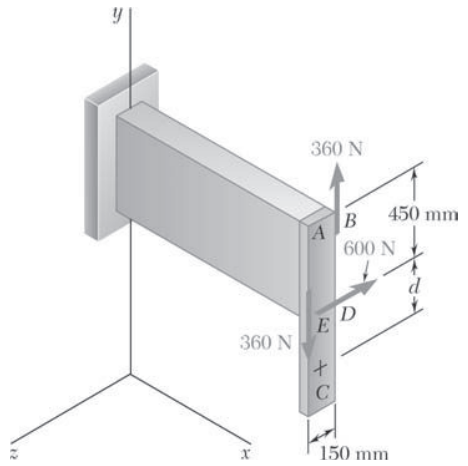


Equivalence requires

$$M_B = d_{AE}Q \quad (0.3 \text{ m})[(250 \text{ N})\sin 50°]$$
$$= [(0.2 \text{ m})\sin 50°]Q$$
$$Q = 375 \text{ N}$$

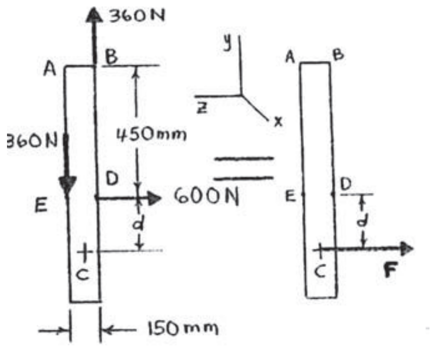
Adding the forces at *B*: $\mathbf{F}_A = 375 \text{ N} \nwarrow 25.0°$ $\qquad \mathbf{F}_B = 625 \text{ N} \searrow 25.0° \blacktriangleleft$

PROBLEM 3.87



A force and a couple are applied as shown to the end of a cantilever beam. (*a*) Replace this system with a single force **F** applied at Point *C*, and determine the distance *d* from *C* to a line drawn through Points *D* and *E*. (*b*) Solve Part *a* if the directions of the two 360-N forces are reversed.

SOLUTION



(*a*) We have $\Sigma\mathbf{F}$: $\mathbf{F} = (360\text{ N})\mathbf{j} - (360\text{ N})\mathbf{j} - (600\text{ N})\mathbf{k}$

or $\mathbf{F} = -(600\text{ N})\mathbf{k}$ ◀

and ΣM_D: $(360\text{ N})(0.15\text{ m}) = (600\text{ N})(d)$

$d = 0.09\text{ m}$

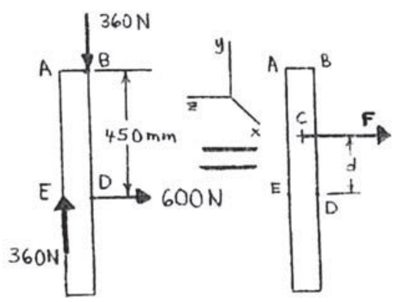
or $d = 90.0\text{ mm}$ below *ED* ◀

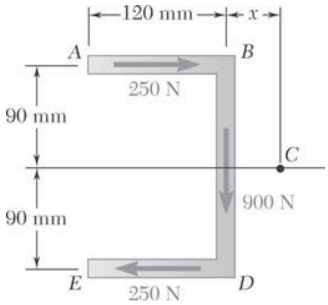
(*b*) We have from Part *a* $\mathbf{F} = -(600\text{ N})\mathbf{k}$ ◀

and ΣM_D: $-(360\text{ N})(0.15\text{ m}) = -(600\text{ N})(d)$

$d = 0.09\text{ m}$

or $d = 90.0\text{ mm}$ above *ED* ◀



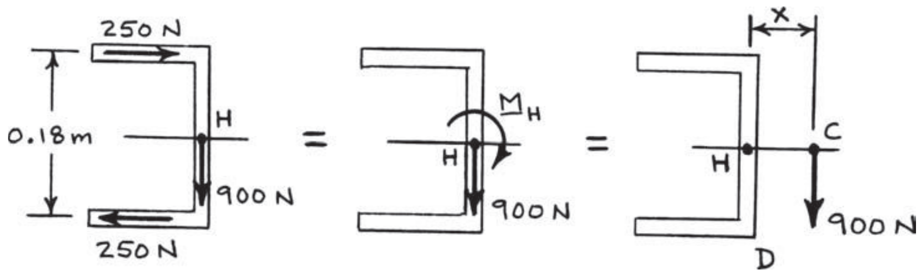


PROBLEM 3.88

The shearing forces exerted on the cross section of a steel channel can be represented by a 900-N vertical force and two 250-N horizontal forces as shown. Replace this force and couple with a single force **F** applied at Point C, and determine the distance x from C to line BD. (Point C is defined as the *shear center* of the section.)

SOLUTION

Replace the 250-N forces with a couple and move the 900-N force to Point C such that its moment about H is equal to the moment of the couple

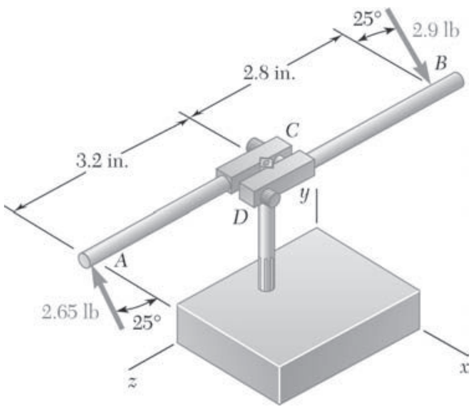


$$M_H = (0.18)(250\text{ N})$$
$$= 45\text{ N}\cdot\text{m}$$

Then $$M_H = x(900\text{ N})$$

or $$45\text{ N}\cdot\text{m} = x(900\text{ N})$$
$$x = 0.05\text{ m}$$

$\mathbf{F} = 900\text{ N}\downarrow \quad x = 50.0\text{ mm}$ ◀



PROBLEM 3.89

While tapping a hole, a machinist applies the horizontal forces shown to the handle of the tap wrench. Show that these forces are equivalent to a single force, and specify, if possible, the point of application of the single force on the handle.

SOLUTION

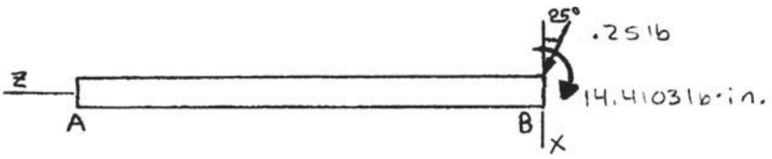
Since the forces at A and B are parallel, the force at B can be replaced with the sum of two forces with one of the forces equal in magnitude to the force at A except with an opposite sense, resulting in a force-couple.

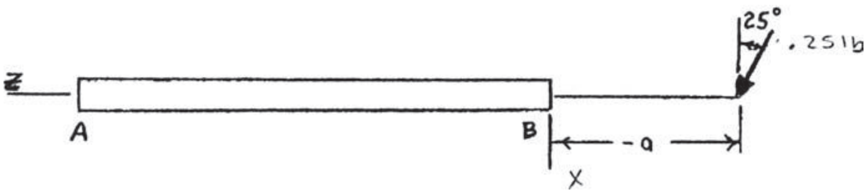
Have $F_B = 2.9 \text{ lb} - 2.65 \text{ lb} = 0.25 \text{ lb}$, where the 2.65 lb force be part of the couple. Combining the two parallel forces,

$$M_{\text{couple}} = (2.65 \text{ lb})[(3.2 \text{ in.} + 2.8 \text{ in.})\cos 25°]$$

$$= 14.4103 \text{ lb}\cdot\text{in.}$$

and $$\mathbf{M}_{\text{couple}} = 14.4103 \text{ lb}\cdot\text{in.}$$





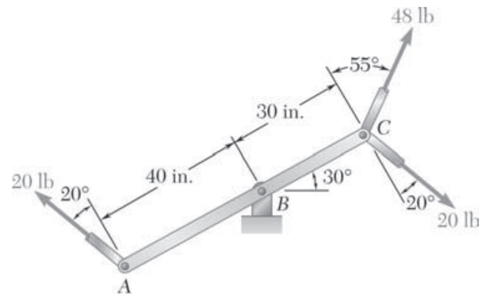
A single equivalent force will be located in the negative z-direction

Based on ΣM_B: $-14.4103 \text{ lb}\cdot\text{in.} = [(.25 \text{ lb})\cos 25°](a)$

$$a = 63.600 \text{ in.}$$

$$\mathbf{F}' = (.25 \text{ lb})(\cos 25°\mathbf{i} + \sin 25°\mathbf{k})$$

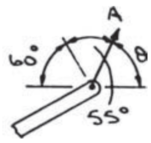
$\mathbf{F}' = (0.227 \text{ lb})\mathbf{i} + (0.1057 \text{ lb})\mathbf{k}$ and is applied on an extension of handle BD at a distance of 63.6 in. to the right of B ◀



PROBLEM 3.90

Three control rods attached to a lever *ABC* exert on it the forces shown. (*a*) Replace the three forces with an equivalent force-couple system at *B*. (*b*) Determine the single force that is equivalent to the force-couple system obtained in Part *a*, and specify its point of application on the lever.

SOLUTION



(*a*) First note that the two 20-lb forces form *A* couple. Then

$$\mathbf{F} = 48 \text{ lb} \measuredangle \theta$$

where $\theta = 180° - (60° + 55°) = 65°$

and

$$M = \Sigma M_B$$
$$= (30 \text{ in.})(48 \text{ lb})\cos 55° - (70 \text{ in.})(20 \text{ lb})\cos 20°$$
$$= -489.62 \text{ lb} \cdot \text{in}$$

The equivalent force-couple system at *B* is

$$\mathbf{F} = 48.0 \text{ lb} \measuredangle 65° \qquad \mathbf{M} = 490 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

(*b*) The single equivalent force $\mathbf{F}'$ is equal to **F**. Further, since the sense of **M** is clockwise, $\mathbf{F}'$ must be applied between *A* and *B*. For equivalence.

$$\Sigma M_B: \quad M = -aF' \cos 55°$$

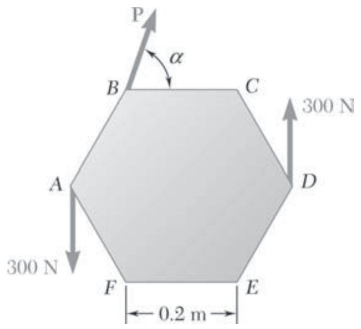
where *a* is the distance from *B* to the point of application of $\mathbf{F}'$. Then

$$-489.62 \text{ lb} \cdot \text{in.} = -a(48.0 \text{ lb})\cos 55°$$

or $a = 17.78$ in. $\qquad \mathbf{F}' = 48.0 \text{ lb} \measuredangle 65.0° \blacktriangleleft$

and is applied to the lever 17.78 in.

To the left of pin *B* ◀

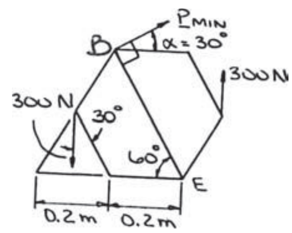


PROBLEM 3.91

A hexagonal plate is acted upon by the force **P** and the couple shown. Determine the magnitude and the direction of the smallest force **P** for which this system can be replaced with a single force at *E*.

SOLUTION

From the statement of the problem, it follows that $\Sigma M_E = 0$ for the given force-couple system. Further, for $\mathbf{P}_{\min}$, must require that **P** be perpendicular to $\mathbf{r}_{B/E}$. Then

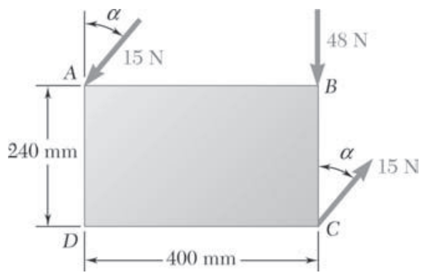


$$\Sigma M_E: \quad (0.2 \sin 30° + 0.2)\text{m} \times 300 \text{ N}$$
$$+ (0.2 \text{ m}) \sin 30° \times 300 \text{ N}$$
$$- (0.4 \text{ m}) P_{\min} = 0$$

or

$$P_{\min} = 300 \text{ N}$$

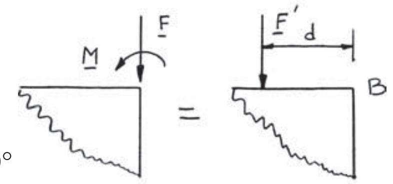
$\mathbf{P}_{\min} = 300 \text{ N} \measuredangle 30.0°$ ◀



PROBLEM 3.92

A rectangular plate is acted upon by the force and couple shown. This system is to be replaced with a single equivalent force. (*a*) For $\alpha = 40°$, specify the magnitude and the line of action of the equivalent force. (*b*) Specify the value of α if the line of action of the equivalent force is to intersect line *CD* 300 mm to the right of *D*.

SOLUTION



(*a*) The given force-couple system (*F*, *M*) at *B* is

$$\mathbf{F} = 48\text{ N}\downarrow$$

and $M = \Sigma M_B = (0.4\text{ m})(15\text{ N})\cos 40° + (0.24\text{ m})(15\text{ N})\sin 40°$

or $\mathbf{M} = 6.9103\text{ N}\cdot\text{m}\circlearrowleft$

The single equivalent force F' is equal to **F**. Further for equivalence

$$\Sigma M_B:\quad M = dF'$$

or $$6.9103\text{ N}\cdot\text{m} = d \times 48\text{ N}$$

or $$d = 0.14396\text{ m}$$ $F' = 48\text{ N}$ ◀

and the line of action of F' intersects line *AB* 144 mm to the right of *A*. ◀

(*b*) Following the solution to Part *a* but with $d = 0.1$ m and α unknown, have

$$\Sigma M_B:\quad (0.4\text{ m})(15\text{ N})\cos\alpha + (0.24\text{ m})(15\text{ N})\sin\alpha$$

$$= (0.1\text{ m})(48\text{ N})$$

or $5\cos\alpha + 3\sin\alpha = 4$

Rearranging and squaring $25\cos^2\alpha = (4 - 3\sin\alpha)^2$

Using $\cos^2\alpha = 1 - \sin^2\alpha$ and expanding

$$25(1 - \sin^2\alpha) = 16 - 24\sin\alpha + 9\sin^2\alpha$$

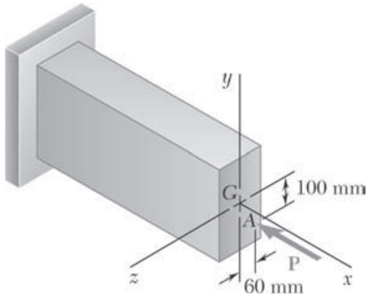
or $34\sin^2\alpha - 24\sin\alpha - 9 = 0$

Then $$\sin\alpha = \frac{24 \pm \sqrt{(-24)^2 - 4(34)(-9)}}{2(34)}$$

$$\sin\alpha = 0.97686 \quad\text{or}\quad \sin\alpha = -0.27098$$

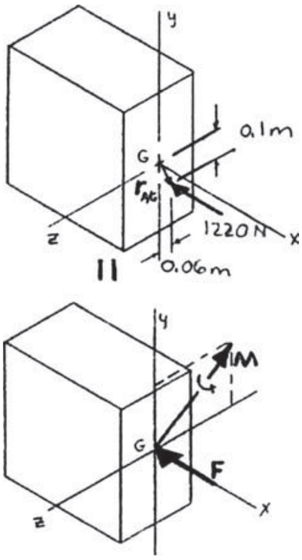
$$\alpha = 77.7° \quad\text{or}\quad \alpha = -15.72°$$ ◀

PROBLEM 3.93



An eccentric, compressive 1220-N force **P** is applied to the end of a cantilever beam. Replace **P** with an equivalent force-couple system at G.

SOLUTION



We have

$$\Sigma \mathbf{F}: \quad -(1220 \text{ N})\mathbf{i} = \mathbf{F}$$

$$\mathbf{F} = -(1220 \text{ N})\mathbf{i} \quad \blacktriangleleft$$

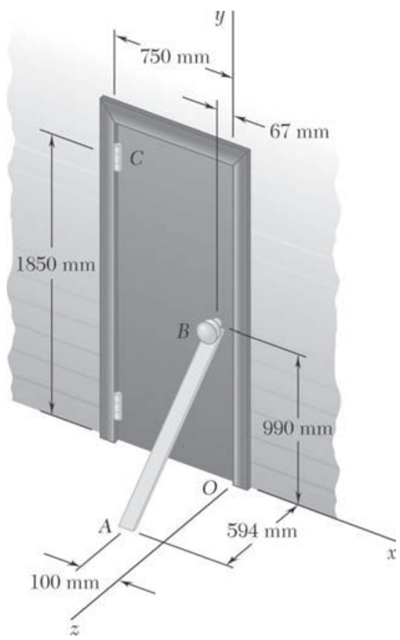
Also, we have

$$\Sigma \mathbf{M}_G: \quad \mathbf{r}_{A/G} \times \mathbf{P} = \mathbf{M}$$

$$1220 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -.1 & -.06 \\ -1 & 0 & 0 \end{vmatrix} \text{N} \cdot \text{m} = \mathbf{M}$$

$$\mathbf{M} = (1220 \text{ N} \cdot \text{m})[(-0.06)(-1)\mathbf{j} - (-0.1)(-1)\mathbf{k}]$$

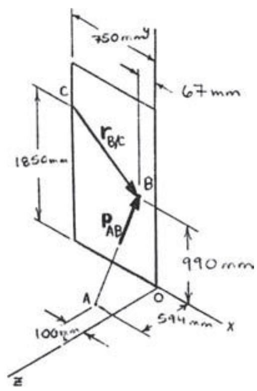
$$\text{or} \quad \mathbf{M} = (73.2 \text{ N} \cdot \text{m})\mathbf{j} - (122 \text{ N} \cdot \text{m})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.94

To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at B a 175-N force directed along line AB. Replace that force with an equivalent force-couple system at C.

SOLUTION



We have

$$\Sigma\mathbf{F}: \quad \mathbf{P}_{AB} = \mathbf{F}_C$$

where

$$\mathbf{P}_{AB} = \boldsymbol{\lambda}_{AB} P_{AB}$$

$$= \frac{(33\text{ mm})\mathbf{i} + (990\text{ mm})\mathbf{j} - (594\text{ mm})\mathbf{k}}{1155.00\text{ mm}}(175\text{ N})$$

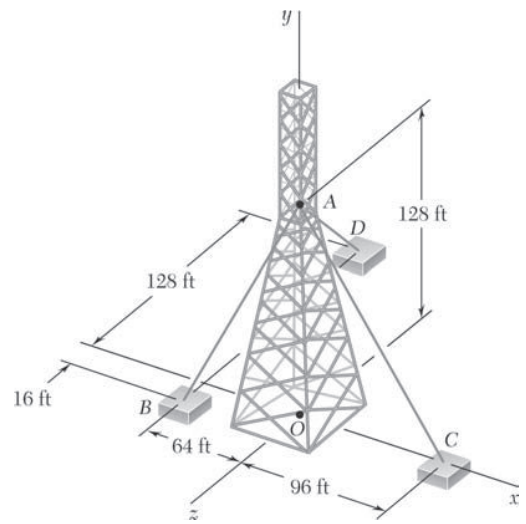
or $\mathbf{F}_C = (5.00\text{ N})\mathbf{i} + (150\text{ N})\mathbf{j} - (90.0\text{ N})\mathbf{k}$ ◀

We have $\quad \Sigma\mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{P}_{AB} = \mathbf{M}_C$

$$\mathbf{M}_C = 5\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.683 & -0.860 & 0 \\ 1 & 30 & -18 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (5)\{(-0.860)(-18)\mathbf{i} - (0.683)(-18)\mathbf{j}$$
$$+ [(0.683)(30) - (0.860)(1)]\mathbf{k}\}$$

or $\mathbf{M}_C = (77.4\text{ N}\cdot\text{m})\mathbf{i} + (61.5\text{ N}\cdot\text{m})\mathbf{j} + (106.8\text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.95

An antenna is guyed by three cables as shown. Knowing that the tension in cable *AB* is 288 lb, replace the force exerted at *A* by cable *AB* with an equivalent force-couple system at the center *O* of the base of the antenna.

SOLUTION

We have $$d_{AB} = \sqrt{(-64)^2 + (-128)^2 + (16)^2} = 144 \text{ ft}$$

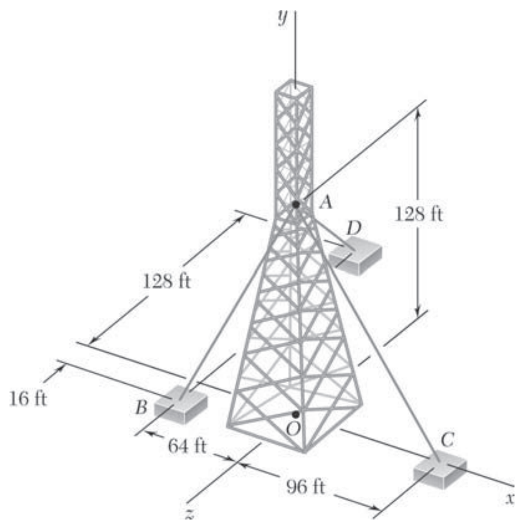
Then $$\mathbf{T}_{AB} = \frac{288 \text{ lb}}{144}(-64\mathbf{i} - 128\mathbf{j} + 16\mathbf{k})$$
$$= (32 \text{ lb})(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k})$$

Now $$\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AB}$$
$$= 128\mathbf{j} \times 32(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k})$$
$$= (4096 \text{ lb}\cdot\text{ft})\mathbf{i} + (16{,}384 \text{ lb}\cdot\text{ft})\mathbf{k}$$

The equivalent force-couple system at *O* is

$$\mathbf{F} = -(128.0 \text{ lb})\mathbf{i} - (256 \text{ lb})\mathbf{j} + (32.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M} = (4.10 \text{ kip}\cdot\text{ft})\mathbf{i} + (16.38 \text{ kip}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.96

An antenna is guyed by three cables as shown. Knowing that the tension in cable AD is 270 lb, replace the force exerted at A by cable AD with an equivalent force-couple system at the center O of the base of the antenna.

SOLUTION

We have $$d_{AD} = \sqrt{(-64)^2 + (-128)^2 + (-128)^2}$$
$$= 192 \text{ ft}$$

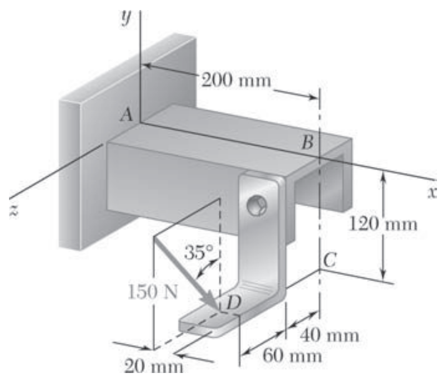
Then $$\mathbf{T}_{AD} = \frac{270 \text{ lb}}{192}(-64\mathbf{i} - 128\mathbf{j} + 128\mathbf{k})$$
$$= (90 \text{ lb})(-\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

Now $$\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AD}$$
$$= 128\mathbf{j} \times 90(-\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$
$$= -(23,040 \text{ lb}\cdot\text{ft})\mathbf{i} + (11,520 \text{ lb}\cdot\text{ft})\mathbf{k}$$

The equivalent force-couple system at O is

$$\mathbf{F} = -(90.0 \text{ lb})\mathbf{i} - (180.0 \text{ lb})\mathbf{j} - (180.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M} = -(23.0 \text{ kip}\cdot\text{ft})\mathbf{i} + (11.52 \text{ kip}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.97

Replace the 150-N force with an equivalent force-couple system at A.

SOLUTION

Equivalence requires $\quad \Sigma\mathbf{F}: \quad \mathbf{F} = (150\text{ N})(-\cos 35°\mathbf{j} - \sin 35°\mathbf{k})$

$$= -(122.873\text{ N})\mathbf{j} - (86.036\text{ N})\mathbf{k}$$

$$\Sigma\mathbf{M}_A: \quad \mathbf{M} = \mathbf{r}_{D/A} \times \mathbf{F}$$

where $\quad \mathbf{r}_{D/A} = (0.18\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.1\text{ m})\mathbf{k}$

Then

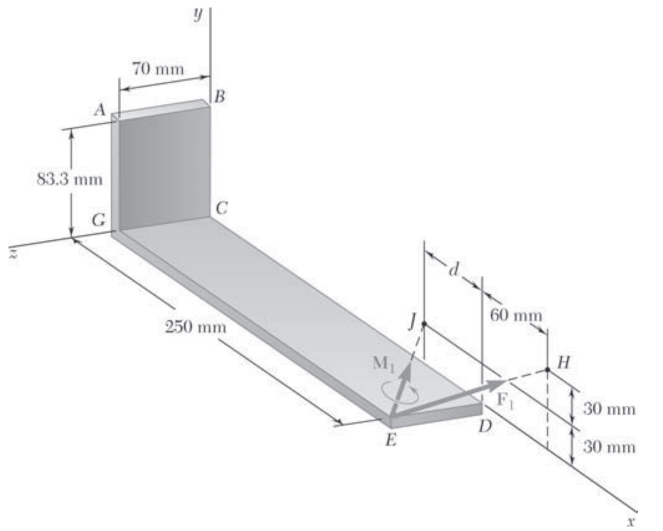
$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.18 & -0.12 & 0.1 \\ 0 & -122.873 & -86.036 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= [(-0.12)(-86.036) - (0.1)(-122.873)]\mathbf{i}$$
$$+ [-(0.18)(-86.036)]\mathbf{j}$$
$$+ [(0.18)(-122.873)]\mathbf{k}$$
$$= (22.6\text{ N}\cdot\text{m})\mathbf{i} + (15.49\text{ N}\cdot\text{m})\mathbf{j} - (22.1\text{ N}\cdot\text{m})\mathbf{k}$$

The equivalent force-couple system at A is

$$\mathbf{F} = -(122.9\text{ N})\mathbf{j} - (86.0\text{ N})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M} = (22.6\text{ N}\cdot\text{m})\mathbf{i} + (15.49\text{ N}\cdot\text{m})\mathbf{j} - (22.1\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.98

A 77-N force $\mathbf{F}_1$ and a 31-N · m couple $\mathbf{M}_1$ are applied to corner E of the bent plate shown. If $\mathbf{F}_1$ and $\mathbf{M}_1$ are to be replaced with an equivalent force-couple system ($\mathbf{F}_2$, $\mathbf{M}_2$) at corner B and if $(M_2)_z = 0$, determine (*a*) the distance d, (*b*) $\mathbf{F}_2$ and $\mathbf{M}_2$.

SOLUTION

(*a*) We have
$$\Sigma M_{Bz}: \quad M_{2z} = 0$$
$$\mathbf{k} \cdot (\mathbf{r}_{H/B} \times \mathbf{F}_1) + M_{1z} = 0 \qquad (1)$$

where
$$\mathbf{r}_{H/B} = (0.31\text{ m})\mathbf{i} - (0.0233)\mathbf{j}$$
$$\mathbf{F}_1 = \boldsymbol{\lambda}_{EH} F_1$$
$$= \frac{(0.06\text{ m})\mathbf{i} + (0.06\text{ m})\mathbf{j} - (0.07\text{ m})\mathbf{k}}{0.11\text{ m}}(77\text{ N})$$
$$= (42\text{ N})\mathbf{i} + (42\text{ N})\mathbf{j} - (49\text{ N})\mathbf{k}$$
$$M_{1z} = \mathbf{k} \cdot \mathbf{M}_1$$
$$\mathbf{M}_1 = \boldsymbol{\lambda}_{EJ} M_1$$
$$= \frac{-d\mathbf{i} + (0.03\text{ m})\mathbf{j} - (0.07\text{ m})\mathbf{k}}{\sqrt{d^2 + 0.0058}\text{ m}}(31\text{ N}\cdot\text{m})$$

Then from Equation (1),
$$\begin{vmatrix} 0 & 0 & 1 \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(-0.07\text{ m})(31\text{ N}\cdot\text{m})}{\sqrt{d^2 + 0.0058}} = 0$$

Solving for d, Equation (1) reduces to
$$(13.0200 + 0.9786) - \frac{2.17\text{ N}\cdot\text{m}}{\sqrt{d^2 + 0.0058}} = 0$$

From which $d = 0.1350\text{ m}$ or $d = 135.0\text{ mm}$ ◀

PROBLEM 3.98 (Continued)

(*b*) $\mathbf{F}_2 = \mathbf{F}_1 = (42\mathbf{i} + 42\mathbf{j} - 49\mathbf{k})\text{N}$ or $\mathbf{F}_2 = (42\text{ N})\mathbf{i} + (42\text{ N})\mathbf{j} - (49\text{ N})\mathbf{k}$ ◀

$$\mathbf{M}_2 = \mathbf{r}_{H/B} \times \mathbf{F}_1 + \mathbf{M}_1$$

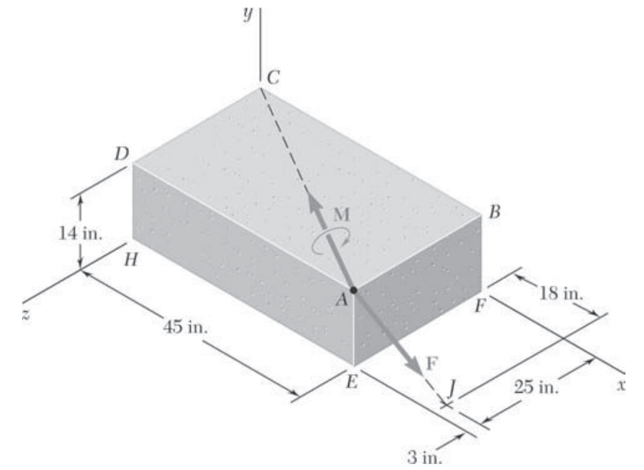
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(0.1350)\mathbf{i} + 0.03\mathbf{j} - 0.07\mathbf{k}}{0.155000}(31\text{ N}\cdot\text{m})$$

$$= (1.14170\mathbf{i} + 15.1900\mathbf{j} + 13.9986\mathbf{k})\text{ N}\cdot\text{m}$$

$$+ (-27.000\mathbf{i} + 6.0000\mathbf{j} - 14.0000\mathbf{k})\text{ N}\cdot\text{m}$$

$$\mathbf{M}_2 = -(25.858\text{ N}\cdot\text{m})\mathbf{i} + (21.190\text{ N}\cdot\text{m})\mathbf{j}$$

or $\mathbf{M}_2 = -(25.9\text{ N}\cdot\text{m})\mathbf{i} + (21.2\text{ N}\cdot\text{m})\mathbf{j}$ ◀



PROBLEM 3.99

A 46-lb force **F** and a 2120-lb · in. couple **M** are applied to corner A of the block shown. Replace the given force-couple system with an equivalent force-couple system at corner H.

SOLUTION

We have $$d_{AJ} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23 \text{ in.}$$

Then $$\mathbf{F} = \frac{46 \text{ lb}}{23}(18\mathbf{i} - 14\mathbf{j} - 3\mathbf{k})$$
$$= (36 \text{ lb})\mathbf{i} - (28 \text{ lb})\mathbf{j} - (6 \text{ lb})\mathbf{k}$$

Also $$d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53 \text{ in.}$$

Then $$\mathbf{M} = \frac{2120 \text{ lb}\cdot\text{in.}}{53}(-45\mathbf{i} - 28\mathbf{k})$$
$$= -(1800 \text{ lb}\cdot\text{in.})\mathbf{i} - (1120 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Now $$\mathbf{M}' = \mathbf{M} + \mathbf{r}_{A/H} \times \mathbf{F}$$

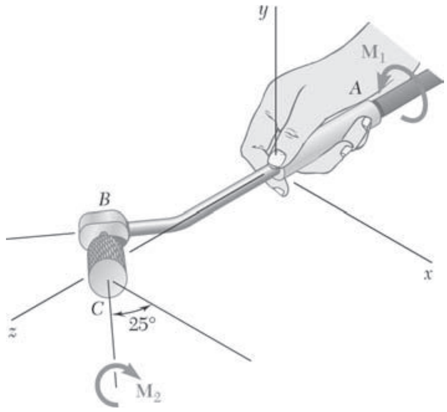
where $$\mathbf{r}_{A/H} = (45 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{j}$$

Then $$\mathbf{M}' = (-1800\mathbf{i} - 1120\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix}$$

$$= (-1800\mathbf{i} - 1120\mathbf{k}) + \{[(14)(-6)]\mathbf{i} + [-(45)(-6)]\mathbf{j} + [(45)(-28) - (14)(36)]\mathbf{k}\}$$
$$= (-1800 - 84)\mathbf{i} + (270)\mathbf{j} + (-1120 - 1764)\mathbf{k}$$
$$= -(1884 \text{ lb}\cdot\text{in.})\mathbf{i} + (270 \text{ lb}\cdot\text{in.})\mathbf{j} - (2884 \text{ lb}\cdot\text{in.})\mathbf{k}$$
$$= -(157 \text{ lb}\cdot\text{ft})\mathbf{i} + (22.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (240 \text{ lb}\cdot\text{ft})\mathbf{k}$$

The equivalent force-couple system at H is $$\mathbf{F}' = (36.0 \text{ lb})\mathbf{i} - (28.0 \text{ lb})\mathbf{j} - (6.00 \text{ lb})\mathbf{k} \blacktriangleleft$$

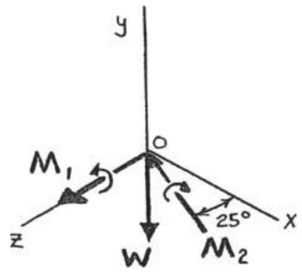
$$\mathbf{M}' = -(157 \text{ lb}\cdot\text{ft})\mathbf{i} + (22.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (240 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.100

The handpiece for a miniature industrial grinder weighs 0.6 lb, and its center of gravity is located on the y axis. The head of the handpiece is offset in the xz plane in such a way that line BC forms an angle of 25° with the x direction. Show that the weight of the handpiece and the two couples $\mathbf{M}_1$ and $\mathbf{M}_2$ can be replaced with a single equivalent force. Further, assuming that $M_1 = 0.68$ lb · in. and $M_2 = 0.65$ lb · in., determine (a) the magnitude and the direction of the equivalent force, (b) the point where its line of action intersects the xz plane.

SOLUTION



First assume that the given force **W** and couples $\mathbf{M}_1$ and $\mathbf{M}_2$ act at the origin.

Now $\qquad \mathbf{W} = -W_{\mathbf{j}}$

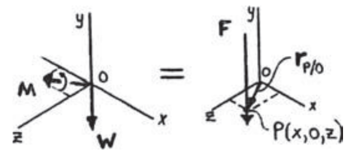
and
$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$
$$= -(M_2 \cos 25°)\mathbf{i} + (M_1 - M_2 \sin 25°)\mathbf{k}$$

Note that since **W** and **M** are perpendicular, it follows that they can be replaced with a single equivalent force.

(a) We have $\qquad F = \mathbf{W} \quad \text{or} \quad \mathbf{F} = -W_{\mathbf{j}} = -(0.6 \text{ lb})\mathbf{j}$ $\qquad$ or $\mathbf{F} = -(0.600 \text{ lb})\mathbf{j}$ ◀

(b) Assume that the line of action of **F** passes through Point $P(x, 0, z)$. Then for equivalence

$$\mathbf{M} = \mathbf{r}_{P/0} \times \mathbf{F}$$



where $\qquad \mathbf{r}_{P/0} = x\mathbf{i} + z\mathbf{k}$

$$-(M_2 \cos 25°)\mathbf{i} + (M_1 - M_2 \sin 25°)\mathbf{k}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 0 & -W & 0 \end{vmatrix} = (Wz)\mathbf{i} - (Wx)\mathbf{k}$$

Equating the **i** and **k** coefficients, $\qquad z = \dfrac{-M_z \cos 25°}{W} \quad$ and $\quad x = -\left(\dfrac{M_1 - M_2 \sin 25°}{W}\right)$

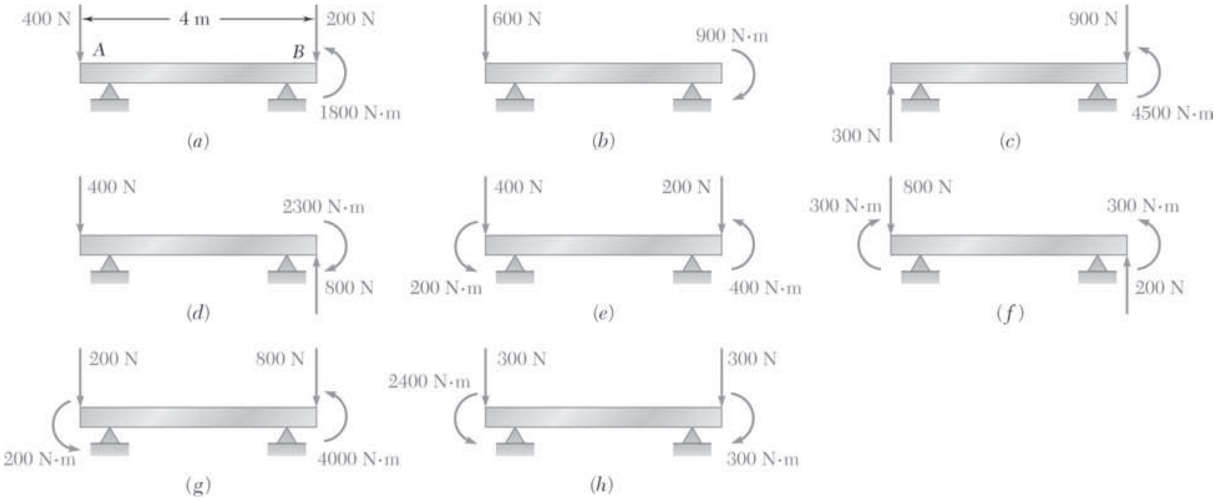
(b) For $\qquad W = 0.6 \text{ lb} \quad M_1 = 0.68 \text{ lb} \cdot \text{in.} \quad M_2 = 0.65 \text{ lb} \cdot \text{in.}$

$$x = \frac{0.68 - 0.65 \sin 25°}{-0.6} = 0.67550 \text{ in.}$$
or $x = 0.675$ in. ◀

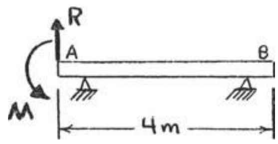
$$z = \frac{-0.65 \cos 25°}{0.6} = -0.98183 \text{ in.}$$
or $z = -0.982$ in. ◀

PROBLEM 3.101

A 4-m-long beam is subjected to a variety of loadings. (*a*) Replace each loading with an equivalent force-couple system at end *A* of the beam. (*b*) Which of the loadings are equivalent?



SOLUTION



(*a*) (*a*) We have ΣF_y: $-400\text{ N} - 200\text{ N} = R_a$

or $\mathbf{R}_a = 600\text{ N}\downarrow$ ◀

and ΣM_A: $1800\text{ N}\cdot\text{m} - (200\text{ N})(4\text{ m}) = M_a$

or $\mathbf{M}_a = 1000\text{ N}\cdot\text{m}$ ◀

(*b*) We have ΣF_y: $-600\text{ N} = R_b$

or $\mathbf{R}_b = 600\text{ N}\downarrow$ ◀

and ΣM_A: $-900\text{ N}\cdot\text{m} = M_b$

or $\mathbf{M}_b = 900\text{ N}\cdot\text{m}$ ◀

(*c*) We have ΣF_y: $300\text{ N} - 900\text{ N} = R_c$

or $\mathbf{R}_c = 600\text{ N}\downarrow$ ◀

and ΣM_A: $4500\text{ N}\cdot\text{m} - (900\text{ N})(4\text{ m}) = M_c$

or $\mathbf{M}_c = 900\text{ N}\cdot\text{m}$ ◀

PROBLEM 3.101 (Continued)

(*d*) We have $\Sigma F_y\!: \quad -400\text{ N} + 800\text{ N} = R_d$

or $\mathbf{R}_d = 400\text{ N}\uparrow$ ◀

and $\Sigma M_A\!: \quad (800\text{ N})(4\text{ m}) - 2300\text{ N}\cdot\text{m} = M_d$

or $\mathbf{M}_d = 900\text{ N}\cdot\text{m}$ ↻ ◀

(*e*) We have $\Sigma F_y\!: \quad -400\text{ N} - 200\text{ N} = R_e$

or $\mathbf{R}_e = 600\text{ N}\downarrow$ ◀

and $\Sigma M_A\!: \quad 200\text{ N}\cdot\text{m} + 400\text{ N}\cdot\text{m} - (200\text{ N})(4\text{ m}) = M_e$

or $\mathbf{M}_e = 200\text{ N}\cdot\text{m}$ ↺ ◀

(*f*) We have $\Sigma F_y\!: \quad -800\text{ N} + 200\text{ N} = R_f$

or $\mathbf{R}_f = 600\text{ N}\downarrow$ ◀

and $\Sigma M_A\!: \quad -300\text{ N}\cdot\text{m} + 300\text{ N}\cdot\text{m} + (200\text{ N})(4\text{ m}) = M_f$

or $\mathbf{M}_f = 800\text{ N}\cdot\text{m}$ ↻ ◀

(*g*) We have $\Sigma F_y\!: \quad -200\text{ N} - 800\text{ N} = R_g$

or $\mathbf{R}_g = 1000\text{ N}\downarrow$ ◀

and $\Sigma M_A\!: \quad 200\text{ N}\cdot\text{m} + 4000\text{ N}\cdot\text{m} - (800\text{ N})(4\text{ m}) = M_g$

or $\mathbf{M}_g = 1000\text{ N}\cdot\text{m}$ ↻ ◀

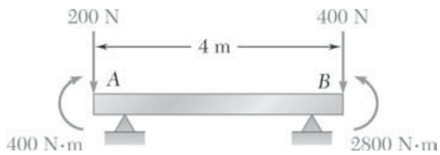
(*h*) We have $\Sigma F_y\!: \quad -300\text{ N} - 300\text{ N} = R_h$

or $\mathbf{R}_h = 600\text{ N}\downarrow$ ◀

and $\Sigma M_A\!: \quad 2400\text{ N}\cdot\text{m} - 300\text{ N}\cdot\text{m} - (300\text{ N})(4\text{ m}) = M_h$

or $\mathbf{M}_h = 900\text{ N}\cdot\text{m}$ ↻ ◀

(*b*) Therefore, loadings (*c*) and (*h*) are equivalent. ◀



PROBLEM 3.102

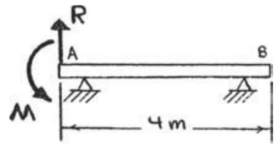
A 4-m-long beam is loaded as shown. Determine the loading of Problem 3.101 which is equivalent to this loading.

SOLUTION

We have ΣF_y: $-200\text{ N} - 400\text{ N} = R$ or $\mathbf{R} = 600\text{ N}\downarrow$ ◀

and ΣM_A: $-400\text{ N}\cdot\text{m} + 2800\text{ N}\cdot\text{m} - (400\text{ N})(4\text{ m}) = M$

or $\mathbf{M} = 800\text{ N}\cdot\text{m}\ \circlearrowleft$



Equivalent to case (*f*) of Problem 3.101 ◀

Problem 3.101 Equivalent force-couples at *A*

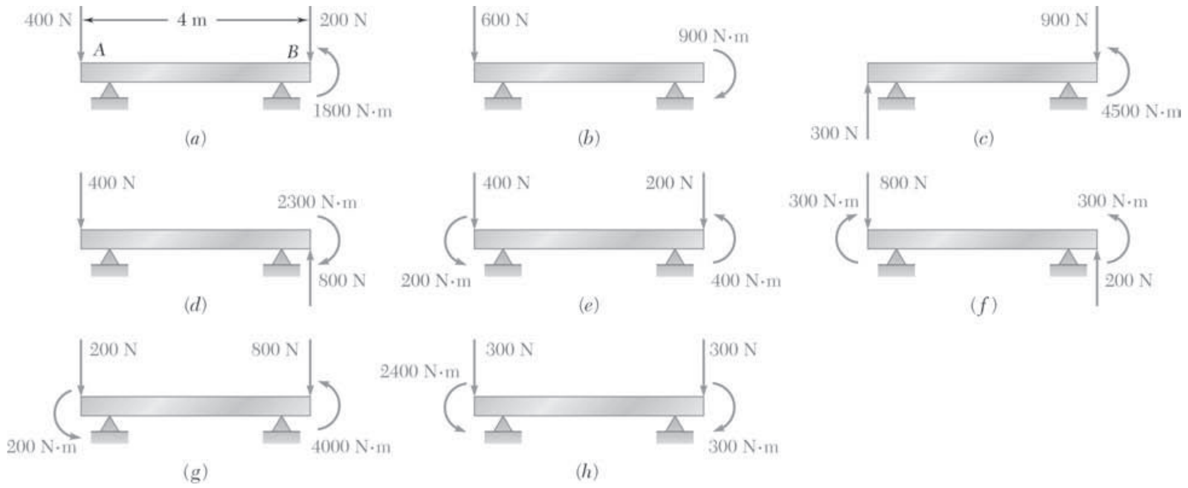
Case	**R**	**M**
(*a*)	$600\text{ N}\downarrow$	$1000\text{ N}\cdot\text{m}\ \circlearrowleft$
(*b*)	$600\text{ N}\downarrow$	$900\text{ N}\cdot\text{m}\ \circlearrowright$
(*c*)	$600\text{ N}\downarrow$	$900\text{ N}\cdot\text{m}\ \circlearrowleft$
(*d*)	$400\text{ N}\uparrow$	$900\text{ N}\cdot\text{m}\ \circlearrowleft$
(*e*)	$600\text{ N}\downarrow$	$200\text{ N}\cdot\text{m}\ \circlearrowright$
(*f*)	$600\text{ N}\downarrow$	$800\text{ N}\cdot\text{m}\ \circlearrowleft$
(*g*)	$1000\text{ N}\downarrow$	$1000\text{ N}\cdot\text{m}\ \circlearrowleft$
(*h*)	$600\text{ N}\downarrow$	$900\text{ N}\cdot\text{m}\ \circlearrowleft$

◀

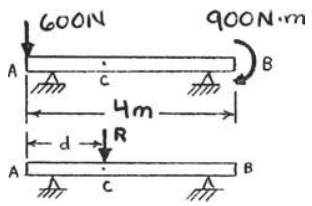
PROBLEM 3.103

Determine the single equivalent force and the distance from Point A to its line of action for the beam and loading of (a) Problem 3.101b, (b) Problem 3.101d, (c) Problem 3.101e.

PROBLEM 3.101 A 4-m-long beam is subjected to a variety of loadings. (a) Replace each loading with an equivalent force-couple system at end A of the beam. (b) Which of the loadings are equivalent?



SOLUTION



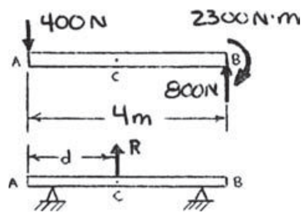
(a) For equivalent single force at distance d from A

We have $\quad \Sigma F_y: \quad -600\text{ N} = R$

or $\mathbf{R} = 600\text{ N}\downarrow$ ◀

and $\quad \Sigma M_C: \quad (600\text{ N})(d) - 900\text{ N}\cdot\text{m} = 0$

or $d = 1.500\text{ m}$ ◀

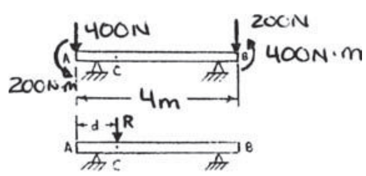


(b) We have $\quad \Sigma F_y: \quad -400\text{ N} + 800\text{ N} = R$

or $\mathbf{R} = 400\text{ N}\uparrow$ ◀

and $\quad \Sigma M_C: \quad (400\text{ N})(d) + (800\text{ N})(4-d)$
$\quad -2300\text{ N}\cdot\text{m} = 0$

or $d = 2.25\text{ m}$ ◀



(c) We have $\quad \Sigma F_y: \quad -400\text{ N} - 200\text{ N} = R$

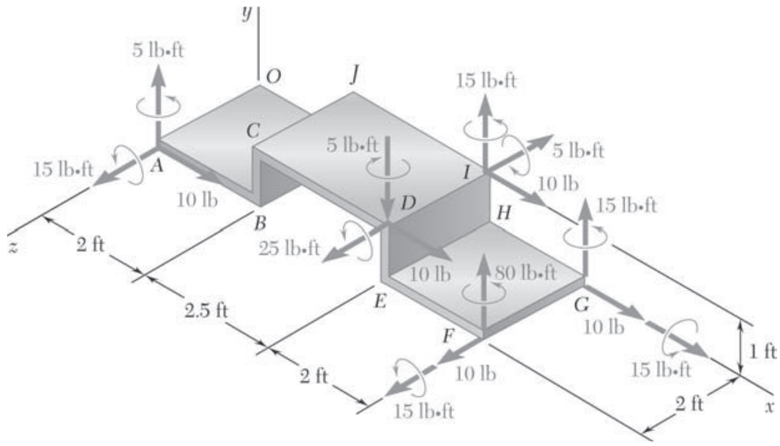
or $\mathbf{R} = 600\text{ N}\downarrow$ ◀

and $\quad \Sigma M_C: \quad 200\text{ N}\cdot\text{m} + (400\text{ N})(d)$
$\quad -(200\text{ N})(4-d) + 400\text{ N}\cdot\text{m} = 0$

or $d = 0.333\text{ m}$ ◀

PROBLEM 3.104

Five separate force-couple systems act at the corners of a piece of sheet metal, which has been bent into the shape shown. Determine which of these systems is equivalent to a force $\mathbf{F} = (10 \text{ lb})\mathbf{i}$ and a couple of moment $\mathbf{M} = (15 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k}$ located at the origin.



SOLUTION

First note that the force-couple system at F cannot be equivalent because of the direction of the force [The force of the other four systems is $(10 \text{ lb})\mathbf{i}$]. Next move each of the systems to the origin O; the forces remain unchanged.

$$A: \quad \mathbf{M}_A = \Sigma \mathbf{M}_O = (5 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k} + (2 \text{ ft})\mathbf{k} \times (10 \text{ lb})\mathbf{i}$$

$$= (25 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k}$$

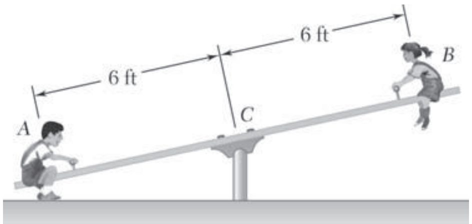
$$D: \quad \mathbf{M}_D = \Sigma \mathbf{M}_O = -(5 \text{ lb} \cdot \text{ft})\mathbf{j} + (25 \text{ lb} \cdot \text{ft})\mathbf{k}$$
$$+ [(4.5 \text{ ft})\mathbf{j} + (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}] \times 10 \text{ lb})\mathbf{i}$$
$$= (15 \text{ lb} \cdot \text{ft})\mathbf{i} + (15 \text{ lb} \cdot \text{ft})\mathbf{k}$$

$$G: \quad \mathbf{M}_G = \Sigma \mathbf{M}_O = (15 \text{ lb} \cdot \text{ft})\mathbf{i} + (15 \text{ lb} \cdot \text{ft})\mathbf{j}$$

$$I: \quad \mathbf{M}_I = \Sigma \mathbf{M}_I = (15 \text{ lb} \cdot \text{ft})\mathbf{j} - (5 \text{ lb} \cdot \text{ft})\mathbf{k}$$
$$+ [(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j}] \times (10 \text{ lb})\mathbf{j}$$

$$= (15 \text{ lb} \cdot \text{ft})\mathbf{j} - (15 \text{ lb} \cdot \text{ft})\mathbf{k}$$

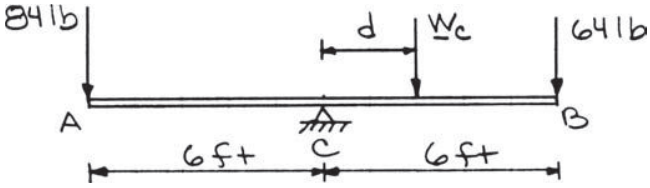
The equivalent force-couple system is the system at corner D. ◀



PROBLEM 3.105

The weights of two children sitting at ends *A* and *B* of a seesaw are 84 lb and 64 lb, respectively. Where should a third child sit so that the resultant of the weights of the three children will pass through *C* if she weighs (*a*) 60 lb, (*b*) 52 lb.

SOLUTION



(*a*) For the resultant weight to act at *C*, $\Sigma M_C = 0 \quad W_C = 60 \text{ lb}$

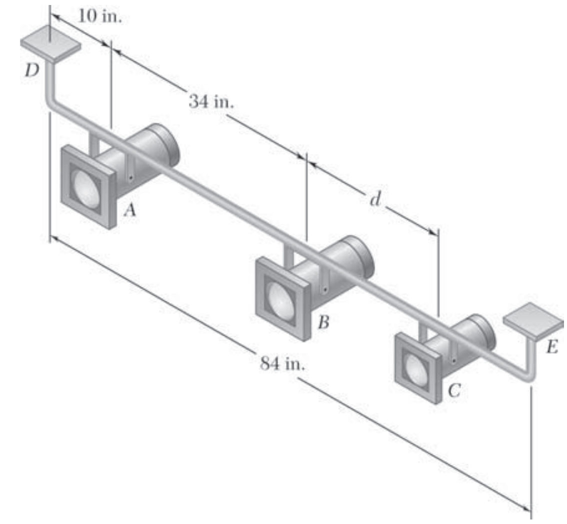
Then $(84 \text{ lb})(6 \text{ ft}) - 60 \text{ lb}(d) - 64 \text{ lb}(6 \text{ ft}) = 0$

$d = 2.00$ ft to the right of C ◀

(*b*) For the resultant weight to act at *C*, $\Sigma M_C = 0 \quad W_C = 52 \text{ lb}$

Then $(84 \text{ lb})(6 \text{ ft}) - 52 \text{ lb}(d) - 64 \text{ lb}(6 \text{ ft}) = 0$

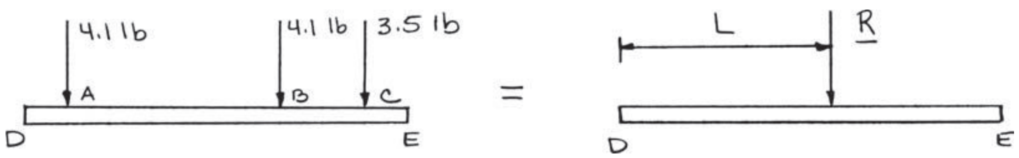
$d = 2.31$ ft to the right of C ◀



PROBLEM 3.106

Three stage lights are mounted on a pipe as shown. The lights at A and B each weigh 4.1 lb, while the one at C weighs 3.5 lb. (*a*) If d = 25 in., determine the distance from D to the line of action of the resultant of the weights of the three lights. (*b*) Determine the value of d so that the resultant of the weights passes through the midpoint of the pipe.

SOLUTION



For equivalence

$$\Sigma F_y: \quad -4.1 - 4.1 - 3.5 = -R \quad \text{or} \quad \mathbf{R} = 11.7 \text{ lb} \downarrow$$

$$\Sigma F_D: \quad -(10 \text{ in.})(4.1 \text{ lb}) - (44 \text{ in.})(4.1 \text{ lb})$$
$$-[(4.4 + d)\text{in.}](3.5 \text{ lb}) = -(L \text{ in.})(11.7 \text{ lb})$$

or $$375.4 + 3.5d = 11.7L \quad (d, L \text{ in in.})$$

(*a*) $$d = 25 \text{ in.}$$

We have $$375.4 + 3.5(25) = 11.7L \quad \text{or} \quad L = 39.6 \text{ in.}$$

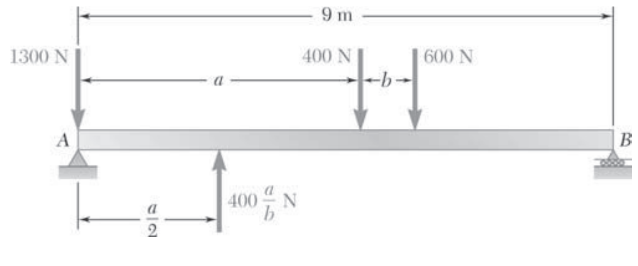
The resultant passes through a Point 39.6 in. to the right of D. ◀

(*b*) $$L = 42 \text{ in.}$$

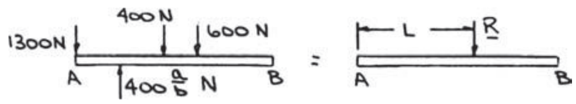
We have $$375.4 + 3.5d = 11.7(42)$$ or $d = 33.1$ in. ◀

PROBLEM 3.107

A beam supports three loads of given magnitude and a fourth load whose magnitude is a function of position. If $b = 1.5$ m and the loads are to be replaced with a single equivalent force, determine (*a*) the value of *a* so that the distance from support *A* to the line of action of the equivalent force is maximum, (*b*) the magnitude of the equivalent force and its point of application on the beam.



SOLUTION



For equivalence

$$\Sigma F_y: \quad -1300 + 400\frac{a}{b} - 400 - 600 = -R$$

or

$$R = \left(2300 - 400\frac{a}{b}\right)\text{ N} \tag{1}$$

$$\Sigma M_A: \quad \frac{a}{2}\left(400\frac{a}{b}\right) - a(400) - (a+b)(600) = -LR$$

or

$$L = \frac{1000a + 600b - 200\dfrac{a^2}{b}}{2300 - 400\dfrac{a}{b}}$$

Then with

$$b = 1.5\text{ m} \quad L = \frac{10a + 9 - \dfrac{4}{3}a^2}{23 - \dfrac{8}{3}a} \tag{2}$$

Where *a*, *L* are in m

(*a*) Find value of *a* to maximize *L*

$$\frac{dL}{da} = \frac{\left(10 - \dfrac{8}{3}a\right)\left(23 - \dfrac{8}{3}a\right) - \left(10a + 9 - \dfrac{4}{3}a^2\right)\left(-\dfrac{8}{3}\right)}{\left(23 - \dfrac{8}{3}a\right)^2}$$

or $$230 - \frac{184}{3}a - \frac{80}{3}a + \frac{64}{9}a^2 + \frac{80}{3}a + 24 - \frac{32}{9}a^2 = 0$$

or $$16a^2 - 276a + 1143 = 0$$

Then $$a = \frac{276 \pm \sqrt{(-276)^2 - 4(16)(1143)}}{2(16)}$$

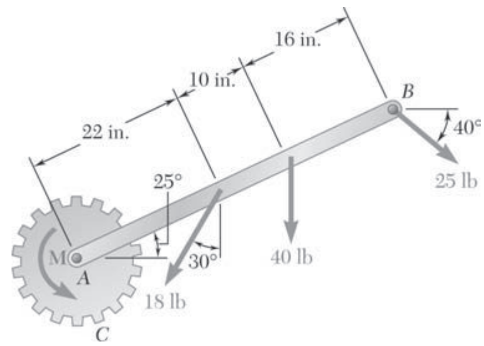
or $a = 10.3435$ m and $a = 6.9065$ m

Since $AB = 9$ m, a must be less than 9 m $\qquad a = 6.91 \text{ m} \blacktriangleleft$

(*b*) Using Eq. (1) $$R = 2300 - 400\frac{6.9065}{1.5}$$ or $R = 458 \text{ N} \blacktriangleleft$

and using Eq. (2) $$L = \frac{10(6.9065) + 9 - \frac{4}{3}(6.9065)^2}{23 - \frac{8}{3}(6.9065)} = 3.16 \text{ m}$$

R is applied 3.16 m to the right of *A*. ◀

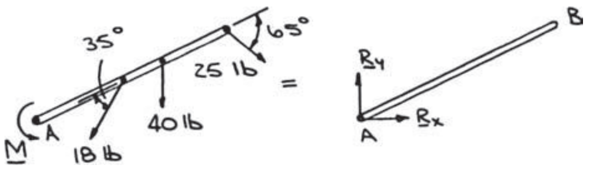


PROBLEM 3.108

Gear C is rigidly attached to arm AB. If the forces and couple shown can be reduced to a single equivalent force at A, determine the equivalent force and the magnitude of the couple $\mathbf{M}$.

SOLUTION

We have



For equivalence

$$\Sigma F_x: \quad -18\sin 30° + 25\cos 40° = R_x$$

or $\quad R_x = 10.1511 \text{ lb}$

$$\Sigma F_y: \quad -18\cos 30° - 40 - 25\sin 40° = R_y$$

or $\quad R_y = -71.658 \text{ lb}$

Then $\quad R = \sqrt{(10.1511)^2 + (71.658)^2}$

$$= 72.416$$

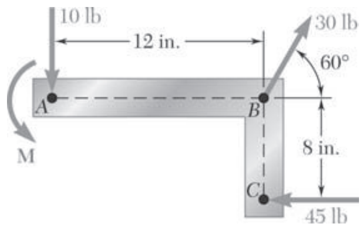
$$\tan\theta = \frac{71.658}{10.1511}$$

or $\quad \theta = 81.9° \qquad \mathbf{R} = 72.4 \text{ lb} \searrow 81.9°$ ◀

Also $\quad \Sigma M_A: \quad M - (22 \text{ in.})(18 \text{ lb})\sin 35° - (32 \text{ in.})(40 \text{ lb})\cos 25°$

$$- (48 \text{ in.})(25 \text{ lb})\sin 65° = 0$$

$M = 2474.8 \text{ lb}\cdot\text{in.} \qquad$ or $\quad M = 206 \text{ lb}\cdot\text{ft}$ ◀



PROBLEM 3.109

A couple of magnitude $M = 54$ lb · in. and the three forces shown are applied to an angle bracket. (*a*) Find the resultant of this system of forces. (*b*) Locate the points where the line of action of the resultant intersects line *AB* and line *BC*.

SOLUTION

(*a*) We have $\Sigma \mathbf{F}: \quad \mathbf{R} = (-10\mathbf{j}) + (30 \cos 60°)\mathbf{i}$

$$+30 \sin 60° \mathbf{j} + (-45\mathbf{i})$$

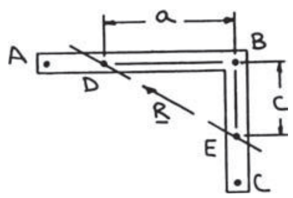
$$= -(30 \text{ lb})\mathbf{i} + (15.9808 \text{ lb})\mathbf{j}$$

or $\mathbf{R} = 34.0$ lb $\searrow$ 28.0° ◀

(*b*) First reduce the given forces and couple to an equivalent force-couple system $(\mathbf{R}, \mathbf{M}_B)$ at *B*.

We have $\Sigma M_B: \quad M_B = (54 \text{ lb} \cdot \text{in}) + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb})$

$$= -186 \text{ lb} \cdot \text{in.}$$



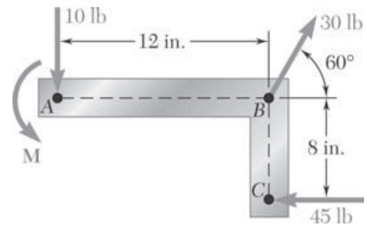
Then with **R** at *D* $\quad \Sigma M_B: \quad -186 \text{ lb} \cdot \text{in} = a(15.9808 \text{ lb})$

or $\quad a = 11.64$ in.

and with **R** at *E* $\quad \Sigma M_B: \quad -186 \text{ lb} \cdot \text{in} = C(30 \text{ lb})$

or $\quad C = 6.2$ in.

The line of action of **R** intersects line *AB* 11.64 in. to the left of *B* and intersects line *BC* 6.20 in. below *B*. ◀



PROBLEM 3.110

A couple **M** and the three forces shown are applied to an angle bracket. Find the moment of the couple if the line of action of the resultant of the force system is to pass through (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*.

SOLUTION

In each case, must have $\mathbf{M}_\mathbf{i}^R = 0$

(*a*) $+\circlearrowleft M_A^B = \Sigma M_A = M + (12 \text{ in.})[(30 \text{ lb})\sin 60°] - (8 \text{ in.})(45 \text{ lb}) = 0$

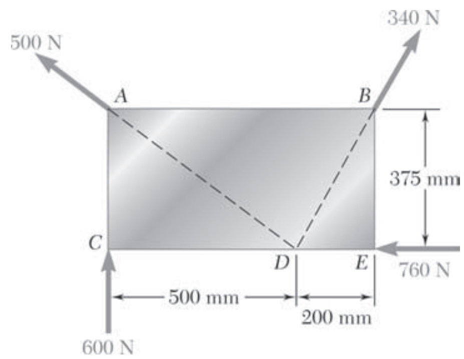
$M = +48.231 \text{ lb} \cdot \text{in.}$ $\qquad \mathbf{M} = 48.2 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*b*) $+\circlearrowleft M_B^R = \Sigma M_B = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb}) = 0$

$M = +240 \text{ lb} \cdot \text{in.}$ $\qquad \mathbf{M} = 240 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*c*) $+\circlearrowleft M_C^R = \Sigma M_C = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})[(30 \text{ lb})\cos 60°] = 0$

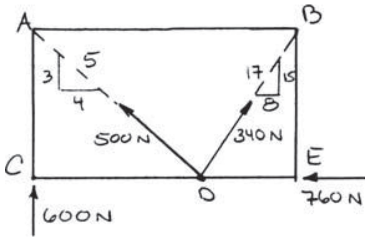
$M = 0$ $\qquad \mathbf{M} = 0$ ◀



PROBLEM 3.111

Four forces act on a 700 × 375-mm plate as shown. (*a*) Find the resultant of these forces. (*b*) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION



(*a*)

$$\mathbf{R} = \Sigma\mathbf{F}$$
$$= (-400\text{ N} + 160\text{ N} - 760\text{ N})\mathbf{i} + (600\text{ N} + 300\text{ N} + 300\text{ N})\mathbf{j}$$
$$= -(1000\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}$$

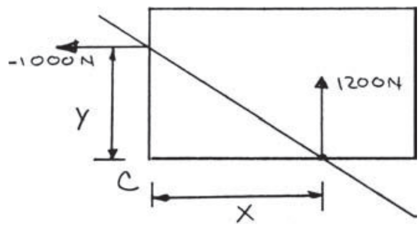
$$R = \sqrt{(1000\text{ N})^2 + (1200\text{ N})^2}$$
$$= 1562.09\text{ N}$$

$$\tan\theta = \left(-\frac{1200\text{ N}}{1000\text{ N}}\right)$$
$$= -1.20000$$
$$\theta = -50.194°$$

$\mathbf{R} = 1562\text{ N} \measuredangle 50.2°$ ◀

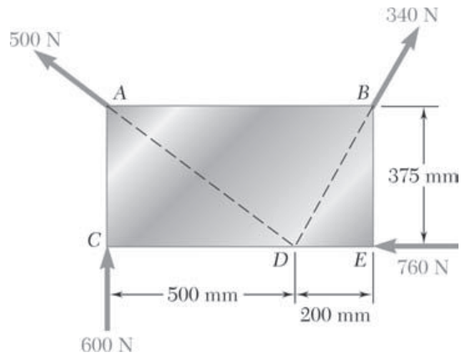
(*b*)

$$\mathbf{M}_C^R = \Sigma\mathbf{r}\times\mathbf{F}$$
$$= (0.5\text{ m})\mathbf{i}\times(300\text{ N} + 300\text{ N})\mathbf{j}$$
$$= (300\text{ N}\cdot\text{m})\mathbf{k}$$



$$(300\text{ N}\cdot\text{m})\mathbf{k} = x\mathbf{i}\times(1200\text{ N})\mathbf{j}$$
$$x = 0.25000\text{ m}$$
$$x = 250\text{ mm}$$
$$(300\text{ N}\cdot\text{m}) = y\mathbf{j}\times(-1000\text{ N})\mathbf{i}$$
$$y = 0.30000\text{ m}$$
$$y = 300\text{ mm}$$

Intersection 250 mm to right of *C* and 300 mm above *C* ◀

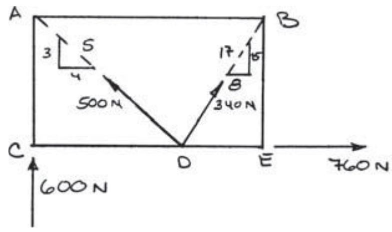


PROBLEM 3.112

Solve Problem 3.111, assuming that the 760-N force is directed to the right.

PROBLEM 3.111 Four forces act on a 700 × 375-mm plate as shown. (*a*) Find the resultant of these forces. (*b*) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION



(*a*)

$$\mathbf{R} = \Sigma \mathbf{F}$$
$$= (-400\text{N} + 160\text{ N} + 760\text{ N})\mathbf{i}$$
$$+ (600\text{ N} + 300\text{ N} + 300\text{ N})\mathbf{j}$$
$$= (520\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}$$

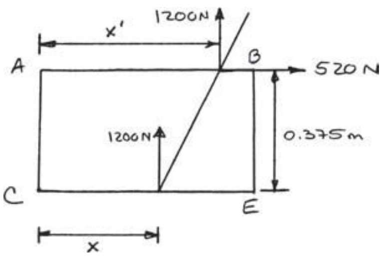
$$R = \sqrt{(520\text{ N})^2 + (1200\text{ N})^2} = 1307.82\text{ N}$$

$$\tan\theta = \left(\frac{1200\text{ N}}{520\text{ N}}\right) = 2.3077$$

$$\theta = 66.5714°$$

$\mathbf{R} = 1308\text{ N} \measuredangle 66.6°$ ◀

(*b*)



$$\mathbf{M}_C^R = \Sigma \mathbf{r} \times \mathbf{F}$$
$$= (0.5\text{ m})\mathbf{i} \times (300\text{ N} + 300\text{ N})\mathbf{j}$$
$$= (300\text{ N} \cdot \text{m})\mathbf{k}$$

$$(300\text{ N} \cdot \text{m})\mathbf{k} = x\mathbf{i} \times (1200\text{ N})\mathbf{j}$$
$$x = 0.25000\text{ m}$$

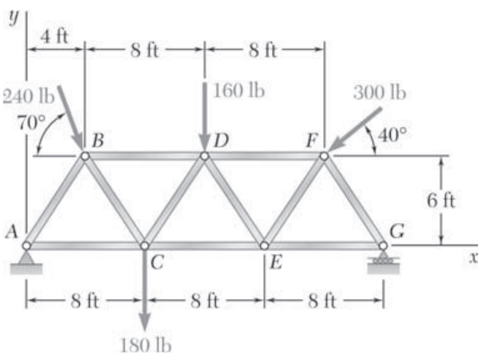
or $\quad x = 0.250\text{ mm}$

$$(300\text{ N} \cdot \text{m})\mathbf{k} = [x'\mathbf{i} + (0.375\text{ m})\mathbf{j}] \times [(520\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}]$$
$$= (1200x' - 195)\mathbf{k}$$

$$x' = 0.41250\text{ m}$$

or $\quad x' = 412.5\text{ mm}$

Intersection 412 mm to the right of *A* and 250 mm to the right of *C* ◀



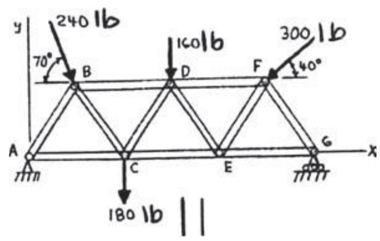
PROBLEM 3.113

A truss supports the loading shown. Determine the equivalent force acting on the truss and the point of intersection of its line of action with a line drawn through Points A and G.

SOLUTION

We have $\quad \mathbf{R} = \Sigma\mathbf{F}$

$$\mathbf{R} = (240\text{ lb})(\cos 70°\mathbf{i} - \sin 70°\mathbf{j}) - (160\text{ lb})\mathbf{j} + (300\text{ lb})(-\cos 40°\mathbf{i} - \sin 40°\mathbf{j}) - (180\text{ lb})\mathbf{j}$$



$$\mathbf{R} = -(147.728\text{ lb})\mathbf{i} - (758.36\text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(147.728)^2 + (758.36)^2}$$

$$= 772.62\text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{-758.36}{-147.728}\right)$$

$$= 78.977°$$

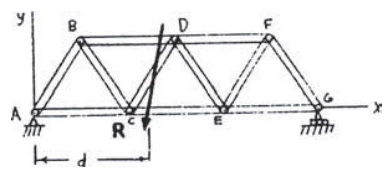
or $\mathbf{R} = 773\text{ lb} \swarrow 79.0°$ ◀

We have $\quad \Sigma M_A = dR_y$

where

$$\Sigma M_A = -[240\text{ lb}\cos 70°](6\text{ ft}) - [240\text{ lb}\sin 70°](4\text{ ft}) - (160\text{ lb})(12\text{ ft}) + [300\text{ lb}\cos 40°](6\text{ ft}) - [300\text{ lb}\sin 40°](20\text{ ft}) - (180\text{ lb})(8\text{ ft})$$

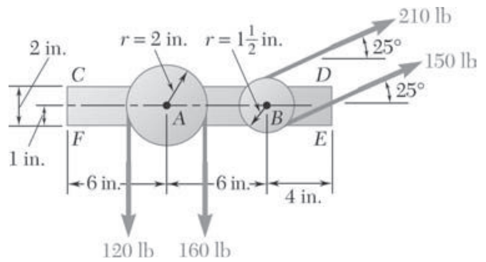
$$= -7232.5\text{ lb}\cdot\text{ft}$$



$$d = \frac{-7232.5\text{ lb}\cdot\text{ft}}{-758.36\text{ lb}}$$

$$= 9.5370\text{ ft}$$

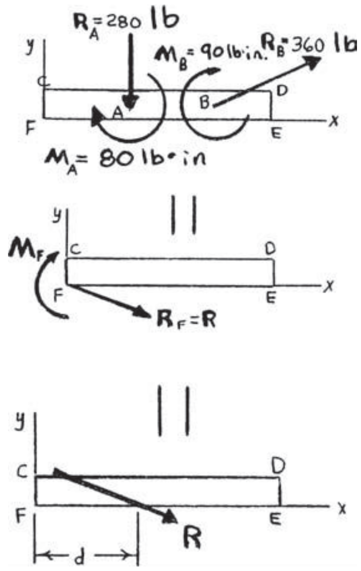
or $d = 9.54$ ft to the right of A ◀



PROBLEM 3.114

Pulleys A and B are mounted on bracket $CDEF$. The tension on each side of the two belts is as shown. Replace the four forces with a single equivalent force, and determine where its line of action intersects the bottom edge of the bracket.

SOLUTION



Equivalent force-couple at A due to belts on pulley A

We have $\qquad \Sigma\mathbf{F}: \quad -120 \text{ lb} - 160 \text{ lb} = R_A$

$$\mathbf{R}_A = 280 \text{ lb} \downarrow$$

We have $\qquad \Sigma\mathbf{M}_A: \quad -40 \text{ lb}(2 \text{ in.}) = M_A$

$$\mathbf{M}_A = 80 \text{ lb} \cdot \text{in.} \circlearrowright$$

Equivalent force-couple at B due to belts on pulley B

We have $\qquad \Sigma\mathbf{F}: \quad (210 \text{ lb} + 150 \text{ lb}) \measuredangle 25° = \mathbf{R}_B$

$$\mathbf{R}_B = 360 \text{ lb} \measuredangle 25°$$

We have $\qquad \Sigma\mathbf{M}_B: \quad -60 \text{ lb}(1.5 \text{ in.}) = M_B$

$$\mathbf{M}_B = 90 \text{ lb} \cdot \text{in.} \circlearrowright$$

Equivalent force-couple at F

We have $\qquad \Sigma\mathbf{F}: \quad \mathbf{R}_F = (-280 \text{ lb})\mathbf{j} + (360 \text{ lb})(\cos 25°\mathbf{i} + \sin 25°\mathbf{j})$

$$= (326.27 \text{ lb})\mathbf{i} - (127.857 \text{ lb})\mathbf{j}$$

$$R = R_F$$

$$= \sqrt{R_{Fx}^2 + R_{Fy}^2}$$

$$= \sqrt{(326.27)^2 + (127.857)^2}$$

$$= 350.43 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{R_{Fy}}{R_{Fx}}\right)$$

$$= \tan^{-1}\left(\frac{-127.857}{326.27}\right)$$

$$= -21.399°$$

or $\mathbf{R}_F = \mathbf{R} = 350 \text{ lb} \searrow 21.4°$ ◀

PROBLEM 3.114 (Continued)

We have $\Sigma \mathbf{M}_F$: $M_F = -(280 \text{ lb})(6 \text{ in.}) - 80 \text{ lb} \cdot \text{in.}$

$$-[(360 \text{ lb})\cos 25°](1.0 \text{ in.})$$
$$+[(360 \text{ lb})\sin 25°](12 \text{ in.}) - 90 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M}_F = -(350.56 \text{ lb} \cdot \text{in.})\mathbf{k}$$

To determine where a single resultant force will intersect line FE,

$$M_F = dR_y$$

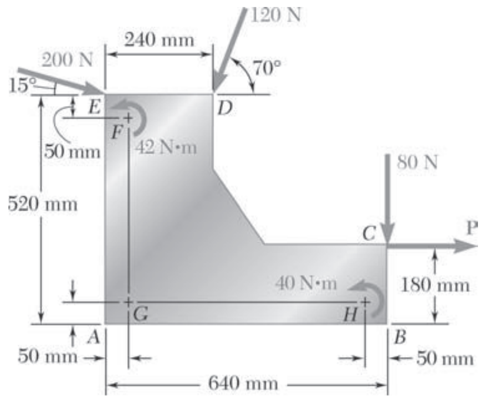
$$d = \frac{M_F}{R_y}$$

$$= \frac{-350.56 \text{ lb} \cdot \text{in.}}{-127 \cdot 857 \text{ lb}}$$

$$= 2.7418 \text{ in.}$$

or $d = 2.74$ in. ◀

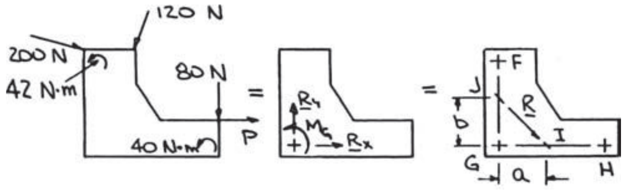
PROBLEM 3.115



A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (*a*) on line *FG*, (*b*) on line *GH*.

SOLUTION

We have



First replace the applied forces and couples with an equivalent force-couple system at *G*.

Thus $\quad \Sigma F_x: \quad 200\cos 15° - 120\cos 70° + P = R_x$

or $\quad R_x = (152.142 + P)\text{ N}$

$\Sigma F_y: \quad -200\sin 15° - 120\sin 70° - 80 = R_y$

or $\quad R_y = -244.53\text{ N}$

$$\begin{aligned}\Sigma M_G: \quad & -(0.47\text{ m})(200\text{ N})\cos 15° + (0.05\text{ m})(200\text{ N})\sin 15° \\ & + (0.47\text{ m})(120\text{ N})\cos 70° - (0.19\text{ m})(120\text{ N})\sin 70° \\ & - (0.13\text{ m})(P\text{ N}) - (0.59\text{ m})(80\text{ N}) + 42\text{ N}\cdot\text{m} \\ & + 40\text{ N}\cdot\text{m} = M_G\end{aligned}$$

or $\quad M_G = -(55.544 + 0.13P)\text{ N}\cdot\text{m} \quad (1)$

Setting $P = 0$ in Eq. (1):

Now with **R** at *I* $\quad \Sigma M_G: \quad -55.544\text{ N}\cdot\text{m} = -a(244.53\text{ N})$

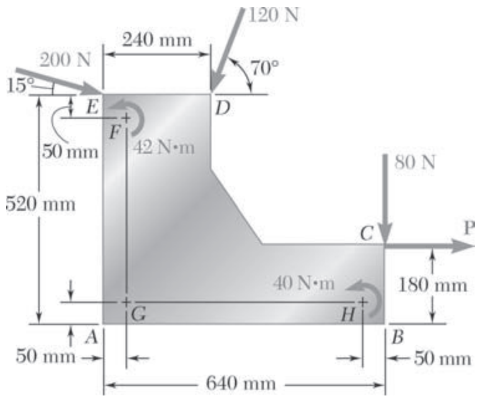
or $\quad a = 0.227\text{ m}$

and with **R** at *J* $\quad \Sigma M_G: \quad -55.544\text{ N}\cdot\text{m} = -b(152.142\text{ N})$

or $\quad b = 0.365\text{ m}$

(*a*) The rivet hole is 0.365 m above *G*. ◀

(*b*) The rivet hole is 0.227 m to the right of *G*. ◀



PROBLEM 3.116

Solve Problem 3.115, assuming that $P = 60$ N.

PROBLEM 3.115 A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (*a*) on line *FG*, (*b*) on line *GH*.

SOLUTION

See the solution to Problem 3.115 leading to the development of Equation (1)

$$M_G = -(55.544 + 0.13P)\ \text{N} \cdot \text{m}$$

and $\qquad R_x = (152.142 + P)\ \text{N}$

For $\qquad P = 60\ \text{N}$

We have $\qquad R_x = (152.142 + 60)$

$$= 212.14\ \text{N}$$

$$M_G = -[55.544 + 0.13(60)]$$

$$= -63.344\ \text{N} \cdot \text{m}$$

Then with **R** at *I* $\qquad \Sigma M_G: \quad -63.344\ \text{N} \cdot \text{m} = -a(244.53\ \text{N})$

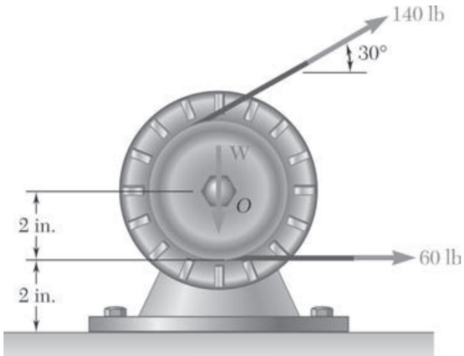
or $\qquad a = 0.259\ \text{m}$

and with **R** at *J* $\qquad \Sigma M_G: \quad -63.344\ \text{N} \cdot \text{m} = -b(212.14\ \text{N})$

or $\qquad b = 0.299\ \text{m}$

(*a*) The rivet hole is 0.299 m above *G*. ◀

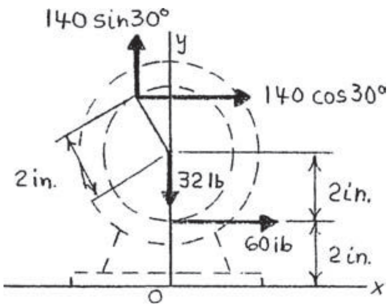
(*b*) The rivet hole is 0.259 m to the right of *G*. ◀



PROBLEM 3.117

A 32-lb motor is mounted on the floor. Find the resultant of the weight and the forces exerted on the belt, and determine where the line of action of the resultant intersects the floor.

SOLUTION



We have

$$\Sigma \mathbf{F}: \quad (60 \text{ lb})\mathbf{i} - (32 \text{ lb})\mathbf{j} + (140 \text{ lb})(\cos 30° \mathbf{i} + \sin 30° \mathbf{j}) = \mathbf{R}$$

$$\mathbf{R} = (181.244 \text{ lb})\mathbf{i} + (38.0 \text{ lb})\mathbf{j}$$

$$\text{or} \quad \mathbf{R} = 185.2 \text{ lb} \measuredangle 11.84° \blacktriangleleft$$

We have $\quad \Sigma M_O: \quad \Sigma M_O = xR_y$

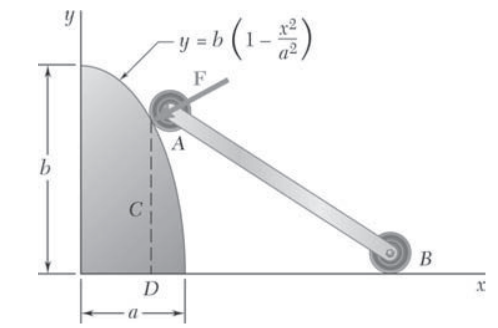
$$-[(140 \text{ lb})\cos 30°][(4 + 2\cos 30°)\text{in.}] - [(140 \text{ lb})\sin 30°][(2 \text{ in.})\sin 30°]$$

$$-(60 \text{ lb})(2 \text{ in.}) = x(38.0 \text{ lb})$$

$$x = \frac{1}{38.0}(-694.97 - 70.0 - 120) \text{ in.}$$

and $\quad x = -23.289$ in.

Or, resultant intersects the base (x axis) 23.3 in. to the left of the vertical centerline (y axis) of the motor. ◀



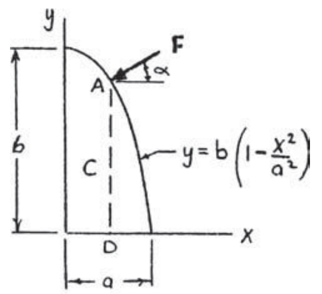
PROBLEM 3.118

As follower AB rolls along the surface of member C, it exerts a constant force **F** perpendicular to the surface. (*a*) Replace **F** with an equivalent force-couple system at the Point D obtained by drawing the perpendicular from the point of contact to the x axis. (*b*) For $a = 1$ m and $b = 2$ m, determine the value of x for which the moment of the equivalent force-couple system at D is maximum.

SOLUTION

(*a*) The slope of any tangent to the surface of member C is

$$\frac{dy}{dx} = \frac{d}{dx}\left[b\left(1 - \frac{x^2}{a^2}\right)\right] = \frac{-2b}{a^2}x$$



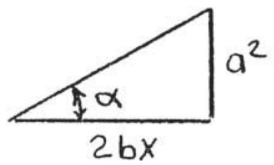
Since the force **F** is perpendicular to the surface,

$$\tan\alpha = -\left(\frac{dy}{dx}\right)^{-1} = \frac{a^2}{2b}\left(\frac{1}{x}\right)$$

For equivalence

$$\Sigma F: \quad \mathbf{F} = \mathbf{R}$$

$$\Sigma M_D: \quad (F\cos\alpha)(y_A) = M_D$$



where

$$\cos\alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}$$

$$y_A = b\left(1 - \frac{x^2}{a^2}\right)$$

$$M_D = \frac{2Fb^2\left(x - \dfrac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}}$$

Therefore, the equivalent force-couple system at D is

$$\mathbf{R} = F \;\measuredangle\; \tan^{-1}\left(\frac{a^2}{2bx}\right) \blacktriangleleft$$

$$\mathbf{M} = \frac{2Fb^2\left(x - \dfrac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}} \blacktriangleleft$$

PROBLEM 3.118 (Continued)

(*b*) To maximize *M*, the value of *x* must satisfy $\frac{dM}{dx} = 0$

where, for $a = 1\text{ m}, \quad b = 2\text{ m}$

$$M = \frac{8F(x - x^3)}{\sqrt{1 + 16x^2}}$$

$$\frac{dM}{dx} = 8F \frac{\sqrt{1 + 16x^2}(1 - 3x^2) - (x - x^3)\left[\frac{1}{2}(32x)(1 + 16x^2)^{-1/2}\right]}{(1 + 16x^2)} = 0$$

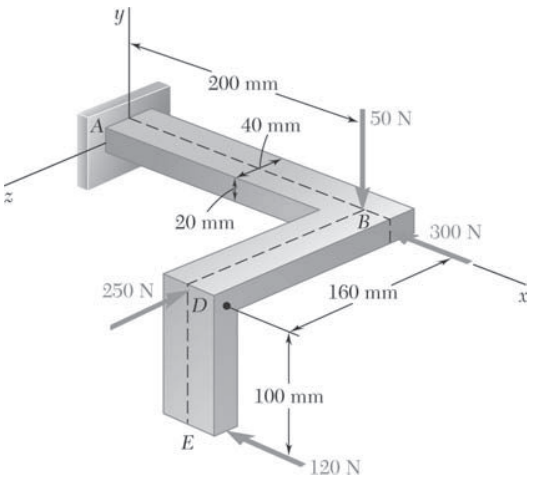
$$(1 + 16x^2)(1 - 3x^2) - 16x(x - x^3) = 0$$

or

$$32x^4 + 3x^2 - 1 = 0$$

$$x^2 = \frac{-3 \pm \sqrt{9 - 4(32)(-1)}}{2(32)} = 0.136011\text{ m}^2 \quad \text{and} \quad -0.22976\text{ m}^2$$

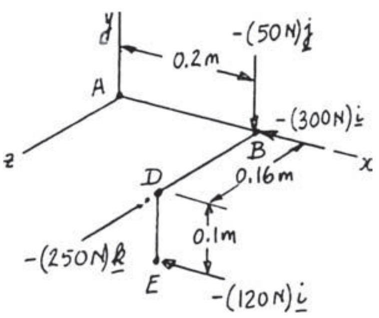
Using the positive value of x^2 $\quad x = 0.36880\text{ m}$ $\quad$ or $x = 369\text{ mm}$ ◀



PROBLEM 3.119

Four forces are applied to the machine component *ABDE* as shown. Replace these forces by an equivalent force-couple system at *A*.

SOLUTION



$$\mathbf{R} = -(50\text{ N})\mathbf{j} - (300\text{ N})\mathbf{i} - (120\text{ N})\mathbf{i} - (250\text{ N})\mathbf{k}$$

$$\mathbf{R} = -(420\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k}$$

$$\mathbf{r}_B = (0.2\text{ m})\mathbf{i}$$

$$\mathbf{r}_D = (0.2\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{k}$$

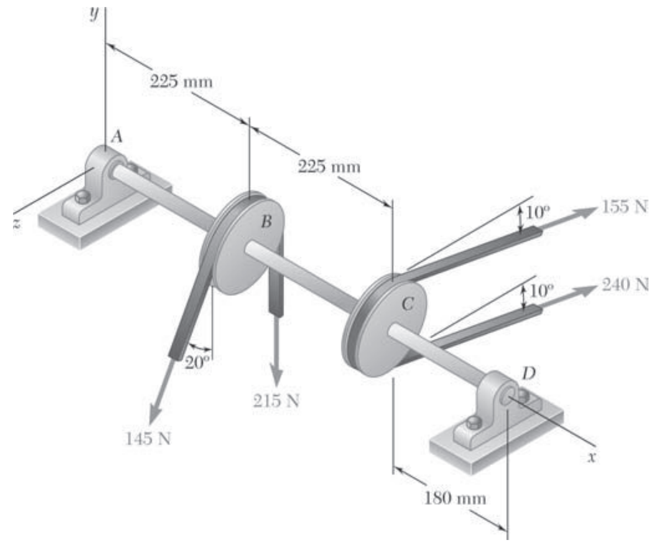
$$\mathbf{r}_E = (0.2\text{ m})\mathbf{i} - (0.1\text{ m})\mathbf{j} + (0.16\text{ m})\mathbf{k}$$

$$\begin{aligned}
\mathbf{M}_A^R &= \mathbf{r}_B \times [-(300\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j}] \\
&\quad + \mathbf{r}_D \times (-250\text{ N})\mathbf{k} + \mathbf{r} \times (-120\text{ N})\mathbf{i} \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & 0 & 0 \\ -300\text{ N} & -50\text{ N} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & 0 & 0.16\text{ m} \\ 0 & 0 & -250\text{ N} \end{vmatrix} \\
&\quad + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & -0.1\text{ m} & 0.16\text{ m} \\ -120\text{ N} & 0 & 0 \end{vmatrix} \\
&= -(10\text{ N}\cdot\text{m})\mathbf{k} + (50\text{ N}\cdot\text{m})\mathbf{j} - (19.2\text{ N}\cdot\text{m})\mathbf{j} - (12\text{ N}\cdot\text{m})\mathbf{k}
\end{aligned}$$

Force-couple system at *A* is

$$\mathbf{R} = -(420\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k} \quad \mathbf{M}_A^R = (30.8\text{ N}\cdot\text{m})\mathbf{j} - (220\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

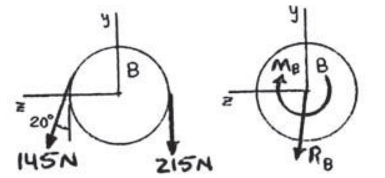
PROBLEM 3.120



Two 150-mm-diameter pulleys are mounted on line shaft AD. The belts at B and C lie in vertical planes parallel to the yz plane. Replace the belt forces shown with an equivalent force-couple system at A.

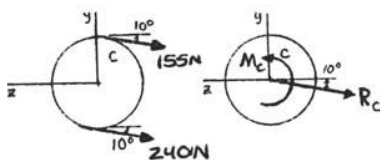
SOLUTION

Equivalent force-couple at each pulley



Pulley B

$$\mathbf{R}_B = (145 \text{ N})(-\cos 20°\mathbf{j} + \sin 20°\mathbf{k}) - 215 \text{ N}\mathbf{j}$$
$$= -(351.26 \text{ N})\mathbf{j} + (49.593 \text{ N})\mathbf{k}$$
$$\mathbf{M}_B = -(215 \text{ N} - 145 \text{ N})(0.075 \text{ m})\mathbf{i}$$
$$= -(5.25 \text{ N}\cdot\text{m})\mathbf{i}$$



Pulley C

$$\mathbf{R}_C = (155 \text{ N} + 240 \text{ N})(-\sin 10°\mathbf{j} - \cos 10°\mathbf{k})$$
$$= -(68.591 \text{ N})\mathbf{j} - (389.00 \text{ N})\mathbf{k}$$
$$\mathbf{M}_C = (240 \text{ N} - 155 \text{ N})(0.075 \text{ m})\mathbf{i}$$
$$= (6.3750 \text{ N}\cdot\text{m})\mathbf{i}$$

Then

$$\mathbf{R} = \mathbf{R}_B + \mathbf{R}_C = -(419.85 \text{ N})\mathbf{j} - (339.41)\mathbf{k}$$

or $\mathbf{R} = (420 \text{ N})\mathbf{j} - (339 \text{ N})\mathbf{k}$ ◀

$$\mathbf{M}_A = \mathbf{M}_B + \mathbf{M}_C + \mathbf{r}_{B/A} \times \mathbf{R}_B + \mathbf{r}_{C/A} \times \mathbf{R}_C$$

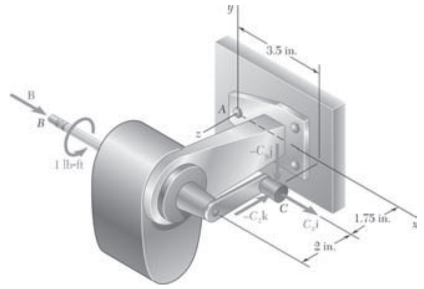
$$= -(5.25 \text{ N}\cdot\text{m})\mathbf{i} + (6.3750 \text{ N}\cdot\text{m})\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.225 & 0 & 0 \\ 0 & -351.26 & 49.593 \end{vmatrix} \text{N}\cdot\text{m}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0 \\ 0 & -68.591 & -389.00 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (1.12500 \text{ N}\cdot\text{m})\mathbf{i} + (163.892 \text{ N}\cdot\text{m})\mathbf{j} - (109.899 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_A = (1.125 \text{ N}\cdot\text{m})\mathbf{i} + (163.9 \text{ N}\cdot\text{m})\mathbf{j} - (109.9 \text{ N}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 3.121



While using a pencil sharpener, a student applies the forces and couple shown. (*a*) Determine the forces exerted at *B* and *C* knowing that these forces and the couple are equivalent to a force-couple system at *A* consisting of the force $\mathbf{R} = (2.6 \text{ lb})\mathbf{i} + R_y\mathbf{j} - (0.7 \text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_A^R = M_x\mathbf{i} + (1.0 \text{ lb}\cdot\text{ft})\mathbf{j} - (0.72 \text{ lb}\cdot\text{ft})\mathbf{k}$. (b) Find the corresponding values of R_y and M_x.

SOLUTION

(*a*) From the statement of the problem, equivalence requires

$$\Sigma\mathbf{F}: \quad \mathbf{B} + \mathbf{C} = \mathbf{R}$$

or

$$\Sigma F_x: \quad B_x + C_x = 2.6 \text{ lb} \qquad (1)$$

$$\Sigma F_y: \quad -C_y = R_y \qquad (2)$$

$$\Sigma F_z: \quad -C_z = -0.7 \text{ lb} \quad \text{or} \quad C_z = 0.7 \text{ lb}$$

and

$$\Sigma\mathbf{M}_A: \quad (\mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{M}_B) + \mathbf{r}_{C/A} \times \mathbf{C} = M_A^R$$

or

$$\Sigma\mathbf{M}_x: \quad (1 \text{ lb}\cdot\text{ft}) + \left(\frac{1.75}{12}\text{ ft}\right)(C_y) = M_x \qquad (3)$$

$$\Sigma M_y: \quad \left(\frac{3.75}{12}\text{ft}\right)(B_x) + \left(\frac{1.75}{12}\text{ft}\right)(C_x) + \left(\frac{3.5}{12}\text{ft}\right)(0.7 \text{ lb}) = 1 \text{ lb}\cdot\text{ft}$$

or

$$3.75B_x + 1.75C_x = 9.55$$

Using Eq. (1)

$$3.75B_x + 1.75(2.6B_x) = 9.55$$

or

$$B_x = 2.5 \text{ lb}$$

and

$$C_x = 0.1 \text{ lb}$$

$$\Sigma M_z: \quad -\left(\frac{3.5}{12}\text{ft}\right)(C_y) = -0.72 \text{ lb}\cdot\text{ft}$$

or

$$C_y = 2.4686 \text{ lb}$$

$$\mathbf{B} = (2.5 \text{ lb})\mathbf{i} \quad \mathbf{C} = (0.1000 \text{ lb})\mathbf{i} - (2.47 \text{ lb})\mathbf{j} - (0.700 \text{ lb})\mathbf{k} \blacktriangleleft$$

(*b*) Eq. (2) $\Rightarrow$

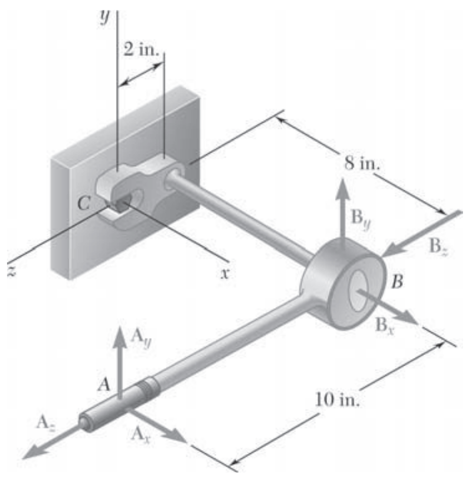
$$R_y = -2.47 \text{ lb} \blacktriangleleft$$

Using Eq. (3)

$$1 + \left(\frac{1.75}{12}\right)(2.4686) = M_x$$

or $M_x = 1.360 \text{ lb}\cdot\text{ft} \blacktriangleleft$

PROBLEM 3.122



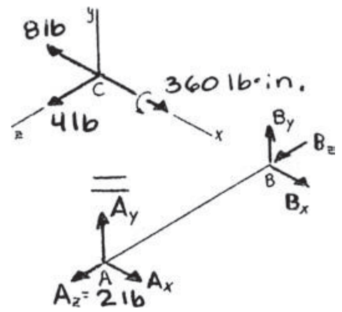
A mechanic uses a crowfoot wrench to loosen a bolt at *C*. The mechanic holds the socket wrench handle at Points *A* and *B* and applies forces at these points. Knowing that these forces are equivalent to a force-couple system at *C* consisting of the force $\mathbf{C} = (8 \text{ lb})\mathbf{i} + (4 \text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_C = (360 \text{ lb} \cdot \text{in.})\mathbf{i}$, determine the forces applied at *A* and at *B* when $A_z = 2$ lb.

SOLUTION

We have $\quad \Sigma \mathbf{F}: \quad \mathbf{A} + \mathbf{B} = \mathbf{C}$

or $\quad F_x: \quad A_x + B_x = 8 \text{ lb}$

$$B_x = -(A_x + 8 \text{ lb}) \tag{1}$$



$\Sigma F_y: \quad A_y + B_y = 0$

or

$$A_y = -B_y \tag{2}$$

$\Sigma F_z: \quad 2 \text{ lb} + B_z = 4 \text{ lb}$

or

$$B_z = 2 \text{ lb} \tag{3}$$

We have $\quad \Sigma \mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{B} + \mathbf{r}_{A/C} \times \mathbf{A} = \mathbf{M}_C$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 2 \\ B_x & B_y & 2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 8 \\ A_x & A_y & 2 \end{vmatrix} \text{lb} \cdot \text{in.} = (360 \text{ lb} \cdot \text{in.})\mathbf{i}$$

or

$$(2B_y - 8A_y)\mathbf{i} + (2B_x - 16 + 8A_x - 16)\mathbf{j}$$

$$+(8B_y + 8A_y)\mathbf{k} = (360 \text{ lb} \cdot \text{in.})\mathbf{i}$$

From **i**-coefficient $\quad 2B_y - 8A_y = 360 \text{ lb} \cdot \text{in.} \quad (4)$

j-coefficient $\quad -2B_x + 8A_x = 32 \text{ lb} \cdot \text{in.} \quad (5)$

k-coefficient $\quad 8B_y + 8A_y = 0 \quad (6)$

PROBLEM 3.122 (Continued)

From Equations (2) and (4): $2B_y - 8(-B_y) = 360$

$$B_y = 36 \text{ lb} \quad A_y = 36 \text{ lb}$$

From Equations (1) and (5): $2(-A_x - 8) + 8A_x = 32$

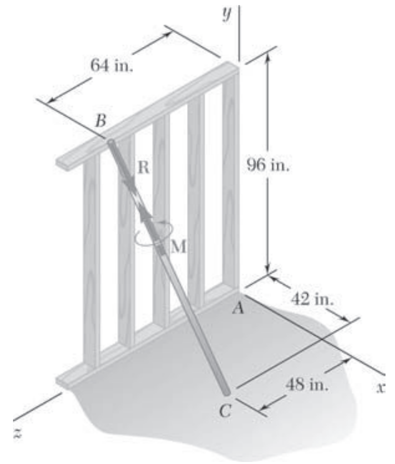
$$A_x = 1.6 \text{ lb}$$

From Equation (1): $B_x = -(1.6 + 8) = -9.6 \text{ lb}$

$$\mathbf{A} = (1.600 \text{ lb})\mathbf{i} - (36.0 \text{ lb})\mathbf{j} + (2.00 \text{ lb})\mathbf{k} \blacktriangleleft$$

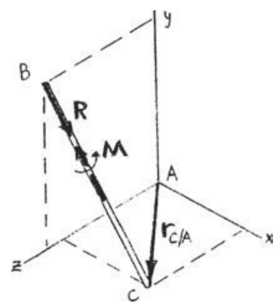
$$\mathbf{B} = -(9.60 \text{ lb})\mathbf{i} + (36.0 \text{ lb})\mathbf{j} + (2.00 \text{ lb})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.123



As an adjustable brace BC is used to bring a wall into plumb, the force-couple system shown is exerted on the wall. Replace this force-couple system with an equivalent force-couple system at A if $R = 21.2$ lb and $M = 13.25$ lb · ft.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{R}_A = \mathbf{R}\lambda_{BC}$

where
$$\boldsymbol{\lambda}_{BC} = \frac{(42 \text{ in.})\mathbf{i} - (96 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}}{106 \text{ in.}}$$

$$\mathbf{R}_A = \frac{21.2 \text{ lb}}{106}(42\mathbf{i} - 96\mathbf{j} - 16\mathbf{k})$$

or
$$\mathbf{R}_A = (8.40 \text{ lb})\mathbf{i} - (19.20 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

We have $\quad \Sigma\mathbf{M}_A: \quad \mathbf{r}_{C/A} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_A$

where
$$\mathbf{r}_{C/A} = (42 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{k} = \frac{1}{12}(42\mathbf{i} + 48\mathbf{k})\text{ft}$$
$$= (3.5 \text{ ft})\mathbf{i} + (4.0 \text{ ft})\mathbf{k}$$

$$\mathbf{R} = (8.40 \text{ lb})\mathbf{i} - (19.50 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k}$$

$$\mathbf{M} = -\lambda_{BC} M$$
$$= \frac{-42\mathbf{i} + 96\mathbf{j} + 16\mathbf{k}}{106}(13.25 \text{ lb} \cdot \text{ft})$$
$$= -(5.25 \text{ lb} \cdot \text{ft})\mathbf{i} + (12 \text{ lb} \cdot \text{ft})\mathbf{j} + (2 \text{ lb} \cdot \text{ft})\mathbf{k}$$

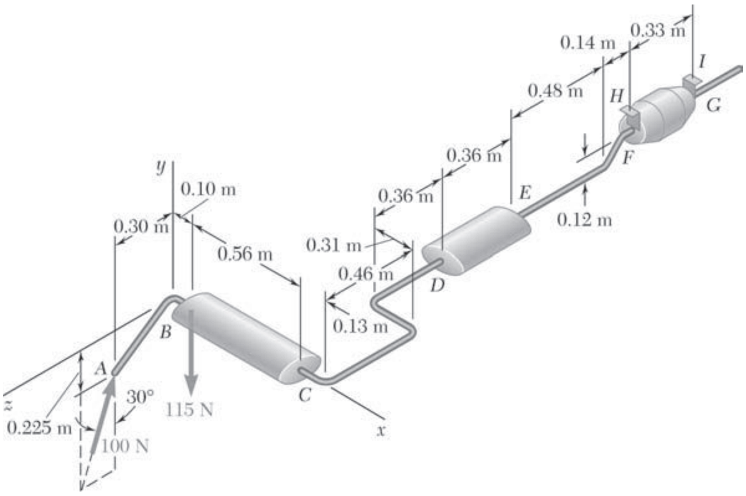
Then
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & 0 & 4.0 \\ 8.40 & -19.20 & -3.20 \end{vmatrix} \text{lb} \cdot \text{ft} + (-5.25\mathbf{i} + 12\mathbf{j} + 2\mathbf{k})\text{lb} \cdot \text{ft} = \mathbf{M}_A$$

$$\mathbf{M}_A = (71.55 \text{ lb} \cdot \text{ft})\mathbf{i} + (56.80 \text{ lb} \cdot \text{ft})\mathbf{j} - (65.20 \text{ lb} \cdot \text{ft})\mathbf{k}$$

or $\mathbf{M}_A = (71.6 \text{ lb} \cdot \text{ft})\mathbf{i} + (56.8 \text{ lb} \cdot \text{ft})\mathbf{j} - (65.2 \text{ lb} \cdot \text{ft})\mathbf{k}$ $\blacktriangleleft$

PROBLEM 3.124

A mechanic replaces a car's exhaust system by firmly clamping the catalytic converter *FG* to its mounting brackets *H* and *I* and then loosely assembling the mufflers and the exhaust pipes. To position the tailpipe *AB*, he pushes in and up at *A* while pulling down at *B*. (*a*) Replace the given force system with an equivalent force-couple system at *D*. (*b*) Determine whether pipe *CD* tends to rotate clockwise or counterclockwise relative to muffler *DE*, as viewed by the mechanic.



SOLUTION

(*a*) Equivalence requires

$$\Sigma \mathbf{F}: \quad \mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$= (100 \text{ N})(\cos 30°\mathbf{j} - \sin 30° \, \mathbf{k}) - (115 \text{ N})\mathbf{j}$$
$$= -(28.4 \text{ N})\mathbf{j} - (50 \text{ N})\mathbf{k}$$

and $\Sigma \mathbf{M}_D: \quad \mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{F}_A + \mathbf{r}_{B/D} \times \mathbf{F}_B$

where $\mathbf{r}_{A/D} = -(0.48 \text{ m})\mathbf{i} - (0.225 \text{ m})\mathbf{j} + (1.12 \text{ m})\mathbf{k}$

$\mathbf{r}_{B/D} = -(0.38 \text{ m})\mathbf{i} + (0.82 \text{ m})\mathbf{k}$

Then

$$\mathbf{M}_D = 100\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.48 & -0.225 & 1.12 \\ 0 & \cos 30° & -\sin 30° \end{vmatrix} + 115\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.38 & 0 & 0.82 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= 100[(0.225 \sin 30° - 1.12 \cos 30°)\mathbf{i} + (-0.48 \sin 30°)\mathbf{j}$$
$$+ (-0.48 \cos 30°)\mathbf{k}] + 115[(0.82)\mathbf{i} + (0.38)\mathbf{k}]$$
$$= 8.56\mathbf{i} - 24.0\mathbf{j} + 2.13\mathbf{k}$$

PROBLEM 3.124 (Continued)

The equivalent force-couple system at D is

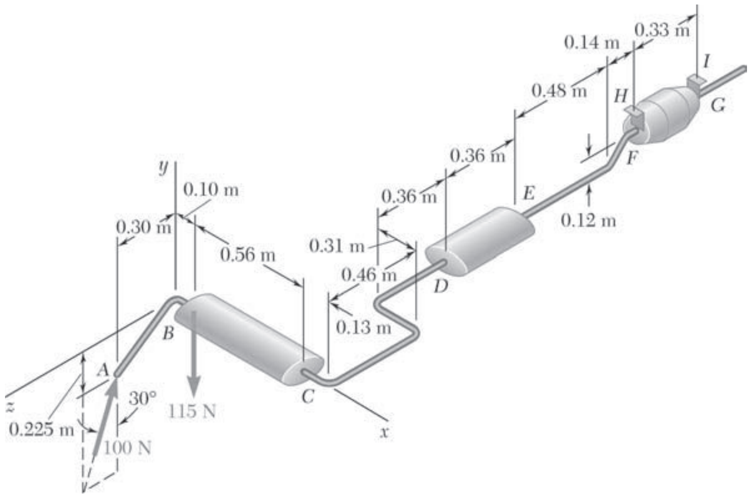
$$\mathbf{R} = -(28.4\ \text{N})\mathbf{j} - (50.0\ \text{N})\mathbf{k} \quad \blacktriangleleft$$

$$M_D = (8.56\ \text{N}\cdot\text{m})\mathbf{i} - (24.0\ \text{N}\cdot\text{m})\mathbf{j} + (2.13\ \text{N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$

(*b*) Since $(M_D)_z$ is positive, pipe *CD* will tend to rotate counterclockwise relative to muffler *DE*. ◀

PROBLEM 3.125

For the exhaust system of Problem 3.124, (*a*) replace the given force system with an equivalent force-couple system at *F*, where the exhaust pipe is connected to the catalytic converter, (*b*) determine whether pipe *EF* tends to rotate clockwise or counterclockwise, as viewed by the mechanic.



SOLUTION

(*a*) Equivalence requires

$$\Sigma \mathbf{F}: \quad \mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$= (100 \text{ N})(\cos 30° \mathbf{j} - \sin 30° \, \mathbf{k}) - (115 \text{ N})\mathbf{j}$$
$$= -(28.4 \text{ N})\mathbf{j} - (50 \text{ N})\mathbf{k}$$

and $\quad \mathbf{M}_F: \quad \mathbf{M}_F = \mathbf{r}_{A/F} \times \mathbf{A} + \mathbf{r}_{B/F} \times \mathbf{B}$

where $\quad \mathbf{r}_{A/F} = -(0.48 \text{ m})\mathbf{i} - (0.345 \text{ m})\mathbf{j} + (2.10 \text{ m})\mathbf{k}$

$\quad \mathbf{r}_{B/F} = -(0.38 \text{ m})\mathbf{i} - (0.12 \text{ m})\mathbf{j} + (1.80 \text{ m})\mathbf{k}$

Then

$$\mathbf{M}_F = 100 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.48 & -0.345 & 2.10 \\ 0 & \cos 30° & -\sin 30° \end{vmatrix} + 115 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.38 & 0.12 & 1.80 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\mathbf{M}_F = 100[(0.345 \sin 30° - 2.10 \cos 30°)\mathbf{i} + (-0.48 \sin 30°)\mathbf{j}$$
$$+ (-0.48 \cos 30°)\mathbf{k}] + 115[(1.80)\mathbf{i} + (0.38)\mathbf{k}]$$
$$= 42.4\mathbf{i} - 24.0\mathbf{j} + 2.13\mathbf{k}$$

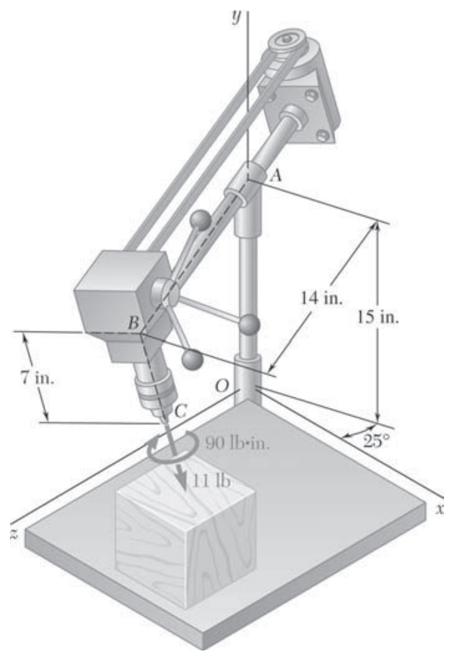
PROBLEM 3.125 (Continued)

The equivalent force-couple system at F is

$$\mathbf{R} = -(28.4\text{ N})\mathbf{j} - (50\text{ N})\mathbf{k} \quad ◀$$

$$\mathbf{M}_F = (42.4\text{ N}\cdot\text{m})\mathbf{i} - (24.0\text{ N}\cdot\text{m})\mathbf{j} + (2.13\text{ N}\cdot\text{m})\mathbf{k} \quad ◀$$

(*b*) Since $(M_F)_z$ is positive, pipe *EF* will tend to rotate counterclockwise relative to the mechanic. ◀



PROBLEM 3.126

The head-and-motor assembly of a radial drill press was originally positioned with arm AB parallel to the z axis and the axis of the chuck and bit parallel to the y axis. The assembly was then rotated 25° about the y axis and 20° about the centerline of the horizontal arm AB, bringing it into the position shown. The drilling process was started by switching on the motor and rotating the handle to bring the bit into contact with the workpiece. Replace the force and couple exerted by the drill press with an equivalent force-couple system at the center O of the base of the vertical column.

SOLUTION

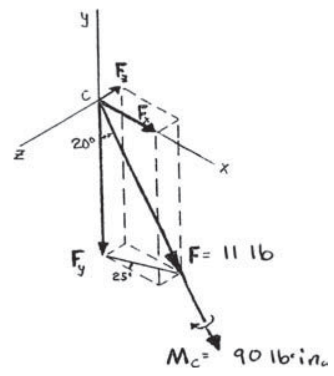
We have $\quad \mathbf{R} = \mathbf{F} = (11 \text{ lb})[(\sin 20° \cos 25°)]\mathbf{i} - (\cos 20°)\mathbf{j} - (\sin 20° \sin 25°)\mathbf{k}]$

$$= (3.4097 \text{ lb})\mathbf{i} - (10.3366 \text{ lb})\mathbf{j} - (1.58998 \text{ lb})\mathbf{k}$$

or $\quad \mathbf{R} = (3.41 \text{ lb})\mathbf{i} - (10.34 \text{ lb})\mathbf{j} - (1.590 \text{ lb})\mathbf{k}$ ◀

We have $\quad \mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F} \times \mathbf{M}_C$

where



$$\mathbf{r}_{B/O} = [(14 \text{ in.}) \sin 25°]\mathbf{i} + (15 \text{ in.})\mathbf{j} + [(14 \text{ in.}) \cos 25°]\mathbf{k}$$

$$= (5.9167 \text{ in.})\mathbf{i} + (15 \text{ in.})\mathbf{j} + (12.6883 \text{ in.})\mathbf{k}$$

$$\mathbf{M}_C = (90 \text{ lb} \cdot \text{in.})[(\sin 20° \cos 25°)\mathbf{i} - (\cos 20°)\mathbf{j} - (\sin 20° \sin 25°)\mathbf{k}]$$

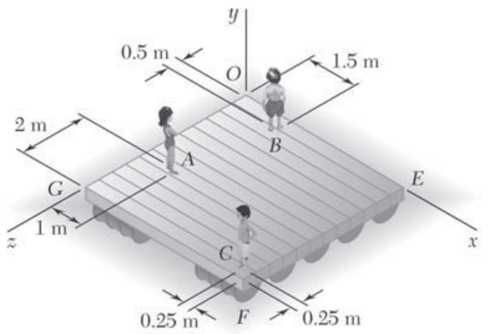
$$= (27.898 \text{ lb} \cdot \text{in.})\mathbf{i} - (84.572 \text{ lb} \cdot \text{in.})\mathbf{j} - (13.0090 \text{ lb} \cdot \text{in.})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5.9167 & 15 & 12.6883 \\ 3.4097 & -10.3366 & 1.58998 \end{vmatrix} \text{lb} \cdot \text{in.}$$

$$+(27.898 - 84.572 - 13.0090) \text{ lb} \cdot \text{in.}$$

$$= (135.202 \text{ lb} \cdot \text{in.})\mathbf{i} - (31.901 \text{ lb} \cdot \text{in.})\mathbf{j} - (125.313 \text{ lb} \cdot \text{in.})\mathbf{k}$$

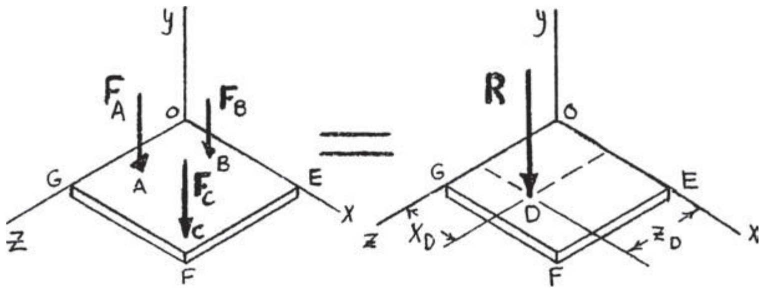
or $\quad \mathbf{M}_O = (135.2 \text{ lb} \cdot \text{in.})\mathbf{i} - (31.9 \text{ lb} \cdot \text{in.})\mathbf{j} - (125.3 \text{ lb} \cdot \text{in.})\mathbf{k}$ ◀



PROBLEM 3.127

Three children are standing on a 5×5-m raft. If the weights of the children at Points *A*, *B*, and *C* are 375 N, 260 N, and 400 N, respectively, determine the magnitude and the point of application of the resultant of the three weights.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$

$$-(375\text{ N})\mathbf{j} - (260\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} = \mathbf{R}$$

$$-(1035\text{ N})\mathbf{j} = \mathbf{R}$$

or $\quad R = 1035\text{ N}$ ◀

We have $\quad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) = R(z_D)$

$$(375\text{ N})(3\text{ m}) + (260\text{ N})(0.5\text{ m}) + (400\text{ N})(4.75\text{ m}) = (1035\text{ N})(z_D)$$

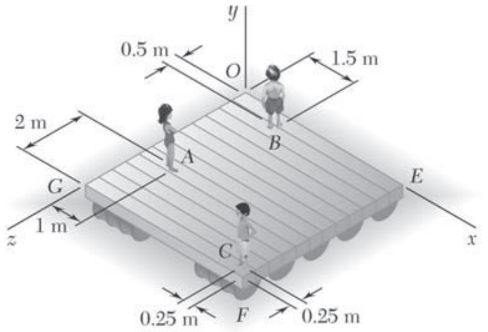
$z_D = 3.0483\text{ m}$ $\quad$ or $\quad z_D = 3.05\text{ m}$ ◀

We have $\quad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) = R(x_D)$

$$375\text{ N}(1\text{ m}) + (260\text{ N})(1.5\text{ m}) + (400\text{ N})(4.75\text{ m}) = (1035\text{ N})(x_D)$$

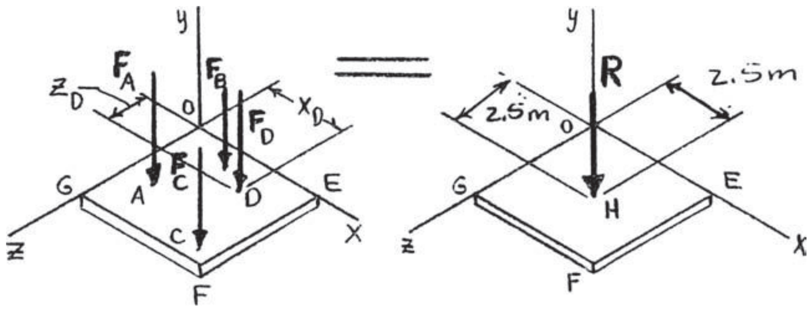
$x_D = 2.5749\text{ m}$ $\quad$ or $\quad x_D = 2.57\text{ m}$ ◀

PROBLEM 3.128



Three children are standing on a 5×5-m raft. The weights of the children at Points *A*, *B*, and *C* are 375 N, 260 N, and 400 N, respectively. If a fourth child of weight 425 N climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$

$$-(375\text{ N})\mathbf{j} - (260\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} - (425\text{ N})\mathbf{j} = \mathbf{R}$$

$$\mathbf{R} = -(1460\text{ N})\mathbf{j}$$

We have $\quad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_H)$

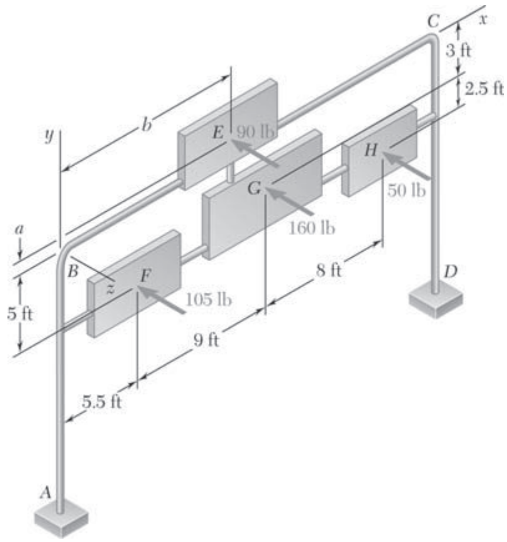
$$(375\text{ N})(3\text{ m}) + (260\text{ N})(0.5\text{ m}) + (400\text{ N})(4.75\text{ m})$$
$$+(425\text{ N})(z_D) = (1460\text{ N})(2.5\text{ m})$$

$z_D = 1.16471\text{ m}$ $\qquad$ or $\quad z_D = 1.165\text{ m}$ ◀

We have $\quad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_H)$

$$(375\text{ N})(1\text{ m}) + (260\text{ N})(1.5\text{ m}) + (400\text{ N})(4.75\text{ m})$$
$$+(425\text{ N})(x_D) = (1460\text{ N})(2.5\text{ m})$$

$x_D = 2.3235\text{ m}$ $\qquad$ or $\quad x_D = 2.32\text{ m}$ ◀

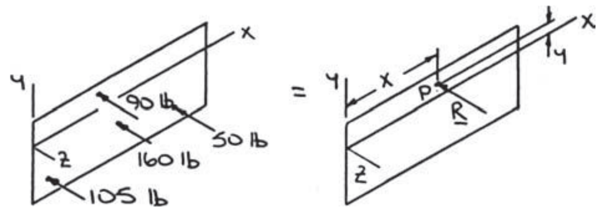


PROBLEM 3.129

Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine the magnitude and the point of application of the resultant of the four wind forces when $a = 1$ ft and $b = 12$ ft.

SOLUTION

We have



Assume that the resultant **R** is applied at Point *P* whose coordinates are (x, y, 0).

Equivalence then requires

$$\Sigma F_z: \quad -105 - 90 - 160 - 50 = -R$$

or $R = 405$ lb ◀

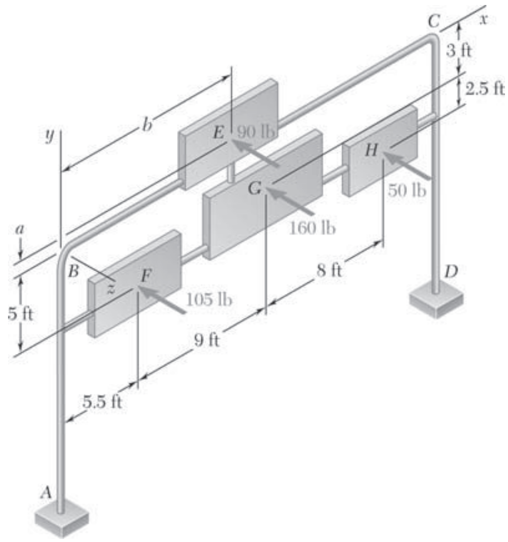
$$\Sigma M_x: \quad (5 \text{ ft})(105 \text{ lb}) - (1 \text{ ft})(90 \text{ lb}) + (3 \text{ ft})(160 \text{ lb}) + (5.5 \text{ ft})(50 \text{ lb}) = -y(405 \text{ lb})$$

or $$y = -2.94 \text{ ft}$$

$$\Sigma M_y: \quad (5.5 \text{ ft})(105 \text{ lb}) + (12 \text{ ft})(90 \text{ lb}) + (14.5 \text{ ft})(160 \text{ lb}) + (22.5 \text{ ft})(50 \text{ lb}) = -x(405 \text{ lb})$$

or $$x = 12.60 \text{ ft}$$

R acts 12.60 ft to the right of member *AB* and 2.94 ft below member *BC*. ◀



PROBLEM 3.130

Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine *a* and *b* so that the point of application of the resultant of the four forces is at *G*.

SOLUTION

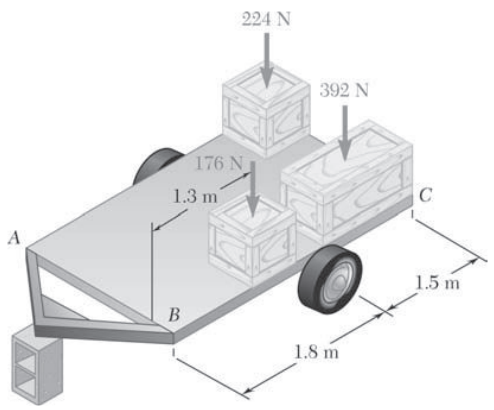
Since **R** acts at *G*, equivalence then requires that $\Sigma \mathbf{M}_G$ of the applied system of forces also be zero. Then

at
$$G: \Sigma M_x: \quad -(a+3)\text{ ft} \times (90\text{ lb}) + (2\text{ ft})(105\text{ lb}) + (2.5\text{ ft})(50\text{ lb}) = 0$$

or $a = 0.722$ ft ◀

$$\Sigma M_y: \quad -(9\text{ ft})(105\text{ ft}) - (14.5 - b)\text{ ft} \times (90\text{ lb}) + (8\text{ ft})(50\text{ lb}) = 0$$

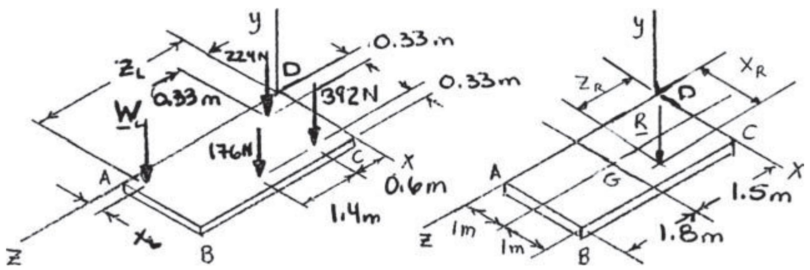
or $b = 20.6$ ft ◀



PROBLEM 3.131*

A group of students loads a 2×3.3-m flatbed trailer with two $0.66 \times 0.66 \times 0.66$-m boxes and one $0.66 \times 0.66 \times 1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.66 \times 0.66 \times 1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION



For the smallest weight on the trailer so that the resultant force of the four weights acts over the axle at the intersection with the center line of the trailer, the added $0.66 \times 0.66 \times 1.2$-m box should be placed adjacent to one of the edges of the trailer with the 0.66×0.66-m side on the bottom. The edges to be considered are based on the location of the resultant for the three given weights.

We have $\Sigma \mathbf{F}$: $-(224\text{ N})\mathbf{j} - (392\text{ N})\mathbf{j} - (176\text{ N})\mathbf{j} = \mathbf{R}$

$$\mathbf{R} = -(792\text{ N})\mathbf{j}$$

We have ΣM_z: $-(224\text{ N})(0.33\text{ m}) - (392\text{ N})(1.67\text{ m}) - (176\text{ N})(1.67\text{ m}) = (-792\text{ N})(x)$

$$x_R = 1.29101\text{ m}$$

We have ΣM_x: $(224\text{ N})(0.33\text{ m}) + (392\text{ N})(0.6\text{ m}) + (176\text{ N})(2.0\text{ m}) = (792\text{ N})(z)$

$$z_R = 0.83475\text{ m}$$

From the statement of the problem, it is known that the resultant of $\mathbf{R}$ from the original loading and the lightest load $\mathbf{W}$ passes through G, the point of intersection of the two center lines. Thus, $\Sigma \mathbf{M}_G = 0$.

Further, since the lightest load $\mathbf{W}$ is to be as small as possible, the fourth box should be placed as far from G as possible without the box overhanging the trailer. These two requirements imply

$$(0.33\text{ m} \le x \le 1\text{ m})(1.5\text{ m} \le z \le 2.97\text{ m})$$

PROBLEM 3.131* (Continued)

With $\quad x_L = 0.33 \text{ m}$

at $\quad G\colon \ \Sigma M_Z\colon \ (1-0.33)\text{ m}\times W_L - (1.29101-1)\text{ m}\times(792\text{ N}) = 0$

or $\quad W_L = 344.00\text{ N}$

Now must check if this is physically possible,

at $\quad G\colon \ \Sigma M_x\colon \ (Z_L - 1.5)\text{m}\times 344\text{ N}) - (1.5-0.83475)\text{m}\times(792\text{ N}) = 0$

or $\quad Z_L = 3.032\text{ m}$

which is **not** acceptable.

With $\quad Z_L = 2.97\text{ m:}$

at $\quad G\colon \ \Sigma M_x\colon \ (2.97-1.5)\text{m}\times W_L - (1.5-0.83475)\text{m}\times(792\text{ N}) = 0$

or $\quad W_L = 358.42\text{ N}$

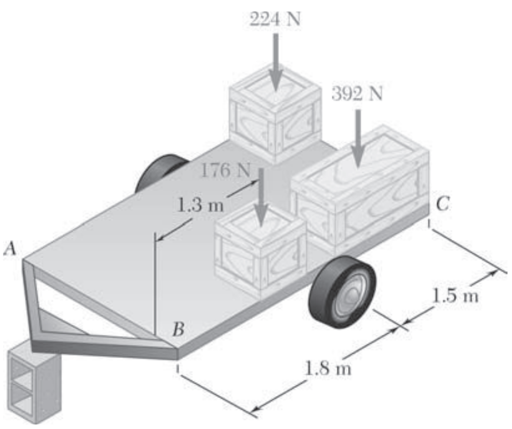
Now check if this is physically possible

at $\quad G\colon \ \Sigma M_z\colon \ (1-X_L)\text{m}\times(358.42\text{ N}) - (1.29101-1)\text{m}\times(792\text{ N}) = 0$

or $\quad X_L = 0.357\text{ m} \quad \text{ok!}$

The minimum weight of the fourth box is $\quad W_L = 358\text{ N}$ ◀

And it is placed on end (A 0.66×0.66-m side down) along side *AB* with the center of the box 0.357 m from side *AD*. ◀



PROBLEM 3.132*

Solve Problem 3.131 if the students want to place as much weight as possible in the fourth box and at least one side of the box must coincide with a side of the trailer.

PROBLEM 3.131* A group of students loads a 2×3.3-m flatbed trailer with two $0.66\times 0.66\times 0.66$-m boxes and one $0.66\times 0.66\times 1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.66\times 0.66\times 1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

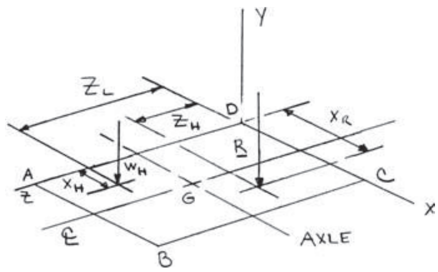
SOLUTION

First replace the three known loads with a single equivalent force **R** applied at coordinate $(X_R,\ 0,\ Z_R)$

Equivalence requires

$$\Sigma F_y:\quad -224-392-176=-R$$

or $$\mathbf{R}=792\text{ N}\downarrow$$



$$\Sigma M_x:\quad (0.33\text{ m})(224\text{ N})+(0.6\text{ m})(392\text{ N})$$
$$+(2\text{ m})(176\text{ N})=z_R(792\text{ N})$$

or $$z_R=0.83475\text{ m}$$

$$\Sigma M_z:\quad -(0.33\text{ m})(224\text{ N})-(1.67\text{ m})(392\text{ N})$$
$$-(1.67\text{ m})(176\text{ N})=x_R(792\text{ N})$$

or $$x_R=1.29101\text{ m}$$

From the statement of the problem, it is known that the resultant of **R** and the heaviest loads $\mathbf{W}_H$ passes through *G*, the point of intersection of the two center lines. Thus,

$$\Sigma\mathbf{M}_G=0$$

Further, since $\mathbf{W}_H$ is to be as large as possible, the fourth box should be placed as close to *G* as possible while keeping one of the sides of the box coincident with a side of the trailer. Thus, the two limiting cases are

$$x_H=0.6\text{ m}\quad\text{or}\quad z_H=2.7\text{ m}$$

PROBLEM 3.132* (Continued)

Now consider these two possibilities

With $x_H = 0.6$ m:

at G: ΣM_z: $(1-0.6)\text{m}\times W_H - (1.29101-1)\text{m}\times(792\text{ N}) = 0$

or $W_H = 576.20$ N

Checking if this is physically possible

at G: ΣM_x: $(z_H - 1.5)\text{m}\times(576.20\text{ N}) - (1.5-0.83475)\text{m}\times(792\text{ N}) = 0$

or $z_H = 2.414$ m

which is acceptable.

With $z_H = 2.7$ m

at G: ΣM_x: $(2.7-1.5)\ W_H - (1.5-0.83475)\text{m}\times(792\text{ N}) = 0$

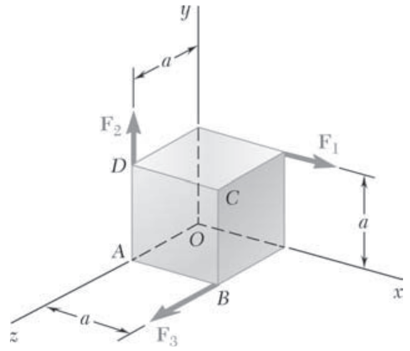
or $W_H = 439$ N

Since this is less than the first case, the maximum weight of the fourth box is

$W_H = 576$ N ◀

and it is placed with a 0.66×1.2-m side down, a 0.66-m edge along side AD, and the center 2.41 m from side DC. ◀

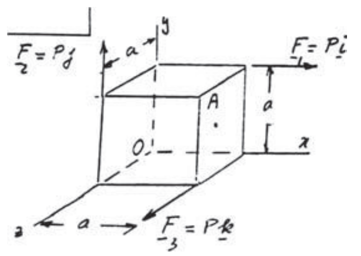
PROBLEM 3.133



Three forces of the same magnitude P act on a cube of side a as shown. Replace the three forces by an equivalent wrench and determine (a) the magnitude and direction of the resultant force $\mathbf{R}$, (b) the pitch of the wrench, (c) the axis of the wrench.

SOLUTION

Force-couple system at O:

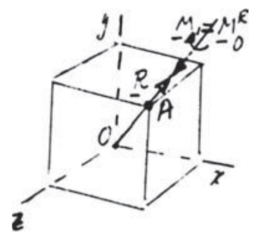


$$\mathbf{R} = P\mathbf{i} + P\mathbf{j} + P\mathbf{k} = P(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

$$\mathbf{M}_O^R = a\mathbf{j} \times P\mathbf{i} + a\mathbf{k} \times P\mathbf{j} + a\mathbf{i} \times P\mathbf{k}$$
$$= -Pa\mathbf{k} - Pa\mathbf{i} - Pa\mathbf{j}$$

$$\mathbf{M}_O^R = -Pa(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Since $\mathbf{R}$ and $\mathbf{M}_O^R$ have the same direction, they form a wrench with $\mathbf{M}_1 = \mathbf{M}_O^R$. Thus, the axis of the wrench is the diagonal OA. We note that



$$\cos\theta_x = \cos\theta_y = \cos\theta_z = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$R = P\sqrt{3} \quad \theta_x = \theta_y = \theta_z = 54.7°$$

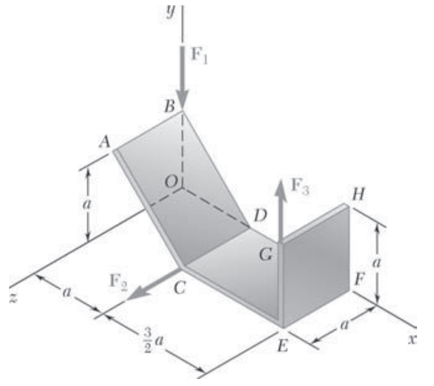
$$M_1 = M_O^R = -Pa\sqrt{3}$$

$$\text{Pitch} = p = \frac{M_1}{R} = \frac{-Pa\sqrt{3}}{P\sqrt{3}} = -a$$

(a) $R = P\sqrt{3} \quad \theta_x = \theta_y = \theta_z = 54.7°$ ◀

(b) $-a$ ◀

(c) Axis of the wrench is diagonal OA ◀



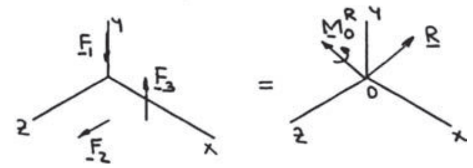
PROBLEM 3.134*

A piece of sheet metal is bent into the shape shown and is acted upon by three forces. If the forces have the same magnitude P, replace them with an equivalent wrench and determine (a) the magnitude and the direction of the resultant force $\mathbf{R}$, (b) the pitch of the wrench, (c) the axis of the wrench.

SOLUTION

First reduce the given forces to an equivalent force-couple system $\left(\mathbf{R},\ \mathbf{M}_O^R\right)$ at the origin.

We have



$$\Sigma\mathbf{F}:\quad -P\mathbf{j}+P\mathbf{j}+P\mathbf{k}=R$$

or
$$\mathbf{R}=P\mathbf{k}$$

$$\Sigma\mathbf{M}_O:\quad -(aP)\mathbf{j}+\left[-(aP)\mathbf{i}+\left(\frac{5}{2}aP\right)\mathbf{k}\right]=M_O^R$$

or
$$\mathbf{M}_O^R=aP\left(-\mathbf{i}-\mathbf{j}+\frac{5}{2}\mathbf{k}\right)$$

(a) Then for the wrench

$$R=P \blacktriangleleft$$

and
$$\boldsymbol{\lambda}_{\text{axis}}=\frac{\mathbf{R}}{R}=\mathbf{k}$$

$$\cos\theta_x=0\quad \cos\theta_y=0\quad \cos\theta_z=1$$

or
$$\theta_x=90^\circ\quad \theta_y=90^\circ\quad \theta_z=0^\circ \quad\blacktriangleleft$$

(b) Now

$$M_1=\boldsymbol{\lambda}_{\text{axis}}\cdot\mathbf{M}_O^R$$
$$=\mathbf{k}\cdot aP\left(-\mathbf{i}-\mathbf{j}+\frac{5}{2}\mathbf{k}\right)$$
$$=\frac{5}{2}aP$$

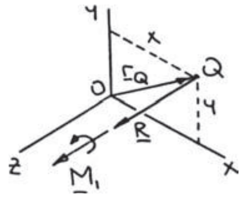
Then
$$P=\frac{\mathbf{M}_1}{R}=\frac{\frac{5}{2}aP}{P}$$

or $P=\dfrac{5}{2}a \blacktriangleleft$

PROBLEM 3.134* (Continued)

(*c*) The components of the wrench are $(\mathbf{R}, \mathbf{M}_1)$, where $\mathbf{M}_1 = M_1 \boldsymbol{\lambda}_{\text{axis}}$, and the axis of the wrench is assumed to intersect the *xy* plane at Point *Q* whose coordinates are (*x*, *y*, 0). Thus require

$$\mathbf{M}_z = \mathbf{r}_Q \times \mathbf{R}_R$$



Where $$\mathbf{M}_z = \mathbf{M}_O \times \mathbf{M}_1$$

Then

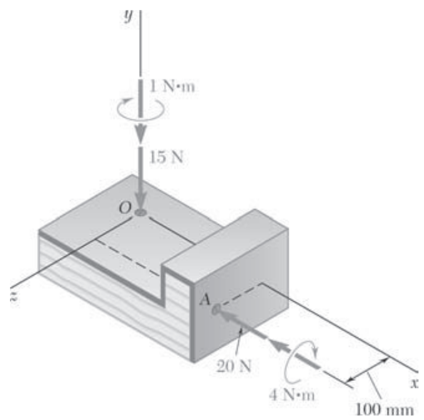
$$aP\left(-\mathbf{i} - \mathbf{j} + \frac{5}{2}\mathbf{k}\right) - \frac{5}{2}aP\mathbf{k} = (x\mathbf{i} + y\mathbf{j}) + P\mathbf{k}$$

Equating coefficients

$$\mathbf{i}: \quad -aP = yP \quad \text{or} \quad y = -a$$

$$\mathbf{j}: \quad -aP = -xP \quad \text{or} \quad x = a$$

The axis of the wrench is parallel to the *z* axis and intersects the *xy* plane at $x = a,\ y = -a.$ ◀



PROBLEM 3.135*

The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION

First, reduce the given force system to a force-couple system.

We have $\Sigma\mathbf{F}:\ -(20\text{ N})\mathbf{i}-(15\text{ N})\mathbf{j}=\mathbf{R}\quad R=25\text{ N}$

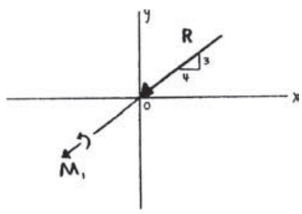
We have $\Sigma\mathbf{M}_O:\ \Sigma(\mathbf{r}_O\times\mathbf{F})+\Sigma\mathbf{M}_C=\mathbf{M}_O^R$

$$\mathbf{M}_O^R=-20\text{ N}(0.1\text{ m})\mathbf{j}-(4\text{ N}\cdot\text{m})\mathbf{i}-(1\text{ N}\cdot\text{m})\mathbf{j}$$
$$=-(4\text{ N}\cdot\text{m})\mathbf{i}-(3\text{ N}\cdot\text{m})\mathbf{j}$$

(*a*) $\mathbf{R}=-(20.0\text{ N})\mathbf{i}-(15.0\text{ N})\mathbf{j}$ ◀

(*b*) We have $M_1=\lambda_R\cdot\mathbf{M}_O^R\quad \lambda=\dfrac{\mathbf{R}}{R}$

$$=(-0.8\mathbf{i}-0.6\mathbf{j})\cdot[-(4\text{ N}\cdot\text{m})]\mathbf{i}-(3\text{ N}\cdot\text{m})\mathbf{j}]$$
$$=5\text{ N}\cdot\text{m}$$



Pitch $p=\dfrac{M_1}{R}=\dfrac{5\text{ N}\cdot\text{m}}{25\text{ N}}=0.200\text{ m}$

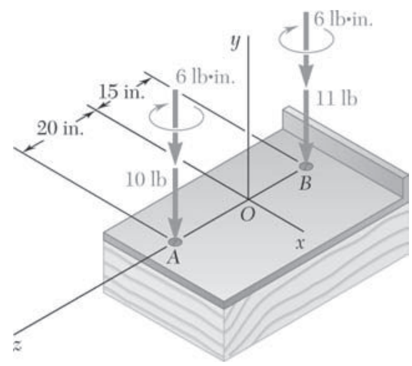
or $p=0.200\text{ m}$ ◀

(*c*) From above note that

$$\mathbf{M}_1=\mathbf{M}_O^R$$

Therefore, the axis of the wrench goes through the origin. The line of action of the wrench lies in the *xy* plane with a slope of

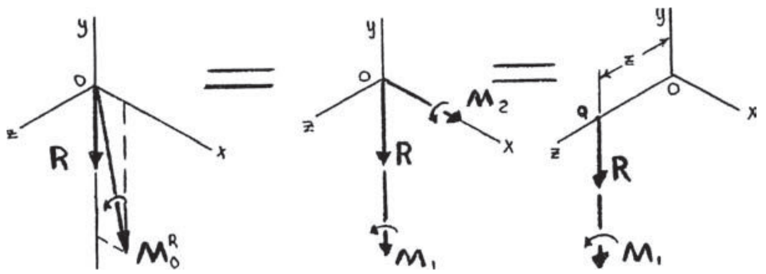
$$y=\frac{3}{4}x\ ◀$$



PROBLEM 3.136*

The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}: \quad -(10\text{ lb})\mathbf{j} - (11\text{ lb})\mathbf{j} = \mathbf{R}$

$$\mathbf{R} = -(21\text{ lb})\mathbf{j}$$

We have $\Sigma\mathbf{M}_O: \quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 20 \\ 0 & -10 & 0 \end{vmatrix} \text{lb} \cdot \text{in.} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -15 \\ 0 & -11 & 0 \end{vmatrix} \text{lb} \cdot \text{in.} - (12\text{ lb} \cdot \text{in})\mathbf{j}$$

$$= (35\text{ lb} \cdot \text{in.})\mathbf{i} - (12\text{ lb} \cdot \text{in.})\mathbf{j}$$

(*a*) $\mathbf{R} = -(21\text{ lb})\mathbf{j}$ or $\mathbf{R} = -(21.0\text{ lb})\mathbf{j}$ ◀

(*b*) We have $M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \quad \boldsymbol{\lambda}_R = \dfrac{\mathbf{R}}{R}$

$$= (-\mathbf{j}) \cdot [(35\text{ lb} \cdot \text{in.})\mathbf{i} - (12\text{ lb} \cdot \text{in.})\mathbf{j}]$$

$$= 12\text{ lb} \cdot \text{in.} \quad \text{and} \quad \mathbf{M}_1 = -(12\text{ lb} \cdot \text{in.})\mathbf{j}$$

and pitch $p = \dfrac{M_1}{R} = \dfrac{12\text{ lb} \cdot \text{in.}}{21\text{ lb}} = 0.57143\text{ in.}$ or $p = 0.571\text{ in.}$ ◀

PROBLEM 3.136* (Continued)

(*c*) We have $\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (35\text{ lb}\cdot\text{in.})\mathbf{i}$$

Require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$(35\text{ lb}\cdot\text{in.})\mathbf{i} = (x\mathbf{i} + z\mathbf{k}) \times [-(21\text{ lb})\mathbf{j}]$$

$$35\mathbf{i} = -(21x)\mathbf{k} + (21z)\mathbf{i}$$

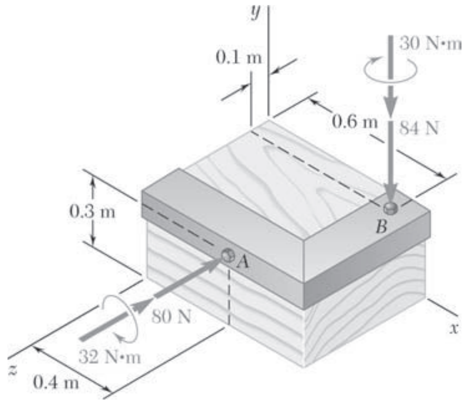
From **i**: $35 = 21z$

$$z = 1.66667\text{ in.}$$

From **k**: $0 = -21x$

$$z = 0$$

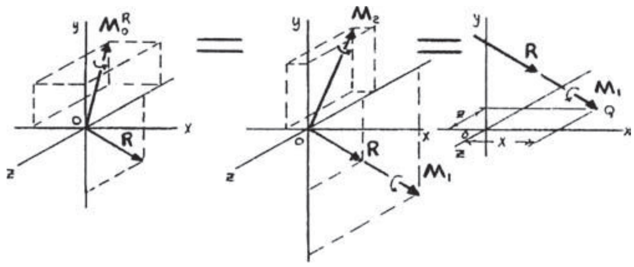
The axis of the wrench is parallel to the *y* axis and intersects the *xz* plane at $x = 0,\ z = 1.667$ in. ◀



PROBLEM 3.137*

Two bolts at A and B are tightened by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (*a*) the resultant **R**, (*b*) the pitch of the single equivalent wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}:\quad -(84\text{ N})\mathbf{j}-(80\text{ N})\mathbf{k}=\mathbf{R}\quad R=116\text{ N}$

and $\Sigma\mathbf{M}_O:\quad \Sigma(\mathbf{r}_O\times\mathbf{F})+\Sigma\mathbf{M}_C=\mathbf{M}_O^R$

$$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0.6 & 0 & .1\\ 0 & 84 & 0\end{vmatrix}+\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0.4 & 0.3 & 0\\ 0 & 0 & 80\end{vmatrix}+(-30\mathbf{j}-32\mathbf{k})\text{N}\cdot\text{m}=\mathbf{M}_O^R$$

$$\mathbf{M}_O^R=-(15.6\text{ N}\cdot\text{m})\mathbf{i}+(2\text{ N}\cdot\text{m})\mathbf{j}-(82.4\text{ N}\cdot\text{m})\mathbf{k}$$

(*a*) $\mathbf{R}=-(84.0\text{ N})\mathbf{j}-(80.0\text{ N})\mathbf{k}$ ◀

(*b*) We have $M_1=\boldsymbol{\lambda}_R\cdot\mathbf{M}_O^R\quad \boldsymbol{\lambda}_R=\dfrac{\mathbf{R}}{R}$

$$=-\frac{-84\mathbf{j}-80\mathbf{k}}{116}\cdot[-(15.6\text{ N}\cdot\text{m})\mathbf{i}+(2\text{ N}\cdot\text{m})\mathbf{j}-(82.4\text{ N}\cdot\text{m})\mathbf{k}]$$

$$=55.379\text{ N}\cdot\text{m}$$

and $\mathbf{M}_1=M_1\lambda_R=-(40.102\text{ N}\cdot\text{m})\mathbf{j}-(38.192\text{ N}\cdot\text{m})\mathbf{k}$

Then pitch $p=\dfrac{M_1}{R}=\dfrac{55.379\text{ N}\cdot\text{m}}{116\text{ N}}=0.47741\text{ m}$ or $p=0.477\text{ m}$ ◀

PROBLEM 3.137* (Continued)

(*c*) We have $\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = [(-15.6\mathbf{i} + 2\mathbf{j} - 82.4\mathbf{k}) - (40.102\mathbf{j} - 38.192\mathbf{k})] \text{ N} \cdot \text{m}$$
$$= -(15.6 \text{ N} \cdot \text{m})\mathbf{i} + (42.102 \text{ N} \cdot \text{m})\mathbf{j} - (44.208 \text{ N} \cdot \text{m})\mathbf{k}$$

Require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$(-15.6\mathbf{i} + 42.102\mathbf{j} - 44.208\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times (84\mathbf{j} - 80\mathbf{k})$$
$$= (84z)\mathbf{i} + (80x)\mathbf{j} - (84x)\mathbf{k}$$

From **i**: $-15.6 = 84z$

$$z = -0.185714 \text{ m}$$

or $z = -0.1857 \text{ m}$

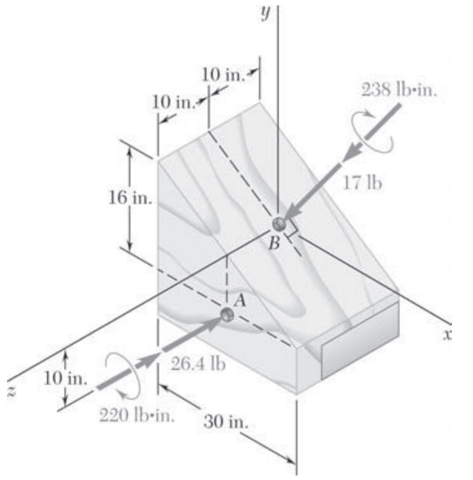
From **k**: $-44.208 = -84x$

$$x = 0.52629 \text{ m}$$

or $x = 0.526 \text{ m}$

The axis of the wrench intersects the *xz* plane at

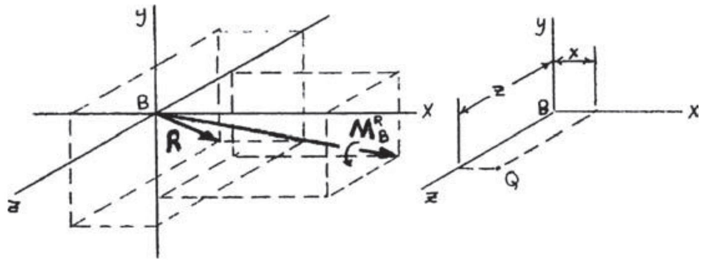
$x = 0.526 \text{ m} \quad y = 0 \quad z = -0.1857 \text{ m}$ ◀



PROBLEM 3.138*

Two bolts at A and B are tightened by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (a) the resultant $\mathbf{R}$, (b) the pitch of the single equivalent wrench, (c) the point where the axis of the wrench intersects the xz plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin at B.

(a) We have $\Sigma\mathbf{F}$: $-(26.4\text{ lb})\mathbf{k}-(17\text{ lb})\left(\frac{8}{17}\mathbf{i}+\frac{15}{17}\mathbf{j}\right)=\mathbf{R}$

$$\mathbf{R}=-(8.00\text{ lb})\mathbf{i}-(15.00\text{ lb})\mathbf{j}-(26.4\text{ lb})\mathbf{k} \blacktriangleleft$$

and $R=31.4\text{ lb}$

We have $\Sigma\mathbf{M}_B$: $\mathbf{r}_{A/B}\times\mathbf{F}_A+\mathbf{M}_A+\mathbf{M}_B=\mathbf{M}_B^R$

$$\mathbf{M}_B^R=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & -10 & 0\\ 0 & 0 & -26.4\end{vmatrix}-220\mathbf{k}-238\left(\frac{8}{17}\mathbf{i}+\frac{15}{17}\mathbf{j}\right)=264\mathbf{i}-220\mathbf{k}-14(8\mathbf{i}+15\mathbf{j})$$

$$\mathbf{M}_B^R=(152\text{ lb}\cdot\text{in.})\mathbf{i}-(210\text{ lb}\cdot\text{in.})\mathbf{j}-(220\text{ lb}\cdot\text{in.})\mathbf{k}$$

(b) We have $M_1=\boldsymbol{\lambda}_R\cdot\mathbf{M}_O^R \quad \boldsymbol{\lambda}_R=\frac{\mathbf{R}}{R}$

$$=\frac{-8.00\mathbf{i}-15.00\mathbf{j}-26.4\mathbf{k}}{31.4}\cdot[(152\text{ lb}\cdot\text{in.})\mathbf{i}-(210\text{ lb}\cdot\text{in.})\mathbf{j}-(220\text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$=246.56\text{ lb}\cdot\text{in.}$$

PROBLEM 3.138* (Continued)

and $\mathbf{M}_1 = M_1\lambda_R = -(62.818 \text{ lb}\cdot\text{in.})\mathbf{i} - (117.783 \text{ lb}\cdot\text{in.})\mathbf{j} - (207.30 \text{ lb}\cdot\text{in.})\mathbf{k}$

Then pitch $p = \dfrac{M_1}{R} = \dfrac{246.56 \text{ lb}\cdot\text{in.}}{31.4 \text{ lb}} = 7.8522 \text{ in.}$ or $p = 7.85 \text{ in.}$ ◀

(*c*) We have $\mathbf{M}_B^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\mathbf{M}_2 = \mathbf{M}_B^R - \mathbf{M}_1 = (152\mathbf{i} - 210\mathbf{j} - 220\mathbf{k}) - (-62.818\mathbf{i} - 117.783\mathbf{j} - 207.30\mathbf{k})$$
$$= (214.82 \text{ lb}\cdot\text{in.})\mathbf{i} - (92.217 \text{ lb}\cdot\text{in.})\mathbf{j} - (12.7000 \text{ lb}\cdot\text{in.})\mathbf{k}$$

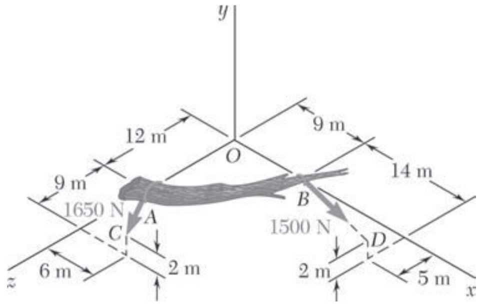
Require $\mathbf{M}_2 = \mathbf{r}_{Q/B} \times \mathbf{R}$

$$214.82\mathbf{i} - 92.217\mathbf{j} - 12.7000\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ -8 & -15 & -26.4 \end{vmatrix}$$
$$= (15z)\mathbf{i} - (8z)\mathbf{j} + (26.4x)\mathbf{j} - (15x)\mathbf{k}$$

From **i**: $214.82 = 15z \qquad z = 14.3213 \text{ in.}$

From **k**: $-12.7000 = -15x \qquad x = 0.84667 \text{ in.}$

The axis of the wrench intersects the *xz* plane at $x = 0.847 \text{ in.} \quad y = 0 \quad z = 14.32 \text{ in.}$ ◀



PROBLEM 3.139*

Two ropes attached at A and B are used to move the trunk of a fallen tree. Replace the forces exerted by the ropes with an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *yz* plane.

SOLUTION

(*a*) First replace the given forces with an equivalent force-couple system $\left(\mathbf{R},\ \mathbf{M}_O^R\right)$ at the origin.

We have

$$d_{AC} = \sqrt{(6)^2 + (2)^2 + (9)^2} = 11 \text{ m}$$

$$d_{BD} = \sqrt{(14)^2 + (2)^2 + (5)^2} = 15 \text{ m}$$

Then

$$T_{AC} = \frac{1650 \text{ N}}{11} = (6\mathbf{i} + 2\mathbf{j} + 9\mathbf{k})$$
$$= (900 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (1350 \text{ N})\mathbf{k}$$

and

$$T_{BD} = \frac{1500 \text{ N}}{15} = (14\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$$
$$= (1400 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (500 \text{ N})\mathbf{k}$$

Equivalence then requires

$$\Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{T}_{AC} + \mathbf{T}_{BD}$$
$$= (900\mathbf{i} + 300\mathbf{j} + 1350\mathbf{k})$$
$$+(1400\mathbf{i} + 200\mathbf{j} + 500\mathbf{k})$$
$$= (2300 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} + (1850 \text{ N})\mathbf{k}$$

$$\Sigma\mathbf{M}_O: \quad \mathbf{M}_O^R = \mathbf{r}_A \times \mathbf{T}_{AC} + \mathbf{r}_B \times \mathbf{T}_{BD}$$
$$= (12 \text{ m})\mathbf{k} \times [(900 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (1350 \text{ N})\mathbf{k}]$$
$$+ (9 \text{ m})\mathbf{i} \times [(1400 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (500 \text{ N})\mathbf{k}]$$
$$= -(3600)\mathbf{i} + (10800 - 4500)\mathbf{j} + (1800)\mathbf{k}$$
$$= -(3600 \text{ N}\cdot\text{m})\mathbf{i} + (6300 \text{ N}\cdot\text{m})\mathbf{j} + (1800 \text{ N}\cdot\text{m})\mathbf{k}$$

The components of the wrench are $(\mathbf{R},\ \mathbf{M}_1)$, where

$$\mathbf{R} = (2300 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} + (1850 \text{ N})\mathbf{k} \quad ◀$$

PROBLEM 3.139* (Continued)

(*b*) We have

$$R = 100\sqrt{(23)^2 + (5)^2 + (18.5)^2} = 2993.7 \text{ N}$$

Let

$$\boldsymbol{\lambda}_{\text{axis}} = \frac{\mathbf{R}}{R} = \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k})$$

Then

$$\begin{aligned} M_1 &= \boldsymbol{\lambda}_{\text{axis}} \cdot \mathbf{M}_O^R \\ &= \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k}) \cdot (-3600\mathbf{i} + 6300\mathbf{j} + 1800\mathbf{k}) \\ &= \frac{1}{0.29937}[(23)(-36) + (5)(63) + (18.5)(18)] \\ &= -601.26 \text{ N} \cdot \text{m} \end{aligned}$$

Finally

$$P = \frac{M_1}{R} = \frac{-601.26 \text{ N} \cdot \text{m}}{2993.7 \text{ N}}$$

or $P = -0.201$ m ◀

(*c*) We have

$$\begin{aligned} M_1 &= M_1 \boldsymbol{\lambda}_{\text{axis}} \\ &= (-601.26 \text{ N} \cdot \text{m}) \times \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k}) \end{aligned}$$

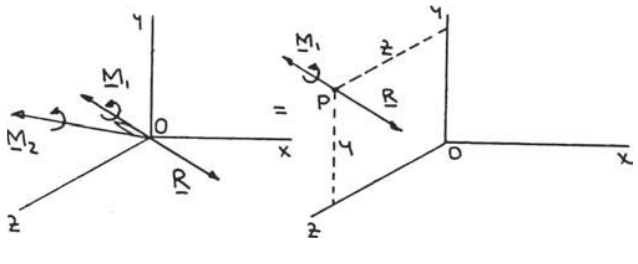
or

$$\mathbf{M}_1 = -(461.93 \text{ N} \cdot \text{m})\mathbf{i} - (100.421 \text{ N} \cdot \text{m})\mathbf{j} - (371.56 \text{ N} \cdot \text{m})\mathbf{k}$$

Now

$$\begin{aligned} \mathbf{M}_2 &= \mathbf{M}_O^R - \mathbf{M}_1 \\ &= (-3600\mathbf{i} + 6300\mathbf{j} + 1800\mathbf{k}) \\ &\quad - (-461.93\mathbf{i} - 100.421\mathbf{j} - 371.56\mathbf{k}) \\ &= -(3138.1 \text{ N} \cdot \text{m})\mathbf{i} + (6400.4 \text{ N} \cdot \text{m})\mathbf{j} + (2171.6 \text{ N} \cdot \text{m})\mathbf{k} \end{aligned}$$

For equivalence



PROBLEM 3.139* (Continued)

Thus require $\mathbf{M}_2 = \mathbf{r}_P \times \mathbf{R}$ $\quad \mathbf{r} = (y\mathbf{j} + z\mathbf{k})$

Substituting

$$-3138.1\mathbf{i} + 6400.4\mathbf{j} + 2171.6\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 2300 & 500 & 1850 \end{vmatrix}$$

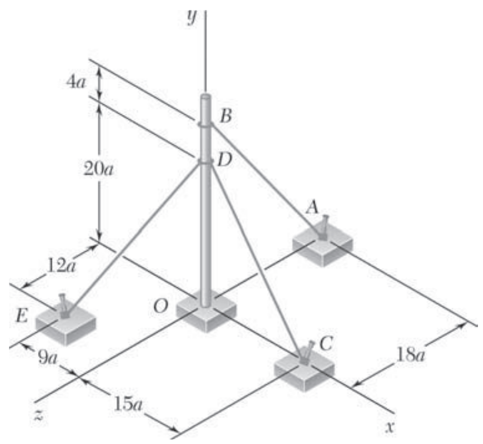
Equating coefficients

$$\mathbf{j}: \quad 6400.4 = 2300z \quad \text{or} \quad z = 2.78 \text{ m}$$

$$\mathbf{k}: \quad 2171.6 = -2300y \quad \text{or} \quad y = -0.944 \text{ m}$$

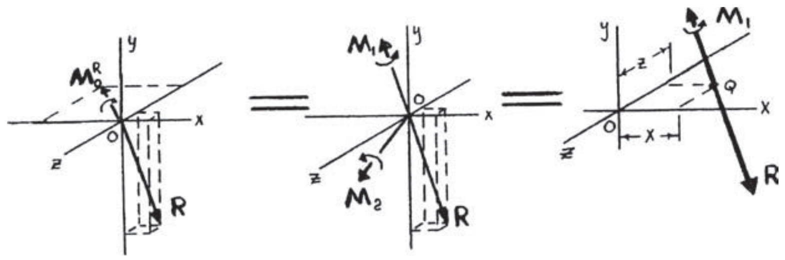
The axis of the wrench intersects the *yz* plane at $y = -0.944$ m $\quad z = 2.78$ m ◀

PROBLEM 3.140*



A flagpole is guyed by three cables. If the tensions in the cables have the same magnitude P, replace the forces exerted on the pole with an equivalent wrench and determine (a) the resultant force **R**, (b) the pitch of the wrench, (c) the point where the axis of the wrench intersects the xz plane.

SOLUTION



(a) First reduce the given force system to a force-couple at the origin.

We have $\Sigma \mathbf{F}: \quad P\boldsymbol{\lambda}_{BA} + P\boldsymbol{\lambda}_{DC} + P\boldsymbol{\lambda}_{DE} = \mathbf{R}$

$$\mathbf{R} = P\left[\left(\frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}\right) + \left(\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}\right) + \left(\frac{-9}{25}\mathbf{i} - \frac{4}{5}\mathbf{j} + \frac{12}{25}\mathbf{k}\right)\right]$$

$$\mathbf{R} = \frac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \blacktriangleleft$$

$$R = \frac{3P}{25}\sqrt{(2)^2 + (20)^2 + (1)^2} = \frac{27\sqrt{5}}{25}P$$

We have $\Sigma \mathbf{M}: \quad \Sigma(\mathbf{r}_O \times P) = \mathbf{M}_O^R$

$$(24a)\mathbf{j} \times \left(\frac{-4P}{5}\mathbf{j} - \frac{3P}{5}\mathbf{k}\right) + (20a)\mathbf{j} \times \left(\frac{3P}{5}\mathbf{i} - \frac{4P}{5}\mathbf{j}\right) + (20a)\mathbf{j} \times \left(\frac{-9P}{25}\mathbf{i} - \frac{4P}{5}\mathbf{j} + \frac{12P}{25}\mathbf{k}\right) = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k})$$

PROBLEM 3.140* (Continued)

(*b*) We have $M_1 = \lambda_R \cdot \mathbf{M}_O^R$

where $\lambda_R = \dfrac{\mathbf{R}}{R} = \dfrac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})\dfrac{25}{27\sqrt{5}P} = \dfrac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$

Then $M_1 = \dfrac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \cdot \dfrac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) = \dfrac{-8Pa}{15\sqrt{5}}$

and pitch $p = \dfrac{M_1}{R} = \dfrac{-8Pa}{15\sqrt{5}}\left(\dfrac{25}{27\sqrt{5}P}\right) = \dfrac{-8a}{81}$ or $p = -0.0988a$ ◀

(*c*) $\mathbf{M}_1 = M_1\lambda_R = \dfrac{-8Pa}{15\sqrt{5}}\left(\dfrac{1}{9\sqrt{5}}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) = \dfrac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k})$

Then $\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \dfrac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) - \dfrac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k}) = \dfrac{8Pa}{675}(-430\mathbf{i} - 20\mathbf{j} - 406\mathbf{k})$

Require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

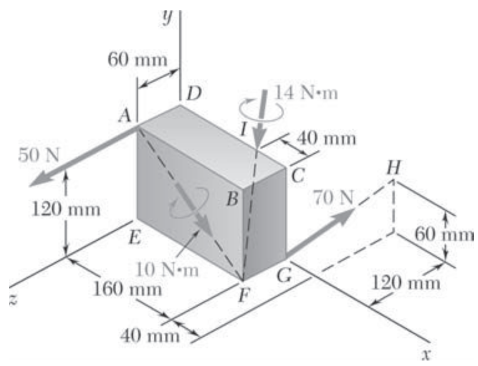
$$\left(\frac{8Pa}{675}\right)(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times \left(\frac{3P}{25}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$$
$$= \left(\frac{3P}{25}\right)[20z\mathbf{i} + (x + 2z)\mathbf{j} - 20x\mathbf{k}]$$

From **i**: $8(-403)\dfrac{Pa}{675} = 20z\left(\dfrac{3P}{25}\right) \quad z = -1.99012a$

From **k**: $8(-406)\dfrac{Pa}{675} = -20x\left(\dfrac{3P}{25}\right) \quad x = 2.0049a$

The axis of the wrench intersects the *xz* plane at

$x = 2.00a,\ z = -1.990a$ ◀



PROBLEM 3.141*

Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz* plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz* plane.

SOLUTION

First, reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}:\ \mathbf{F}_A + \mathbf{F}_G = \mathbf{R}$

$$\mathbf{R} = (50\text{ N})\mathbf{k} + 70\text{ N}\left[\frac{(40\text{ mm})\mathbf{i} + (60\text{ mm})\mathbf{j} - (120\text{ mm})\mathbf{k}}{140\text{ mm}}\right]$$

$$= (20\text{ N})\mathbf{i} + (30\text{ N})\mathbf{j} - (10\text{ N})\mathbf{k}$$

and $R = 37.417\text{ N}$

We have $\Sigma\mathbf{M}_O:\ \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = [(0.12\text{ m})\mathbf{j}\times(50\text{ N})\mathbf{k}] + \{(0.16\text{ m})\mathbf{i}\times[(20\text{ N})\mathbf{i} + (30\text{ N})\mathbf{j} - (60\text{ N})\mathbf{k}]\}$$

$$+ (10\text{ N}\cdot\text{m})\left[\frac{(160\text{ mm})\mathbf{i} - (120\text{ mm})\mathbf{j}}{200\text{ mm}}\right]$$

$$+ (14\text{ N}\cdot\text{m})\left[\frac{(40\text{ mm})\mathbf{i} - (120\text{ mm})\mathbf{j} + (60\text{ mm})\mathbf{k}}{140\text{ mm}}\right]$$

$$\mathbf{M}_0^R = (18\text{ N}\cdot\text{m})\mathbf{i} - (8.4\text{ N}\cdot\text{m})\mathbf{j} + (10.8\text{ N}\cdot\text{m})\mathbf{k}$$

To be able to reduce the original forces and couples to a single equivalent force, **R** and **M** must be perpendicular. Thus, $\mathbf{R}\cdot\mathbf{M} = 0$.

Substituting

$$(20\mathbf{i} + 30\mathbf{j} - 10\mathbf{k})\cdot(18\mathbf{i} - 8.4\mathbf{j} + 10.8\mathbf{k}) \stackrel{?}{=} 0$$

or

$$(20)(18) + (30)(-8.4) + (-10)(10.8) \stackrel{?}{=} 0$$

or

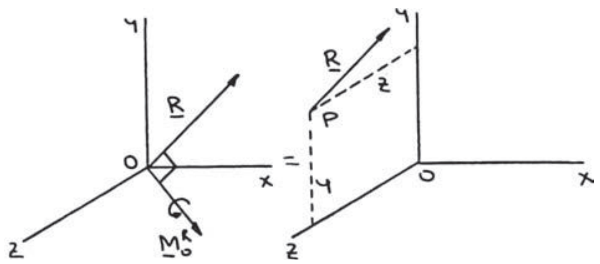
$$0 \stackrel{\checkmark}{=} 0$$

R and **M** are perpendicular so that the given system can be reduced to the single equivalent force

$$\mathbf{R} = (20.0\text{ N})\mathbf{i} + (30.0\text{ N})\mathbf{j} - (10.00\text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.141* (Continued)

Then for equivalence



Thus require $\mathbf{M}_O^R = \mathbf{r}_p \times \mathbf{R} \quad \mathbf{r}_p = y\mathbf{j} + z\mathbf{k}$

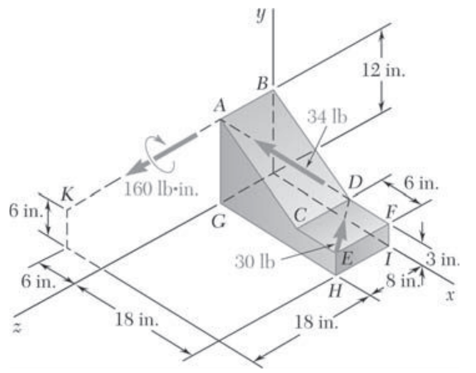
Substituting

$$18\mathbf{i} - 8.4\mathbf{j} + 10.8\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 20 & 30 & -10 \end{vmatrix}$$

Equating coefficients

$$\mathbf{j}: \quad -8.4 = 20z \quad \text{or} \quad z = -0.42 \text{ m}$$
$$\mathbf{k}: \quad 10.8 = -20y \quad \text{or} \quad y = -0.54 \text{ m}$$

The line of action of **R** intersects the yz plane at $x = 0 \quad y = -0.540 \text{ m} \quad z = -0.420 \text{ m}$ ◀



PROBLEM 3.142*

Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz* plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz* plane.

SOLUTION

First determine the resultant of the forces at D. We have

$$d_{DA} = \sqrt{(-12)^2 + (9)^2 + (8)^2} = 17 \text{ in.}$$

$$d_{ED} = \sqrt{(-6)^2 + (0)^2 + (-8)^2} = 10 \text{ in.}$$

Then

$$\mathbf{F}_{DA} = \frac{34 \text{ lb}}{17} = (-12\mathbf{i} + 9\mathbf{j} + 8\mathbf{k})$$
$$= -(24 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} + (16 \text{ lb})\mathbf{k}$$

and

$$\mathbf{F}_{ED} = \frac{30 \text{ lb}}{10} = (-6\mathbf{i} - 8\mathbf{k})$$
$$= -(18 \text{ lb})\mathbf{i} - (24 \text{ lb})\mathbf{k}$$

Then

$$\Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{F}_{DA} + \mathbf{F}_{ED}$$
$$= (-24\mathbf{i} + 18\mathbf{j} + 16\mathbf{k} + (-18\mathbf{i} - 24\mathbf{k})$$
$$= -(42 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} - (8 \text{ lb})\mathbf{k}$$

For the applied couple

$$d_{AK} = \sqrt{(-6)^2 + (-6)^2 + (18)^2} = 6\sqrt{11} \text{ in.}$$

Then

$$\mathbf{M} = \frac{160 \text{ lb} \cdot \text{in.}}{6\sqrt{11}}(-6\mathbf{i} - 6\mathbf{j} + 18\mathbf{k})$$
$$= \frac{160}{\sqrt{11}}[-(1 \text{ lb} \cdot \text{in.})\mathbf{i} - (1 \text{ lb} \cdot \text{in.})\mathbf{j} + (3 \text{ lb} \cdot \text{in.})\mathbf{k}]$$

To be able to reduce the original forces and couple to a single equivalent force, **R** and **M** must be perpendicular. Thus

$$\mathbf{R} \cdot \mathbf{M} \stackrel{?}{=} 0$$

PROBLEM 3.142* (Continued)

Substituting

$$(-42\mathbf{i}+18\mathbf{j}-8\mathbf{k})\cdot\frac{160}{\sqrt{11}}(-\mathbf{i}-\mathbf{j}+3\mathbf{k})\stackrel{?}{=}0$$

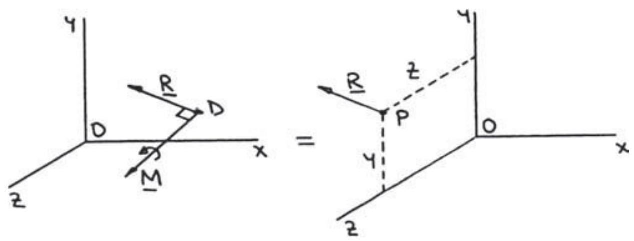
or $$\frac{160}{\sqrt{11}}[(-42)(-1)+(18)(-1)+(-8)(3)]\stackrel{?}{=}0$$

or $$0\stackrel{\checkmark}{=}0$$

R and **M** are perpendicular so that the given system can be reduced to the single equivalent force

$$\mathbf{R}=-(42.0\text{ lb})\mathbf{i}+(18.00\text{ lb})\mathbf{j}-(8.00\text{ lb})\mathbf{k} \quad ◀$$

Then for equivalence



Thus require $\quad \mathbf{M}=\mathbf{r}_{P/D}\times\mathbf{R}$

where $\quad \mathbf{r}_{P/D}=-(12\text{ in.})\mathbf{i}+[(y-3)\text{in.}]\mathbf{j}+(z\text{ in.})\mathbf{k}$

Substituting

$$\frac{160}{\sqrt{11}}(-\mathbf{i}-\mathbf{j}+3\mathbf{k})=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -12 & (y-3) & z\\ -42 & 18 & -8\end{vmatrix}$$

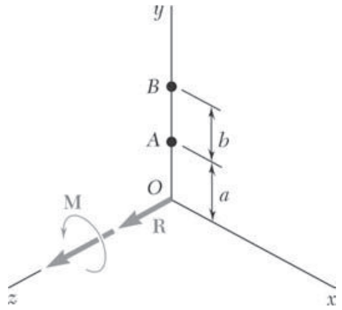
$$\begin{aligned}&=[(y-3)(-8)-(z)(18)]\mathbf{i}\\&\quad+[(z)(-42)-(-12)(-8)]\mathbf{j}\\&\quad+[(-12)(18)-(y-3)(-42)]\mathbf{k}\end{aligned}$$

Equating coefficients

$$\mathbf{j}:\quad -\frac{160}{\sqrt{11}}=-42z-96 \qquad \text{or} \quad z=-1.137\text{ in.}$$

$$\mathbf{k}:\quad \frac{480}{\sqrt{11}}=-216+42(y-3) \quad \text{or} \quad y=11.59\text{ in.}$$

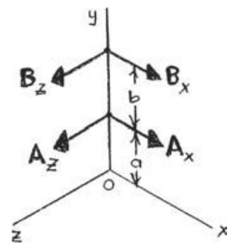
The line of action of **R** intersects the yz plane at $\quad x=0 \quad y=11.59\text{ in.} \quad z=-1.137\text{ in.}$ ◀



PROBLEM 3.143*

Replace the wrench shown with an equivalent system consisting of two forces perpendicular to the y axis and applied respectively at A and B.

SOLUTION



Express the forces at A and B as

$$\mathbf{A} = A_x\mathbf{i} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Then, for equivalence to the given force system

$$\Sigma F_x: \quad A_x + B_x = 0 \tag{1}$$

$$\Sigma F_z: \quad A_z + B_z = R \tag{2}$$

$$\Sigma M_x: \quad A_z(a) + B_z(a+b) = 0 \tag{3}$$

$$\Sigma M_z: \quad -A_x(a) - B_x(a+b) = M \tag{4}$$

From Equation (1), $\qquad B_x = -A_x$

Substitute into Equation (4)

$$-A_x(a) + A_x(a+b) = M$$

$$A_x = \frac{M}{b} \quad \text{and} \quad B_x = -\frac{M}{b}$$

From Equation (2), $\qquad B_z = R - A_z$

and Equation (3), $\qquad A_z a + (R - A_z)(a+b) = 0$

$$A_z = R\left(1 + \frac{a}{b}\right)$$

and $\qquad B_z = R - R\left(1 + \frac{a}{b}\right)$

$$B_z = -\frac{a}{b}R$$

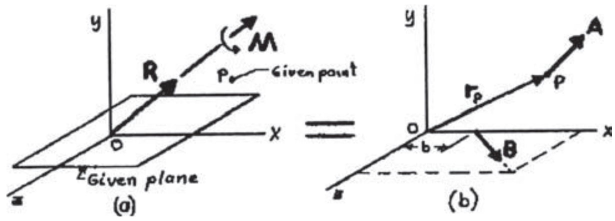
Then
$$\mathbf{A} = \left(\frac{M}{b}\right)\mathbf{i} + R\left(1 + \frac{a}{b}\right)\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -\left(\frac{M}{b}\right)\mathbf{i} - \left(\frac{a}{b}R\right)\mathbf{k} \blacktriangleleft$$

PROBLEM 3.144*

Show that, in general, a wrench can be replaced with two forces chosen in such a way that one force passes through a given point while the other force lies in a given plane.

SOLUTION



First, choose a coordinate system so that the *xy* plane coincides with the given plane. Also, position the coordinate system so that the line of action of the wrench passes through the origin as shown in Figure *a*. Since the orientation of the plane and the components (**R**, **M**) of the wrench are known, it follows that the scalar components of **R** and **M** are known relative to the shown coordinate system.

A force system to be shown as equivalent is illustrated in Figure *b*. Let *A* be the force passing through the given Point *P* and **B** be the force that lies in the given plane. Let *b* be the *x*-axis intercept of **B**.

The known components of the wrench can be expressed as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} \quad \text{and} \quad M = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

while the unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Since the position vector of Point *P* is given, it follows that the scalar components (x, y, z) of the position vector $\mathbf{r}_P$ are also known.

Then, for equivalence of the two systems

$$\Sigma F_x: \quad R_x = A_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R_y = A_y \tag{2}$$

$$\Sigma F_z: \quad R_z = A_z + B_z \tag{3}$$

$$\Sigma M_x: \quad M_x = yA_z - zA_y \tag{4}$$

$$\Sigma M_y: \quad M_y = zA_x - xA_z - bB_z \tag{5}$$

$$\Sigma M_z: \quad M_z = xA_y - yA_x \tag{6}$$

Based on the above six independent equations for the six unknowns $(A_x, A_y, A_z, B_x, B_y, B_z, b)$, there exists a unique solution for **A** and **B**.

From Equation (2) $\qquad A_y = R_y$ ◀

PROBLEM 3.144* (Continued)

Equation (6) $$A_x = \left(\frac{1}{y}\right)(xR_y - M_z) \blacktriangleleft$$

Equation (1) $$B_x = R_x - \left(\frac{1}{y}\right)(xR_y - M_z) \blacktriangleleft$$

Equation (4) $$A_z = \left(\frac{1}{y}\right)(M_x + zR_y) \blacktriangleleft$$

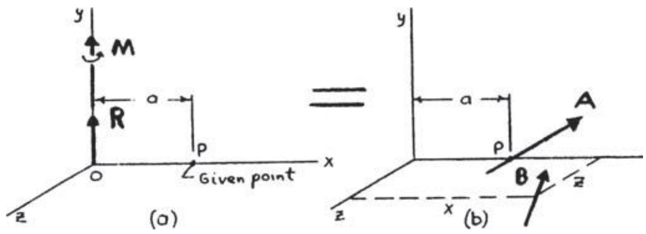
Equation (3) $$B_z = R_z - \left(\frac{1}{y}\right)(M_x + zR_y) \blacktriangleleft$$

Equation (5) $$b = \frac{(xM_x + yM_y + zM_z)}{(M_x - yR_z + zR_y)} \blacktriangleleft$$

PROBLEM 3.145*

Show that a wrench can be replaced with two perpendicular forces, one of which is applied at a given point.

SOLUTION



First, observe that it is always possible to construct a line perpendicular to a given line so that the constructed line also passes through a given point. Thus, it is possible to align one of the coordinate axes of a rectangular coordinate system with the axis of the wrench while one of the other axes passes through the given point.

See Figures *a* and *b*.

We have $\mathbf{R} = R\mathbf{j}$ and $\mathbf{M} = M\mathbf{j}$ and are known.

The unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

The distance *a* is known. It is assumed that force **B** intersects the *xz* plane at (*x*, 0, *z*). The for equivalence

$$\Sigma F_x: \quad 0 = A_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R = A_y + B_y \tag{2}$$

$$\Sigma F_z: \quad 0 = A_z + B_z \tag{3}$$

$$\Sigma M_x: \quad 0 = -zB_y \tag{4}$$

$$\Sigma M_y: \quad M = -aA_z - xB_z + zB_x \tag{5}$$

$$\Sigma M_z: \quad 0 = aA_y + xB_y \tag{6}$$

Since **A** and **B** are made perpendicular,

$$\mathbf{A}\cdot\mathbf{B} = 0 \quad \text{or} \quad A_xB_x + A_yB_y + A_zB_z = 0 \tag{7}$$

There are eight unknowns: $A_x, A_y, A_z, B_x, B_y, B_z, x, z$

But only seven independent equations. Therefore, *there exists an infinite number of solutions.*

Next consider Equation (4): $0 = -zB_y$

If $B_y = 0$, Equation (7) becomes $A_xB_x + A_zB_z = 0$

Using Equations (1) and (3) this equation becomes $A_x^2 + A_z^2 = 0$

PROBLEM 3.145* (Continued)

Since the components of **A** must be real, a nontrivial solution is not possible. Thus, it is required that $B_y \neq 0$, so that from Equation (4), $z = 0$.

To obtain one possible solution, arbitrarily let $A_x = 0$.

(*Note:* Setting A_y, A_z, or B_z equal to zero results in unacceptable solutions.)

The defining equations then become

$$0 = B_x \qquad (1)'$$

$$R = A_y + B_y \qquad (2)$$

$$0 = A_z + B_z \qquad (3)$$

$$M = -aA_z - xB_z \qquad (5)'$$

$$0 = aA_y + xB_y \qquad (6)$$

$$A_y B_y + A_z B_z = 0 \qquad (7)'$$

Then Equation (2) can be written $\quad A_y = R - B_y$

Equation (3) can be written $\quad B_z = -A_z$

Equation (6) can be written $\quad x = -\dfrac{aA_y}{B_y}$

Substituting into Equation (5)′,

$$M = -aA_z - \left(-a\frac{R - B_y}{B_y} \right)(-A_z)$$

or

$$A_z = -\frac{M}{aR} B_y \qquad (8)$$

Substituting into Equation (7)′,

$$(R - B_y)B_y + \left(-\frac{M}{aR} B_y \right)\left(\frac{M}{aR} B_y \right) = 0$$

or

$$B_y = \frac{a^2 R^3}{a^2 R^2 + M^2}$$

Then from Equations (2), (8), and (3)

$$A_y = R - \frac{a^2 R^2}{a^2 R^2 + M^2} = \frac{RM^2}{a^2 R^2 + M^2}$$

$$A_z = -\frac{M}{aR}\left(\frac{a^2 R^3}{a^2 R^2 + M^2} \right) = -\frac{aR^2 M}{a^2 R^2 + M^2}$$

$$B_z = \frac{aR^2 M}{a^2 R^2 + M^2}$$

PROBLEM 3.145* (Continued)

In summary

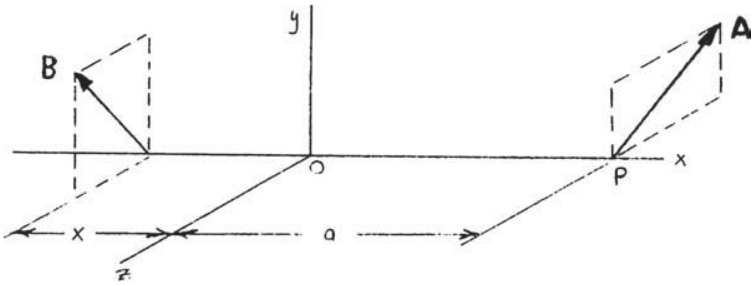
$$\mathbf{A} = \frac{RM}{a^2R^2 + M^2}(M\mathbf{j} - aR\mathbf{k}) \quad \blacktriangleleft$$

$$\mathbf{B} = \frac{aR^2}{a^2R^2 + M^2}(aR\mathbf{j} + M\mathbf{k}) \quad \blacktriangleleft$$

Which shows that it is possible to replace a wrench with two perpendicular forces, one of which is applied at a given point.

Lastly, if $R > 0$ and $M > 0$, it follows from the equations found for **A** and **B** that $A_y > 0$ and $B_y > 0$.

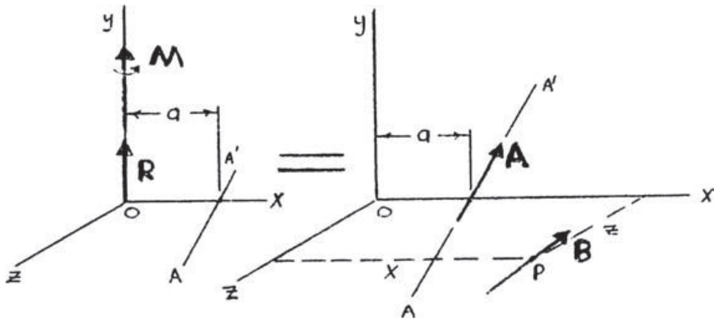
From Equation (6), $x < 0$ (assuming $a > 0$). Then, as a consequence of letting $A_x = 0$, force **A** lies in a plane parallel to the *yz* plane and to the right of the origin, while force **B** lies in a plane parallel to the *yz* plane but to the left to the origin, as shown in the figure below.



PROBLEM 3.146*

Show that a wrench can be replaced with two forces, one of which has a prescribed line of action.

SOLUTION



First, choose a rectangular coordinate system where one axis coincides with the axis of the wrench and another axis intersects the prescribed line of action (AA'). Note that it has been assumed that the line of action of force **B** intersects the xz plane at Point $P(x, 0, z)$. Denoting the known direction of line AA' by

$$\lambda_A = \lambda_x\mathbf{i} + \lambda_y\mathbf{j} + \lambda_z\mathbf{k}$$

it follows that force **A** can be expressed as

$$\mathbf{A} = A\lambda_A = A(\lambda_x\mathbf{i} + \lambda_y\mathbf{j} + \lambda_z\mathbf{k})$$

Force **B** can be expressed as

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Next, observe that since the axis of the wrench and the prescribed line of action AA' are known, it follows that the distance a can be determined. In the following solution, it is assumed that a is known.

Then, for equivalence

$$\Sigma F_x: \quad 0 = A\lambda_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R = A\lambda_y + B_y \tag{2}$$

$$\Sigma F_z: \quad 0 = A\lambda_z + B_z \tag{3}$$

$$\Sigma M_x: \quad 0 = -zB_y \tag{4}$$

$$\Sigma M_y: \quad M = -aA\lambda_z + zB_x - xB_z \tag{5}$$

$$\Sigma M_x: \quad 0 = -aA\lambda_y + xB_y \tag{6}$$

Since there are six unknowns (A, B_x, B_y, B_z, x, z) and six independent equations, it will be possible to obtain a solution.

PROBLEM 3.146* (Continued)

Case 1: Let $z = 0$ to satisfy Equation (4)

Now Equation (2) $\quad A\lambda_y = R - B_y$

Equation (3) $\quad B_z = -A\lambda_z$

Equation (6) $\quad x = -\dfrac{aA\lambda_y}{B_y} = -\left(\dfrac{a}{B_y}\right)(R - B_y)$

Substitution into Equation (5)

$$M = -aA\lambda_z - \left[-\left(\frac{a}{B_y}\right)(R - B_y)(-A\lambda_z)\right]$$

$$A = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y$$

Substitution into Equation (2)

$$R = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y\lambda_y + B_y$$

$$B_y = \frac{\lambda_z aR^2}{\lambda_z aR - \lambda_y M}$$

Then

$$A = -\frac{MR}{\lambda_z aR - \lambda_y M} = \frac{R}{\lambda_y - \dfrac{aR}{M}\lambda_z}$$

$$B_x = -A\lambda_x = \frac{\lambda_x MR}{\lambda_z aR - \lambda_y M}$$

$$B_z = -A\lambda_z = \frac{\lambda_z MR}{\lambda_z aR - \lambda_y M}$$

In summary

$$\mathbf{A} = \frac{P}{\lambda_y - \dfrac{aR}{M}\lambda_z}\lambda_A \quad \blacktriangleleft$$

$$\mathbf{B} = \frac{R}{\lambda_z aR - \lambda_y M}(\lambda_x M\mathbf{i} + \lambda_z aR\mathbf{j} + \lambda_z M\mathbf{k}) \quad \blacktriangleleft$$

and

$$x = a\left(1 - \frac{R}{B_y}\right)$$

$$= a\left[1 - R\left(\frac{\lambda_z aR - \lambda_y M}{\lambda_z aR^2}\right)\right]$$

or $\quad x = \dfrac{\lambda_y M}{\lambda_z R} \quad \blacktriangleleft$

Note that for this case, the lines of action of both **A** and **B** intersect the x axis.

PROBLEM 3.146* (Continued)

Case 2: Let $B_y = 0$ to satisfy Equation (4)

Now Equation (2) $\qquad A = \dfrac{R}{\lambda_y}$

Equation (1) $\qquad B_x = -R\left(\dfrac{\lambda_x}{\lambda_y}\right)$

Equation (3) $\qquad B_z = -R\left(\dfrac{\lambda_z}{\lambda_y}\right)$

Equation (6) $\qquad aA\lambda_y = 0 \qquad$ which requires $a = 0$

Substitution into Equation (5)

$$M = z\left[-R\left(\frac{\lambda_x}{\lambda_y}\right)\right] - x\left[-R\left(\frac{\lambda_z}{\lambda_y}\right)\right] \quad \text{or} \quad \lambda_z x - \lambda_x z = \left(\frac{M}{R}\right)\lambda_y$$

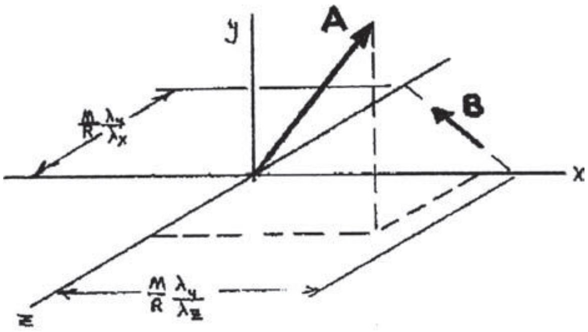
This last expression is the equation for the line of action of force **B**.

In summary

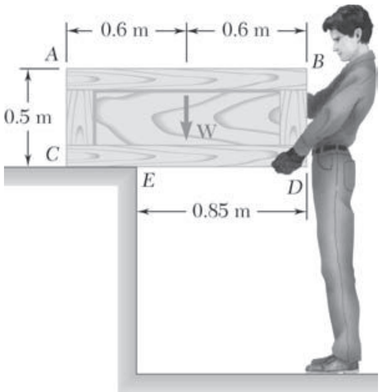
$$\mathbf{A} = \left(\frac{R}{\lambda_y}\right)\lambda_A \qquad ◀$$

$$\mathbf{B} = \left(\frac{R}{\lambda_y}\right)(-\lambda_x\mathbf{i} - \lambda_x\mathbf{k}) \qquad ◀$$

Assuming that $\lambda_x, \lambda_y, \lambda_z > 0$, the equivalent force system is as shown below.



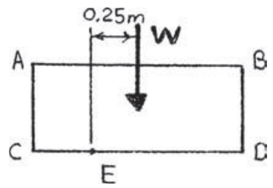
Note that the component of **A** in the *xz* plane is parallel to **B**.



PROBLEM 3.147

A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *B* that creates a moment of equal magnitude and opposite sense about *E*.

SOLUTION

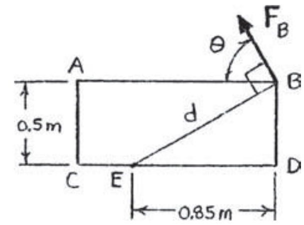


(*a*) By definition $W = mg = 80\text{ kg}(9.81\text{ m/s}^2) = 784.8\text{ N}$

We have $\Sigma M_E: \quad M_E = (784.8\text{ N})(0.25\text{ m})$

$\mathbf{M}_E = 196.2\text{ N}\cdot\text{m}$ ↻ ◀

(*b*) For the force at *B* to be the smallest, resulting in a moment ($\mathbf{M}_E$) about *E*, the line of action of force $\mathbf{F}_B$ must be perpendicular to the line connecting *E* to *B*. The sense of $\mathbf{F}_B$ must be such that the force produces a counterclockwise moment about *E*.



Note: $d = \sqrt{(0.85\text{ m})^2 + (0.5\text{ m})^2} = 0.98615\text{ m}$

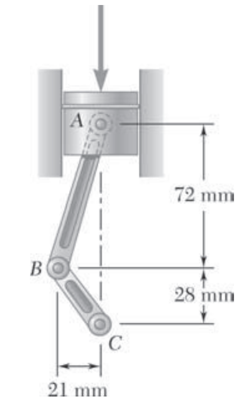
We have $\Sigma M_E: \quad 196.2\text{ N}\cdot\text{m} = F_B(0.98615\text{ m})$

$F_B = 198.954\text{ N}$

and $\theta = \tan^{-1}\left(\dfrac{0.85\text{ m}}{0.5\text{ m}}\right) = 59.534°$

or $\mathbf{F}_B = 199.0\text{ N} \measuredangle\, 59.5°$ ◀

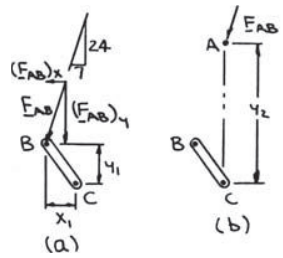
PROBLEM 3.148



It is known that the connecting rod AB exerts on the crank BC a 1.5-kN force directed down and to the left along the centerline of AB. Determine the moment of the force about C.

SOLUTION

Using (*a*)



$$M_C = y_1(F_{AB})_x + x_1(F_{AB})_y$$

$$= (0.028 \text{ m})\left(\frac{7}{25}\times 1500 \text{ N}\right) + (0.021 \text{ m})\left(\frac{24}{25}\times 1500 \text{ N}\right)$$

$$= 42 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 42.0 \text{ N}\cdot\text{m}$ ◀

Using (*b*)

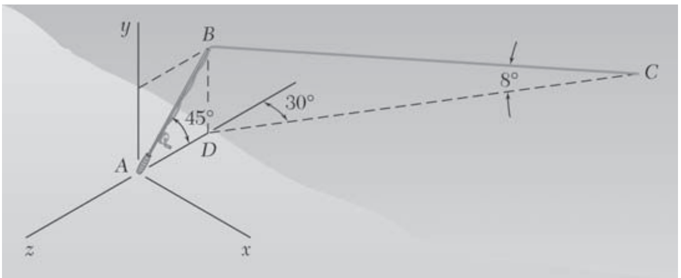
$$M_C = y_2(F_{AB})_x$$

$$= (0.1 \text{ m})\left(\frac{7}{25}\times 1500 \text{ N}\right) = 42 \text{ N}\cdot\text{m}$$

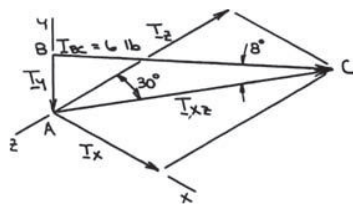
or $\mathbf{M}_C = 42.0 \text{ N}\cdot\text{m}$ ◀

PROBLEM 3.149

A 6-ft-long fishing rod *AB* is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about *A* of the force exerted by the line at *B*.



SOLUTION



We have $T_{xz} = (6\text{ lb})\cos 8° = 5.9416\text{ lb}$

Then $T_x = T_{xz}\sin 30° = 2.9708\text{ lb}$

$T_y = T_{BC}\sin 8° = -0.83504\text{ lb}$

$T_z = T_{xz}\cos 30° = -5.1456\text{ lb}$

Now $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$

where $\mathbf{r}_{B/A} = (6\sin 45°)\mathbf{j} - (6\cos 45°)\mathbf{k}$

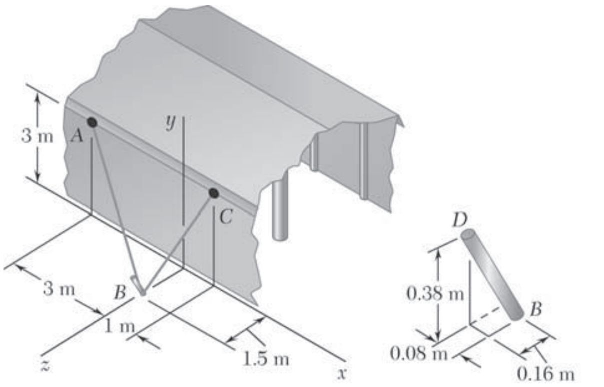
$$= \frac{6\text{ ft}}{\sqrt{2}}(\mathbf{j} - \mathbf{k})$$

Then
$$\mathbf{M}_A = \frac{6}{\sqrt{2}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -1 \\ 2.9708 & -0.83504 & -5.1456 \end{vmatrix}$$

$$= \frac{6}{\sqrt{2}}(-5.1456 - 0.83504)\mathbf{i} - \frac{6}{\sqrt{2}}(2.9708)\mathbf{j} - \frac{6}{\sqrt{2}}(2.9708)\mathbf{k}$$

or $\mathbf{M}_A = -(25.4\text{ lb}\cdot\text{ft})\mathbf{i} - (12.60\text{ lb}\cdot\text{ft})\mathbf{j} - (12.60\text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

PROBLEM 3.150



Detail of the stake at B

Ropes AB and BC are two of the ropes used to support a tent. The two ropes are attached to a stake at B. If the tension in rope AB is 540 N, determine (*a*) the angle between rope AB and the stake, (*b*) the projection on the stake of the force exerted by rope AB at Point B.

SOLUTION

First note
$$BA = \sqrt{(-3)^2 + (3)^2 + (-1.5)^2} = 4.5 \text{ m}$$
$$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2} = 0.42 \text{ m}$$

Then
$$\mathbf{T}_{BA} = \frac{T_{BA}}{4.5}(-3\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k})$$
$$= \frac{T_{BA}}{3}(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$
$$\boldsymbol{\lambda}_{BD} = \frac{\overrightarrow{BD}}{BD} = \frac{1}{0.42}(-0.08\mathbf{i} + 0.38\mathbf{j} + 0.16\mathbf{k})$$
$$= \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k})$$

(*a*) We have
$$\mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD} = T_{BA} \cos\theta$$

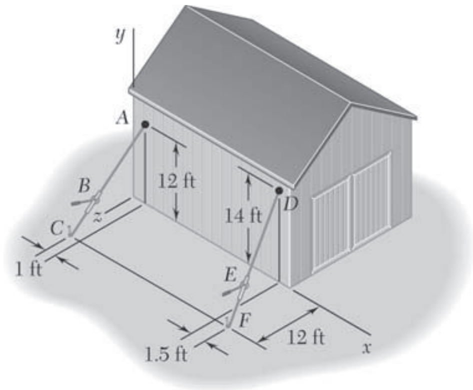
or
$$\frac{T_{BA}}{3}(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) = T_{BA} \cos\theta$$

or
$$\cos\theta = \frac{1}{63}[(-2)(-4) + (2)(19) + (-1)(8)]$$
$$= 0.60317$$

or $\theta = 52.9°$ ◀

(*b*) We have
$$(T_{BA})_{BD} = \mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD}$$
$$= T_{BA} \cos\theta$$
$$= (540 \text{ N})(0.60317)$$

or $(T_{BA})_{BD} = 326$ N ◀

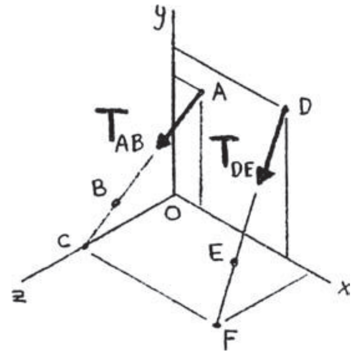


PROBLEM 3.151

A farmer uses cables and winch pullers B and E to plumb one side of a small barn. If it is known that the sum of the moments about the x axis of the forces exerted by the cables on the barn at Points A and D is equal to 4728 lb · ft, determine the magnitude of $\mathbf{T}_{DE}$ when $T_{AB} = 255$ lb.

SOLUTION

The moment about the x axis due to the two cable forces can be found using the z components of each force acting at their intersection with the xy-plane (A and D). The x components of the forces are parallel to the x axis, and the y components of the forces intersect the x axis. Therefore, neither the x or y components produce a moment about the x axis.



We have $\qquad \Sigma M_x: \quad (T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$

where

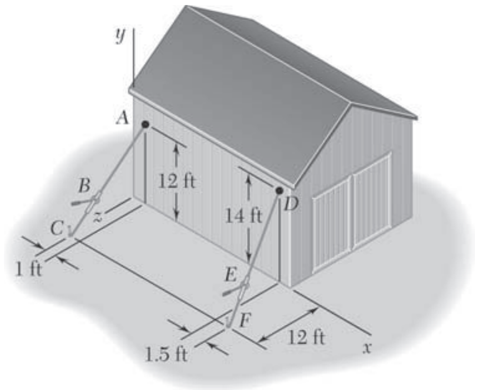
$$\begin{aligned}(T_{AB})_z &= \mathbf{k}\cdot\mathbf{T}_{AB}\\ &= \mathbf{k}\cdot(T_{AB}\lambda_{AB})\\ &= \mathbf{k}\cdot\left[255\text{ lb}\left(\frac{-\mathbf{i}-12\mathbf{j}+12\mathbf{k}}{17}\right)\right]\\ &= 180\text{ lb}\end{aligned}$$

$$\begin{aligned}(T_{DE})_z &= \mathbf{k}\cdot\mathbf{T}_{DE}\\ &= \mathbf{k}\cdot(T_{DE}\lambda_{DE})\\ &= \mathbf{k}\cdot\left[T_{DE}\left(\frac{1.5\mathbf{i}-14\mathbf{j}+12\mathbf{k}}{18.5}\right)\right]\\ &= 0.64865T_{DE}\end{aligned}$$

$$y_A = 12\text{ ft}$$
$$y_D = 14\text{ ft}$$
$$M_x = 4728\text{ lb}\cdot\text{ft}$$

$$(180\text{ lb})(12\text{ ft}) + (0.64865T_{DE})(14\text{ ft}) = 4728\text{ lb}\cdot\text{ft}$$

and $\qquad T_{DE} = 282.79$ lb $\qquad$ or $T_{DE} = 283$ lb ◀



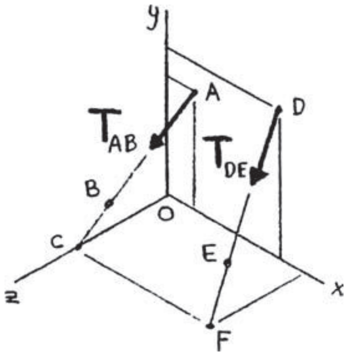
PROBLEM 3.152

Solve Problem 3.151 when the tension in cable AB is 306 lb.

PROBLEM 3.151 A farmer uses cables and winch pullers B and E to plumb one side of a small barn. If it is known that the sum of the moments about the x axis of the forces exerted by the cables on the barn at Points A and D is equal to 4728 lb · ft, determine the magnitude of $\mathbf{T}_{DE}$ when $T_{AB} = 255$ lb.

SOLUTION

The moment about the x axis due to the two cable forces can be found using the z components of each force acting at the intersection with the xy plane (A and D). The x components of the forces are parallel to the x axis, and the y components of the forces intersect the x axis. Therefore, neither the x or y components produce a moment about the x axis.



We have $\Sigma M_x: \quad (T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$

Where

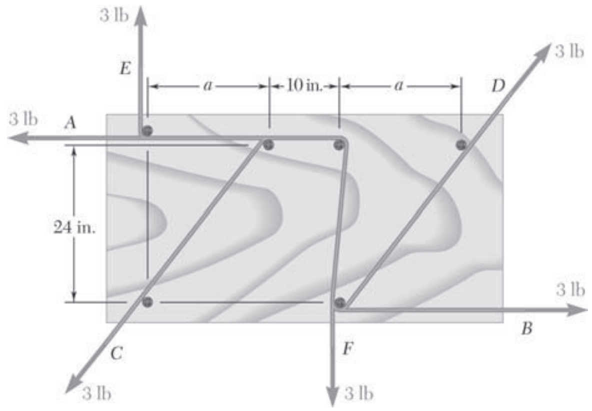
$$\begin{aligned}(T_{AB})_z &= \mathbf{k}\cdot\mathbf{T}_{AB}\\ &= \mathbf{k}\cdot(T_{AB}\boldsymbol{\lambda}_{AB})\\ &= \mathbf{k}\cdot\left[306\text{ lb}\left(\frac{-\mathbf{i}-12\mathbf{j}+12\mathbf{k}}{17}\right)\right]\\ &= 216\text{ lb}\end{aligned}$$

$$\begin{aligned}(T_{DE})_z &= \mathbf{k}\cdot\mathbf{T}_{DE}\\ &= \mathbf{k}\cdot(T_{DE}\boldsymbol{\lambda}_{DE})\\ &= \mathbf{k}\cdot\left[T_{DE}\left(\frac{1.5\mathbf{i}-14\mathbf{j}+12\mathbf{k}}{18.5}\right)\right]\\ &= 0.64865T_{DE}\end{aligned}$$

$$y_A = 12\text{ ft}$$
$$y_D = 14\text{ ft}$$
$$M_x = 4728\text{ lb}\cdot\text{ft}$$

$$(216\text{ lb})(12\text{ ft}) + (0.64865T_{DE})(14\text{ ft}) = 4728\text{ lb}\cdot\text{ft}$$

and $\quad T_{DE} = 235.21$ lb $\quad$ or $T_{DE} = 235$ lb ◀



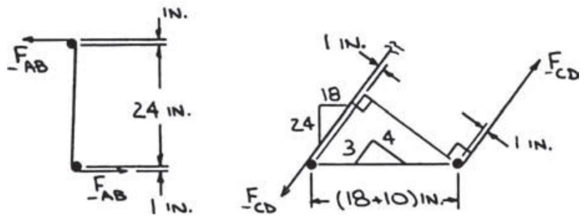
PROBLEM 3.153

A wiring harness is made by routing either two or three wires around 2-in.-diameter pegs mounted on a sheet of plywood. If the force in each wire is 3 lb, determine the resultant couple acting on the plywood when $a = 18$ in. and (a) only wires AB and CD are in place, (b) all three wires are in place.

SOLUTION

In general, $M = \Sigma dF$, where d is the perpendicular distance between the lines of action of the two forces acting on a given wire.

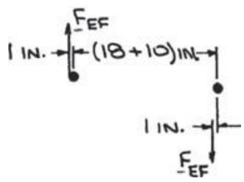
(a)



We have $\quad M = d_{AB}F_{AB} + d_{CD}F_{CD}$

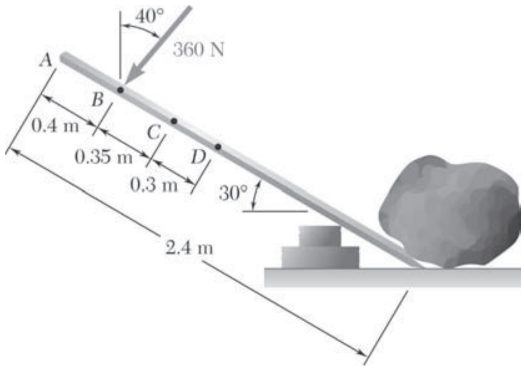
$$= (2 + 24)\text{ in.} \times 3\text{ lb} + \left(2 + \frac{4}{5} \times 28\right)\text{in.} \times 3\text{ lb} \qquad \text{or} \quad \mathbf{M} = 151.2\text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

(b)



We have $\quad M = [d_{AB}F_{AB} + d_{CD}F_{CD}] + d_{EF}F_{EF}$

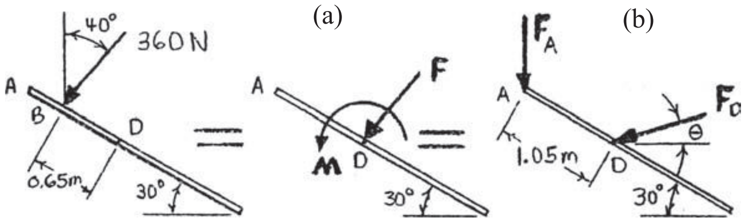
$$= 151.2\text{ lb} \cdot \text{in.} - 28\text{ in.} \times 3\text{ lb} \qquad \text{or} \quad \mathbf{M} = 67.2\text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$



PROBLEM 3.154

A worker tries to move a rock by applying a 360-N force to a steel bar as shown. (*a*) Replace that force with an equivalent force-couple system at *D*. (b) Two workers attempt to move the same rock by applying a vertical force at *A* and another force at *D*. Determine these two forces if they are to be equivalent to the single force of Part *a*.

SOLUTION



(*a*) We have $\Sigma \mathbf{F}$: $360\text{ N}(-\sin 40°\mathbf{i} - \cos 40°\mathbf{j}) = -(231.40\text{ N})\mathbf{i} - (275.78\text{ N})\mathbf{j} = \mathbf{F}$

or $\mathbf{F} = 360\text{ N} \measuredangle 50°$ ◀

We have $\Sigma \mathbf{M}_D$: $\mathbf{r}_{B/D} \times \mathbf{R} = \mathbf{M}$

where

$$\mathbf{r}_{B/D} = -[(0.65\text{ m})\cos 30°]\mathbf{i} + [(0.65\text{ m})\sin 30°]\mathbf{j}$$
$$= -(0.56292\text{ m})\mathbf{i} + (0.32500\text{ m})\mathbf{j}$$

$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.56292 & 0.32500 & 0 \\ -231.40 & -275.78 & 0 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= [155.240 + 75.206)\text{N}\cdot\text{m}]\mathbf{k}$$

$$= (230.45\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 230\text{ N}\cdot\text{m}$ ⤸ ◀

(*b*) We have $\Sigma \mathbf{M}_D$: $\mathbf{M} = \mathbf{r}_{A/D} \times \mathbf{F}_A$

where

$$\mathbf{r}_{B/D} = -[(1.05\text{ m})\cos 30°]\mathbf{i} + [(1.05\text{ m})\sin 30°]\mathbf{j}$$
$$= -(0.90933\text{ m})\mathbf{i} + (0.52500\text{ m})\mathbf{j}$$

$$F_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.90933 & 0.52500 & 0 \\ 0 & -1 & 0 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= [230.45\text{ N}\cdot\text{m}]\mathbf{k}$$

PROBLEM 3.154 (Continued)

or $\quad (0.90933F_A)\mathbf{k} = 230.45\mathbf{k}$

$$F_A = 253.42 \text{ N} \qquad \text{or} \quad \mathbf{F}_A = 253 \text{ N} \downarrow \blacktriangleleft$$

We have $\quad \Sigma\mathbf{F}: \quad \mathbf{F} = \mathbf{F}_A + \mathbf{F}_D$

$$-(231.40 \text{ N})\mathbf{i} - (275.78 \text{ N})\mathbf{j} = -(253.42 \text{ N})\mathbf{j} + F_D(-\cos\theta\mathbf{i} - \sin\theta\mathbf{j})$$

From $\quad \mathbf{i}: \quad 231.40 \text{ N} = F_D \cos\theta \qquad (1)$

$\quad \mathbf{j}: \quad 22.36 \text{ N} = F_D \sin\theta \qquad (2)$

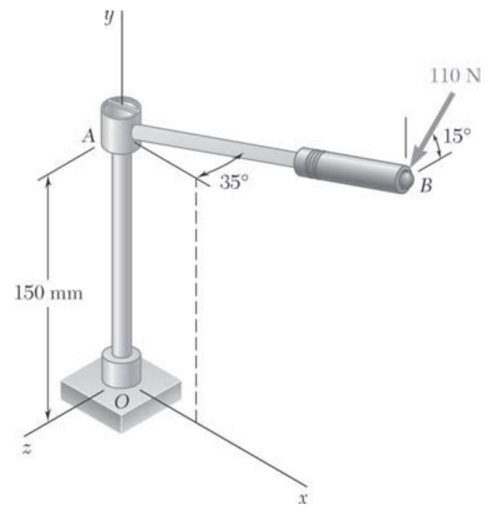
Equation (2) divided by Equation (1)

$$\tan\theta = 0.096629$$

$$\theta = 5.5193° \quad \text{or} \quad \theta = 5.52°$$

Substitution into Equation (1)

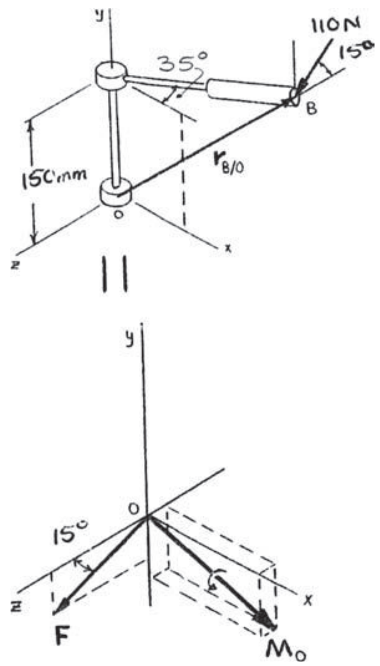
$$F_D = \frac{231.40}{\cos 5.5193°} = 232.48 \text{ N} \qquad \text{or} \quad \mathbf{F}_D = 232 \text{ N} \nearrow 5.52° \blacktriangleleft$$



PROBLEM 3.155

A 110-N force acting in a vertical plane parallel to the yz plane is applied to the 220-mm-long horizontal handle AB of a socket wrench. Replace the force with an equivalent force-couple system at the origin O of the coordinate system.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{P}_B = \mathbf{F}$

where
$$\mathbf{P}_B = 110 \text{ N}[-(\sin 15°)\mathbf{j} + (\cos 15°)\mathbf{k}]$$
$$= -(28.470 \text{ N})\mathbf{j} + (106.252 \text{ N})\mathbf{k}$$

or $\mathbf{F} = -(28.5 \text{ N})\mathbf{j} + (106.3 \text{ N})\mathbf{k}$ ◀

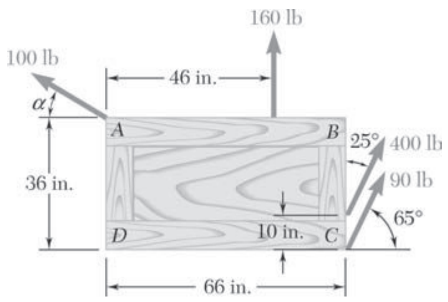
We have $\quad \Sigma M_O: \quad \mathbf{r}_{B/O} \times \mathbf{P}_B = \mathbf{M}_O$

where
$$\mathbf{r}_{B/O} = [(0.22\cos 35°)\mathbf{i} + (0.15)\mathbf{j} - (0.22\sin 35°)\mathbf{k}]\text{m}$$
$$= (0.180213 \text{ m})\mathbf{i} + (0.15 \text{ m})\mathbf{j} - (0.126187 \text{ m})\mathbf{k}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.180213 & 0.15 & 0.126187 \\ 0 & -28.5 & 106.3 \end{vmatrix} \text{N}\cdot\text{m} = \mathbf{M}_O$$

$$\mathbf{M}_O = [(12.3487)\mathbf{i} - (19.1566)\mathbf{j} - (5.1361)\mathbf{k}]\text{N}\cdot\text{m}$$

or $\mathbf{M}_O = (12.35 \text{ N}\cdot\text{m})\mathbf{i} - (19.16 \text{ N}\cdot\text{m})\mathbf{j} - (5.13 \text{ N}\cdot\text{m})\mathbf{k}$ ◀

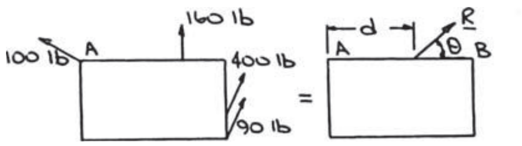


PROBLEM 3.156

Four ropes are attached to a crate and exert the forces shown. If the forces are to be replaced with a single equivalent force applied at a point on line AB, determine (a) the equivalent force and the distance from A to the point of application of the force when $\alpha = 30°$, (b) the value of α so that the single equivalent force is applied at Point B.

SOLUTION

We have



(a) For equivalence $\quad \Sigma F_x: \quad -100\cos 30° + 400\cos 65° + 90\cos 65° = R_x$

or $\quad R_x = 120.480 \text{ lb}$

$\Sigma F_y: \quad 100\sin\alpha + 160 + 400\sin 65° + 90\sin 65° = R_y$

or $\quad R_y = (604.09 + 100\sin\alpha) \text{ lb} \qquad (1)$

With $\alpha = 30°$ $\quad R_y = 654.09 \text{ lb}$

Then $\quad R = \sqrt{(120.480)^2 + (654.09)^2} \qquad \tan\theta = \dfrac{654.09}{120.480}$

$= 665 \text{ lb} \qquad$ or $\theta = 79.6°$

Also $\quad \Sigma M_A: \quad (46 \text{ in.})(160 \text{ lb}) + (66 \text{ in.})(400 \text{ lb})\sin 65°$

$+(26 \text{ in.})(400 \text{ lb})\cos 65° + (66 \text{ in.})(90 \text{ lb})\sin 65°$

$+(36 \text{ in.})(90 \text{ lb})\cos 65° = d(654.09 \text{ lb})$

or $\quad \Sigma M_A = 42{,}435 \text{ lb}\cdot\text{in.}$ and $d = 64.9 \text{ in.}$ $\qquad \mathbf{R} = 665 \text{ lb} \measuredangle 79.6°$ ◀

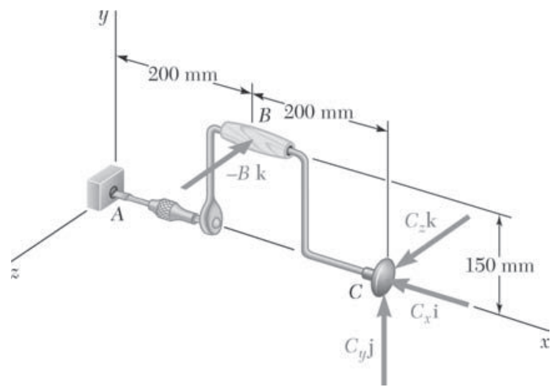
and **R** is applied 64.9 in. To the right of A. ◀

(b) We have $d = 66$ in.

Then $\quad \Sigma M_A: \quad 42{,}435 \text{ lb}\cdot\text{in} = (66 \text{ in.})R_y$

or $\quad R_y = 642.95 \text{ lb}$

Using Eq. (1) $\quad 642.95 = 604.09 + 100\sin\alpha \qquad$ or $\alpha = 22.9°$ ◀



PROBLEM 3.157

A blade held in a brace is used to tighten a screw at A. (a) Determine the forces exerted at B and C, knowing that these forces are equivalent to a force-couple system at A consisting of $\mathbf{R} = -(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k}$ and $\mathbf{M}_A^R = -(12\text{ N}\cdot\text{m})\mathbf{i}$. ($b$) Find the corresponding values of R_y and R_z. (c) What is the orientation of the slot in the head of the screw for which the blade is least likely to slip when the brace is in the position shown?

SOLUTION

(a) Equivalence requires $\Sigma\mathbf{F}$: $\mathbf{R} = \mathbf{B} + \mathbf{C}$

or

$$-(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = -B\mathbf{k} + (-C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k})$$

Equating the **i** coefficients $\quad \mathbf{i}: \; -30\text{ N} = -C_x \quad \text{or} \quad C_x = 30\text{ N}$

Also $\quad \Sigma\mathbf{M}_A: \; \mathbf{M}_A^R = \mathbf{r}_{B/A}\times\mathbf{B} + \mathbf{r}_{C/A}\times\mathbf{C}$

or

$$-(12\text{ N}\cdot\text{m})\mathbf{i} = [(0.2\text{ m})\mathbf{i} + (0.15\text{ m})\mathbf{j}]\times(-B)\mathbf{k} + (0.4\text{ m})\mathbf{i}\times[-(30\text{ N})\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k}]$$

Equating coefficients

$\mathbf{i}: \; -12\text{ N}\cdot\text{m} = -(0.15\text{ m})B \quad \text{or} \quad B = 80\text{ N}$

$\mathbf{k}: \; 0 = (0.4\text{ m})C_y \quad \text{or} \quad C_y = 0$

$\mathbf{j}: \; 0 = (0.2\text{ m})(80\text{ N}) - (0.4\text{ m})C_z \quad \text{or} \quad C_z = 40\text{ N}$

$$\mathbf{B} = -(80.0\text{ N})\mathbf{k} \quad \mathbf{C} = -(30.0\text{ N})\mathbf{i} + (40.0\text{ N})\mathbf{k} \; ◀$$

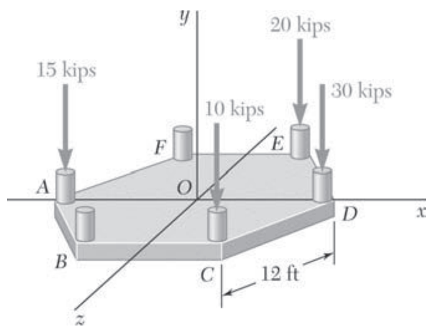
(b) Now we have for the equivalence of forces

$$-(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = -(80\text{ N})\mathbf{k} + [(-30\text{ N})\mathbf{i} + (40\text{ N})\mathbf{k}]$$

Equating coefficients $\quad \mathbf{j}: \; R_y = 0 \qquad R_y = 0 \; ◀$

$\mathbf{k}: \; R_z = -80 + 40 \qquad \text{or} \quad R_z = -40.0\text{ N} \; ◀$

(c) First note that $\mathbf{R} = -(30\text{ N})\mathbf{i} - (40\text{ N})\mathbf{k}$. Thus, the screw is best able to resist the lateral force R_z when the slot in the head of the screw is vertical. ◀



PROBLEM 3.158

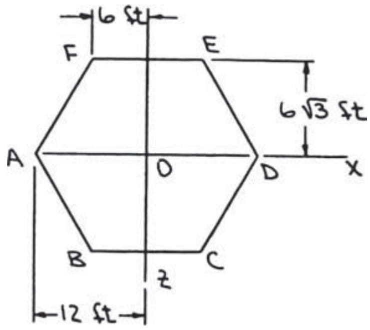
A concrete foundation mat in the shape of a regular hexagon of side 12 ft supports four column loads as shown. Determine the magnitudes of the additional loads that must be applied at B and F if the resultant of all six loads is to pass through the center of the mat.

SOLUTION

From the statement of the problem it can be concluded that the six applied loads are equivalent to the resultant **R** at O. It then follows that

$$\Sigma \mathbf{M}_O = 0 \quad \text{or} \quad \Sigma M_x = 0 \quad \Sigma M_z = 0$$

For the applied loads.



Then $\quad \Sigma \mathbf{M}_x = 0\colon \quad (6\sqrt{3}\text{ ft})F_B + (6\sqrt{3}\text{ ft})(10\text{ kips}) - (6\sqrt{3}\text{ ft})(20\text{ kips})$
$\quad -(6\sqrt{3}\text{ ft})F_F = 0$

or $\quad F_B - F_F = 10 \qquad (1)$

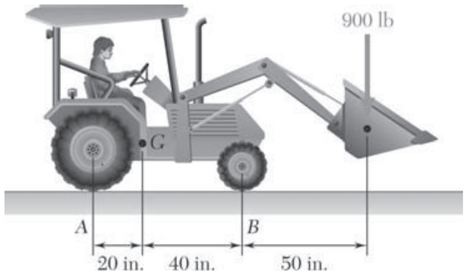
$\Sigma \mathbf{M}_z = 0\colon \quad (12\text{ ft})(15\text{ kips}) + (6\text{ ft})F_B - (6\text{ ft})(10\text{ kips})$
$\quad -(12\text{ ft})(30\text{ kips}) - (6\text{ ft})(20\text{ kips}) + (6\text{ ft})F_F = 0$

or $\quad F_B + F_F = 60 \qquad (2)$

Then $(1) + (2) \Rightarrow \qquad \mathbf{F}_B = 35.0\text{ kips} \downarrow$ ◀

and $\qquad \mathbf{F}_F = 25.0\text{ kips} \downarrow$ ◀

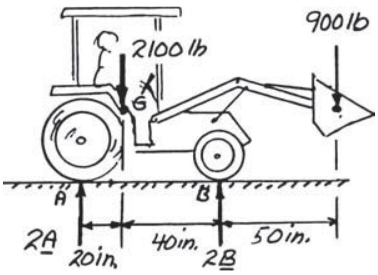
CHAPTER 4



PROBLEM 4.1

A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION



(*a*) Rear wheels $\quad +\circlearrowleft \Sigma M_B = 0: \quad +(2100 \text{ lb})(40 \text{ in.}) - (900 \text{ lb})(50 \text{ in.}) + 2A(60 \text{ in.}) = 0$

$$A = +325 \text{ lb} \qquad \mathbf{A} = 325 \text{ lb} \uparrow \blacktriangleleft$$

(*b*) Front wheels $\quad +\circlearrowleft \Sigma M_A: \quad -(2100 \text{ lb})(20 \text{ in.}) - (900 \text{ lb})(110 \text{ in.}) - 2B(60 \text{ in.}) = 0$

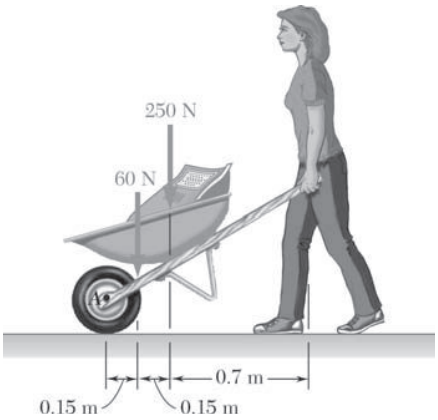
$$B = +1175 \text{ lb} \qquad \mathbf{B} = 1175 \text{ lb} \uparrow \blacktriangleleft$$

Check: $\quad +\Sigma F_y = 0: \quad 2A + 2B - 2100 \text{ lb} - 900 \text{ lb} = 0$

$$2(325 \text{ lb}) + 2(1175 \text{ lb}) - 2100 \text{ lb} - 900 = 0$$

$$0 = 0 \quad \text{(Checks)}$$

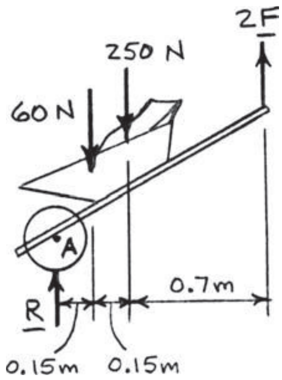
PROBLEM 4.2



A gardener uses a 60-N wheelbarrow to transport a 250-N bag of fertilizer. What force must she exert on each handle?

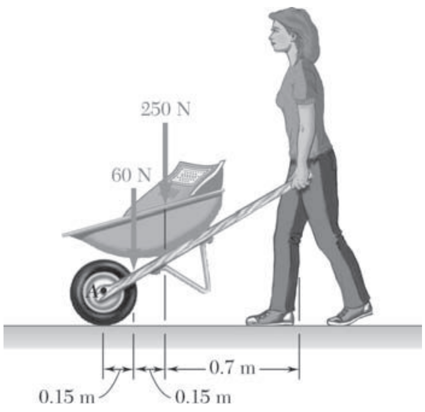
SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (2F)(1\text{ m}) - (60\text{ N})(0.15\text{ m}) - (250\text{ N})(0.3\text{ m}) = 0$$

$$\mathbf{F} = 42.0\text{ N}\uparrow \blacktriangleleft$$



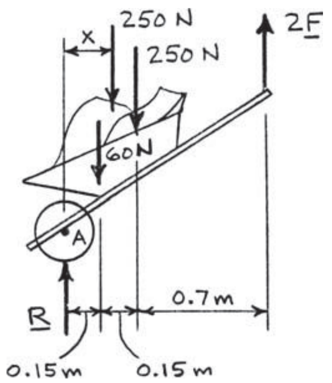
PROBLEM 4.3

The gardener of Problem 4.2 wishes to transport a second 250-N bag of fertilizer at the same time as the first one. Determine the maximum allowable horizontal distance from the axle A of the wheelbarrow to the center of gravity of the second bag if she can hold only 75 N with each arm.

PROBLEM 4.2 A gardener uses a 60-N wheelbarrow to transport a 250-N bag of fertilizer. What force must she exert on each handle?

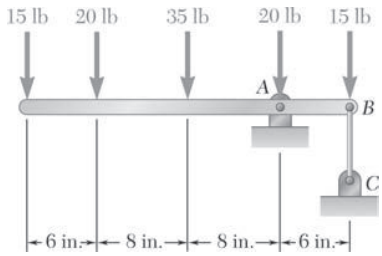
SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad 2(75\text{ N})(1\text{ m}) - (60\text{ N})(0.15\text{ m}) - (250\text{ N})(0.3\text{ m}) - (250\text{ N})x = 0$$

$$x = 0.264\text{ m} \blacktriangleleft$$

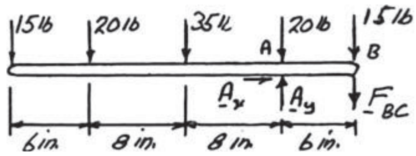


PROBLEM 4.4

For the beam and loading shown, determine (*a*) the reaction at A, (*b*) the tension in cable BC.

SOLUTION

Free-Body Diagram:



(*a*) Reaction at A: $\Sigma F_x = 0$: $A_x = 0$

$$+\circlearrowleft \Sigma M_B = 0: \quad (15\text{ lb})(28\text{ in.}) + (20\text{ lb})(22\text{ in.}) + (35\text{ lb})(14\text{ in.}) + (20\text{ lb})(6\text{ in.}) - A_y(6\text{ in.}) = 0$$

$$A_y = +245\text{ lb} \qquad \mathbf{A} = 245\text{ lb}\uparrow \blacktriangleleft$$

(*b*) Tension in BC

$$+\circlearrowleft \Sigma M_A = 0: \quad (15\text{ lb})(22\text{ in.}) + (20\text{ lb})(16\text{ in.}) + (35\text{ lb})(8\text{ in.}) - (15\text{ lb})(6\text{ in.}) - F_{BC}(6\text{ in.}) = 0$$

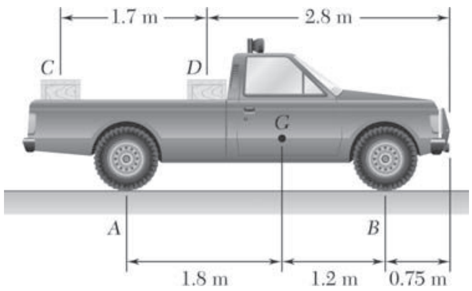
$$F_{BC} = +140.0\text{ lb} \qquad F_{BC} = 140.0\text{ lb} \blacktriangleleft$$

Check:

$$+\uparrow \Sigma F_y = 0: \quad -15\text{ lb} - 20\text{ lb} = 35\text{ lb} - 20\text{ lb} + A - F_{BC} = 0$$

$$-105\text{ lb} + 245\text{ lb} - 140.0 = 0$$

$$0 = 0 \quad \text{(Checks)}$$

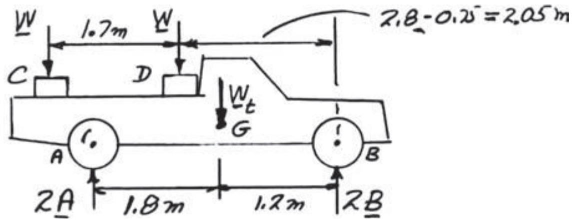


PROBLEM 4.5

Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION

Free-Body Diagram:



$$W = (350 \text{ kg})(9.81 \text{ m/s}^2) = 3.434 \text{ kN}$$

$$W_t = (1400 \text{ kg})(9.81 \text{ m/s}^2) = 13.734 \text{ kN}$$

(*a*) Rear wheels

$$+\circlearrowleft \Sigma M_B = 0: \quad W(1.7 \text{ m} + 2.05 \text{ m}) + W(2.05 \text{ m}) + W_t(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$$

$$(3.434 \text{ kN})(3.75 \text{ m}) + (3.434 \text{ kN})(2.05 \text{ m}) + (13.734 \text{ kN})(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$$

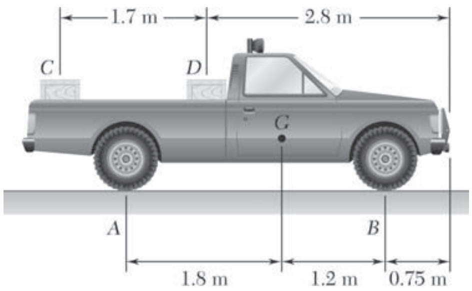
$$A = +6.0663 \text{ kN} \qquad \mathbf{A} = 6.07 \text{ kN} \uparrow \blacktriangleleft$$

(*b*) Front wheels

$$+\uparrow \Sigma F_y = 0: \quad -W - W - W_t + 2A + 2B = 0$$

$$-3.434 \text{ kN} - 3.434 \text{ kN} - 13.734 \text{ kN} + 2(6.0663 \text{ kN}) + 2B = 0$$

$$B = +4.2347 \text{ kN} \qquad \mathbf{B} = 4.23 \text{ kN} \uparrow \blacktriangleleft$$



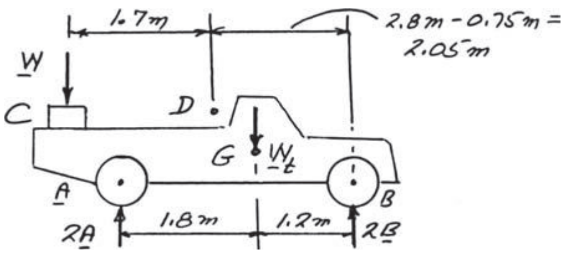
PROBLEM 4.6

Solve Problem 4.5, assuming that crate D is removed and that the position of crate C is unchanged.

PROBLEM 4.5 Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels A, (*b*) front wheels B.

SOLUTION

Free-Body Diagram:



$$W = (350\text{ kg})(9.81\text{ m/s}^2) = 3.434\text{ kN}$$
$$W_t = (1400\text{ kg})(9.81\text{ m/s}^2) = 13.734\text{ kN}$$

(*a*) <u>Rear wheels</u> $+\circlearrowleft \Sigma M_B = 0\!: \quad W(1.7\text{ m} + 2.05\text{ m}) + W_t(1.2\text{ m}) - 2A(3\text{ m}) = 0$

$$(3.434\text{ kN})(3.75\text{ m}) + (13.734\text{ kN})(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

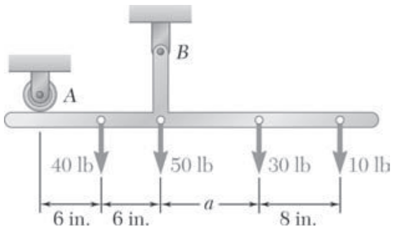
$$A = +4.893\text{ kN} \qquad \mathbf{A} = 4.89\text{ kN}\uparrow \blacktriangleleft$$

(*b*) <u>Front wheels</u> $+\uparrow \Sigma M_y = 0\!: \quad -W - W_t + 2A + 2B = 0$

$$-3.434\text{ kN} - 13.734\text{ kN} + 2(4.893\text{ kN}) + 2B = 0$$

$$B = +3.691\text{ kN} \qquad \mathbf{B} = 3.69\text{ kN}\uparrow \blacktriangleleft$$

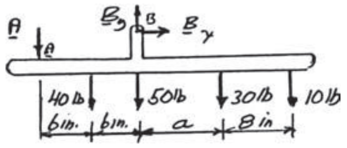
PROBLEM 4.7



A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (40 \text{ lb})(6 \text{ in.}) - (30 \text{ lb})a - (10 \text{ lb})(a + 8 \text{ in.}) + (12 \text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12} \qquad (1)$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(40 \text{ lb})(6 \text{ in.}) - (50 \text{ lb})(12 \text{ in.}) - (30 \text{ lb})(a + 12 \text{ in.})$$
$$-(10 \text{ lb})(a + 20 \text{ in.}) + (12 \text{ in.})B_y = 0$$

$$B_y = \frac{(1400 + 40a)}{12}$$

Since

$$B_x = 0 \quad B = \frac{(1400 + 40a)}{12} \qquad (2)$$

(a) For $a = 10$ in.

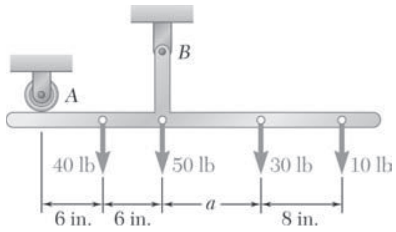
Eq. (1): $$A = \frac{(40 \times 10 - 160)}{12} = +20.0 \text{ lb} \qquad \mathbf{A} = 20.0 \text{ lb}\downarrow \blacktriangleleft$$

Eq. (2): $$B = \frac{(1400 + 40 \times 10)}{12} = +150.0 \text{ lb} \qquad \mathbf{B} = 150.0 \text{ lb}\downarrow \blacktriangleleft$$

(b) For $a = 7$ in.

Eq. (1): $$A = \frac{(40 \times 7 - 160)}{12} = +10.00 \text{ lb} \qquad \mathbf{A} = 10.00 \text{ lb}\downarrow \blacktriangleleft$$

Eq. (2): $$B = \frac{(1400 + 40 \times 7)}{12} = +140.0 \text{ lb} \qquad \mathbf{B} = 140.0 \text{ lb}\uparrow \blacktriangleleft$$



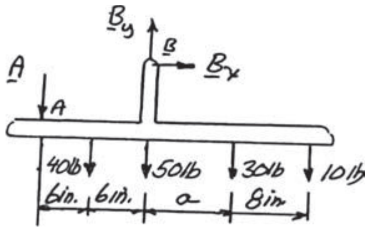
PROBLEM 4.8

For the bracket and loading of Problem 4.7, determine the smallest distance a if the bracket is not to move.

PROBLEM 4.7 A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:



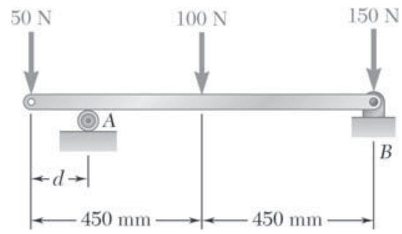
For no motion, reaction at A must be downward or zero; smallest distance a for no motion corresponds to $\mathbf{A} = 0$.

$$+\circlearrowleft \Sigma M_B = 0: \quad (40 \text{ lb})(6 \text{ in.}) - (30 \text{ lb})a - (10 \text{ lb})(a + 8 \text{ in.}) + (12 \text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12}$$

$$A = 0: \quad (40a - 160) = 0$$

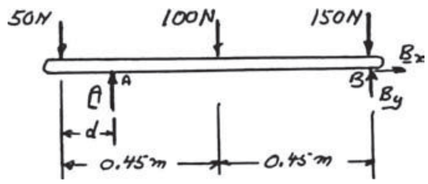
$a = 4.00$ in. ◀



PROBLEM 4.9

The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance d for which the beam is safe.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$B = B_y$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (50\text{ N})d - (100\text{ N})(0.45\text{ m} - d) - (150\text{ N})(0.9\text{ m} - d) + B(0.9\text{ m} - d) = 0$$

$$50d - 45 + 100d - 135 + 150d + 0.9B - Bd$$

$$d = \frac{180\text{ N}\cdot\text{m} - (0.9\text{ m})B}{300A - B} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (50\text{ N})(0.9\text{ m}) - A(0.9\text{ m} - d) + (100\text{ N})(0.45\text{ m}) = 0$$

$$45 - 0.9A + Ad + 45 = 0$$

$$d = \frac{(0.9\text{ m})A - 90\text{ N}\cdot\text{m}}{A} \qquad (2)$$

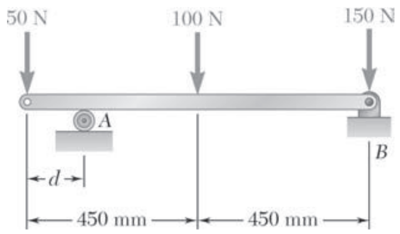
Since $B \le 180$ N, Eq. (1) yields.

$$d \ge \frac{180 - (0.9)180}{300 - 180} = \frac{18}{120} = 0.15\text{ m} \qquad d \ge 150.0\text{ mm} \ \lhd$$

Since $A \le 180$ N, Eq. (2) yields.

$$d \le \frac{(0.9)180 - 90}{180} = \frac{72}{180} = 0.40\text{ m} \qquad d \le 400\text{ mm} \ \lhd$$

Range: $150.0\text{ mm} \le d \le 400\text{ mm}$ ◀

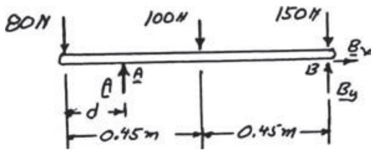


PROBLEM 4.10

Solve Problem 4.9 if the 50-N load is replaced by an 80-N load.

PROBLEM 4.9 The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance d for which the beam is safe.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$B = B_y$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (80\text{ N})d - (100\text{ N})(0.45\text{ m} - d) - (150\text{ N})(0.9\text{ m} - d) + B(0.9\text{ m} - d) = 0$$

$$80d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$$

$$d = \frac{180\text{ N}\cdot\text{m} - 0.9B}{330\text{N} - B} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (80\text{ N})(0.9\text{ m}) - A(0.9\text{ m} - d) + (100\text{ N})(0.45\text{ m}) = 0$$

$$d = \frac{0.9A - 117}{A} \qquad (2)$$

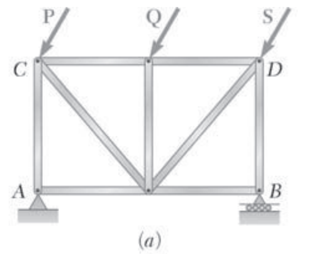
Since $B \leq 180$ N, Eq. (1) yields.

$$d \geq (180 - 0.9\times 180)/(330 - 180) = \frac{18}{150} = 0.12\text{ m} \qquad d = 120.0\text{ mm} \ \triangleleft$$

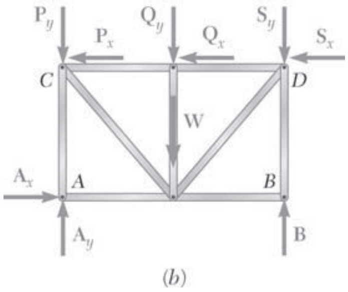
Since $A \leq 180$ N, Eq. (2) yields.

$$d \leq (0.9\times 180 - 112)/180 = \frac{45}{180} = 0.25\text{ m} \qquad d = 250\text{ mm} \ \triangleleft$$

Range: $120.0\text{ mm} \leq d \leq 250\text{ mm}$ ◀



(a)

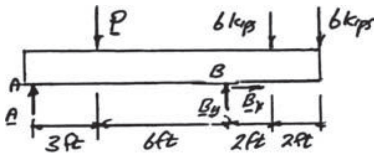


(b)

PROBLEM 4.11

For the beam of Sample Problem 4.2, determine the range of values of P for which the beam will be safe, knowing that the maximum allowable value of each of the reactions is 30 kips and that the reaction at A must be directed upward.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\mathbf{B} = B_y \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -P(3 \text{ ft}) + B(9 \text{ ft}) - (6 \text{ kips})(11 \text{ ft}) - (6 \text{ kips})(13 \text{ ft}) = 0$$

$$P = 3B - 48 \text{ kips} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -A(9 \text{ ft}) + P(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (6 \text{ kips})(4 \text{ ft}) = 0$$

$$P = 1.5A + 6 \text{ kips} \qquad (2)$$

Since $B \leq 30$ kips, Eq. (1) yields.

$$P \leq (3)(30 \text{ kips}) - 48 \text{ kips} \qquad P \leq 42.0 \text{ kips} \triangleleft$$

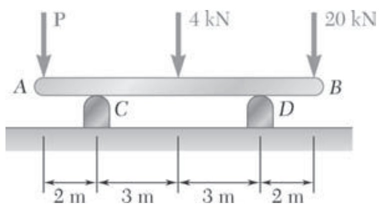
Since $0 \leq A \leq 30$ kips. Eq. (2) yields.

$$0 + 6 \text{ kips} \leq P \leq (1.5)(30 \text{ kips})1.6 \text{ kips}$$

$$6.00 \text{ kips} \leq P \leq 51.0 \text{ kips} \triangleleft$$

Range of values of P for which beam will be safe:

$$6.00 \text{ kips} \leq P \leq 42.0 \text{ kips} \blacktriangleleft$$

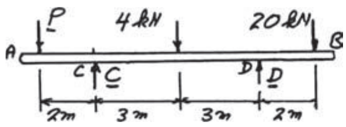


PROBLEM 4.12

The 10-m beam AB rests upon, but is not attached to, supports at C and D. Neglecting the weight of the beam, determine the range of values of P for which the beam will remain in equilibrium.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad P(2\text{ m}) - (4\text{ kN})(3\text{ m}) - (20\text{ kN})(8\text{ m}) + D(6\text{ m}) = 0$$

$$P = 86\text{ kN} - 3D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad P(8\text{ m}) + (4\text{ kN})(3\text{ m}) - (20\text{ kN})(2\text{ m}) - C(6\text{ m}) = 0$$

$$P = 3.5\text{ kN} + 0.75C \qquad (2)$$

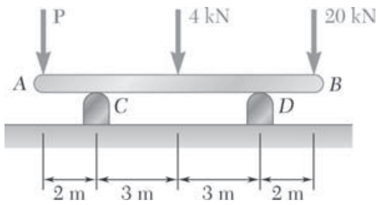
For no motion $C \geq 0$ and $D \geq 0$

For $C \geq 0$ from (2) $P \leq 3.50$ kN

For $D \geq 0$ from (1) $P \leq 86.0$ kN

Range of P for no motion:

$3.50\text{ kN} \leq P \leq 86.0\text{ kN}$ ◀

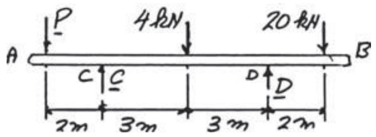


PROBLEM 4.13

The maximum allowable value of each of the reactions is 50 kN, and each reaction must be directed upward. Neglecting the weight of the beam, determine the range of values of P for which the beam is safe.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad P(2\text{ m}) - (4\text{ kN})(3\text{ m}) - (20\text{ kN})(8\text{ m}) + D(6\text{ m}) = 0$$

$$P = 86\text{ kN} - 3D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad P(8\text{ m}) + (4\text{ kN})(3\text{ m}) - (20\text{ kN})(2\text{ m}) - C(6\text{ m}) = 0$$

$$P = 3.5\text{ kN} + 0.75C \qquad (2)$$

For $C \geq 0$, from (2): $\quad P \geq 3.50\text{ kN}$ ◁

For $D \geq 0$, from (1): $\quad P \leq 86.0\text{ kN}$ ◁

For $C \leq 50\text{ kN}$, from (2):

$$P \leq 3.5\text{ kN} + 0.75(50\text{ kN})$$

$$P \leq 41.0\text{ kN}$$ ◁

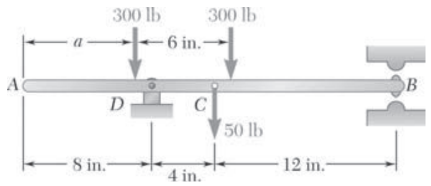
For $D \leq 50\text{ kN}$, from (1):

$$P \geq 86\text{ kN} - 3(50\text{ kN})$$

$$P \geq -64.0\text{ kN}$$ ◁

Comparing the four criteria, we find

$$3.50\text{ kN} \leq P \leq 41.0\text{ kN}$$ ◀

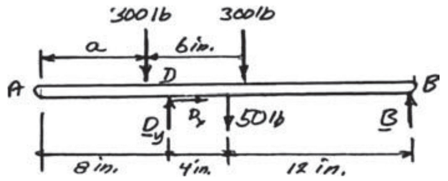


PROBLEM 4.14

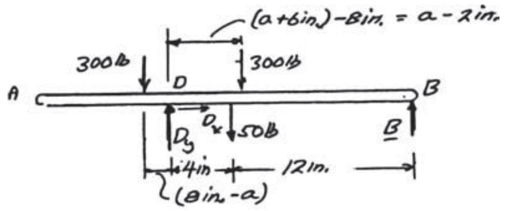
For the beam and loading shown, determine the range of the distance a for which the reaction at B does not exceed 100 lb downward or 200 lb upward.

SOLUTION

Assume B is positive when directed $\uparrow$



Sketch showing distance from D to forces.



$$+\circlearrowleft \Sigma M_D = 0: \quad (300\text{ lb})(8\text{ in.} - a) - (300\text{ lb})(a - 2\text{ in.}) - (50\text{ lb})(4\text{ in.}) + 16B = 0$$

$$-600a + 2800 + 16B = 0$$

$$a = \frac{(2800 + 16B)}{600} \qquad (1)$$

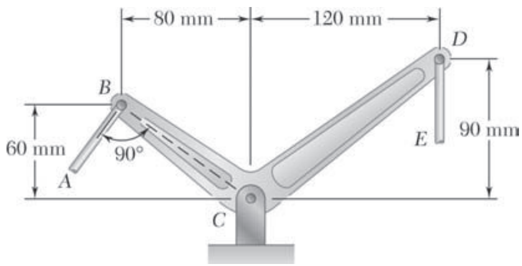
For $B = 100\text{ lb}\downarrow = -100\text{ lb}$, Eq. (1) yields:

$$a \geq \frac{[2800 + 16(-100)]}{600} = \frac{1200}{600} = 2\text{ in.} \qquad a \geq 2.00\text{ in.} \triangleleft$$

For $B = 200\uparrow = +200\text{ lb}$, Eq. (1) yields:

$$a \leq \frac{[2800 + 16(200)]}{600} = \frac{6000}{600} = 10\text{ in.} \qquad a \leq 10.00\text{ in.} \triangleleft$$

Required range: $2.00\text{ in.} \leq a \leq 10.00\text{ in.}$ ◀

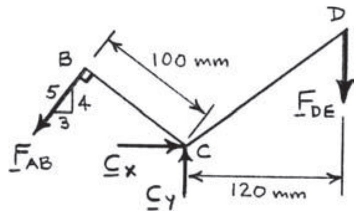


PROBLEM 4.15

Two links AB and DE are connected by a bell crank as shown. Knowing that the tension in link AB is 720 N, determine (*a*) the tension in link DE, (*b*) the reaction at C.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad F_{AB}(100 \text{ mm}) - F_{DE}(120 \text{ mm}) = 0$$

$$F_{DE} = \frac{5}{6} F_{AB} \qquad (1)$$

(*a*) For

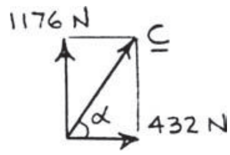
$$F_{AB} = 720 \text{ N}$$

$$F_{DE} = \frac{5}{6}(720 \text{ N}) \qquad F_{DE} = 600 \text{ N} \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(720 \text{ N}) + C_x = 0$$

$$C_x = +432 \text{ N}$$



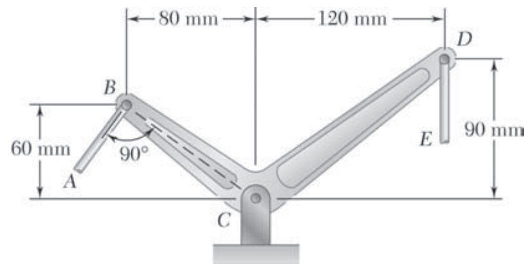
$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(720 \text{ N}) + C_y - 600 \text{ N} = 0$$

$$C_y = +1176 \text{ N}$$

$$C = 1252.84 \text{ N}$$

$$\alpha = 69.829°$$

$$\mathbf{C} = 1253 \text{ N} \measuredangle 69.8° \blacktriangleleft$$



PROBLEM 4.16

Two links AB and DE are connected by a bell crank as shown. Determine the maximum force that may be safely exerted by link AB on the bell crank if the maximum allowable value for the reaction at C is 1600 N.

SOLUTION

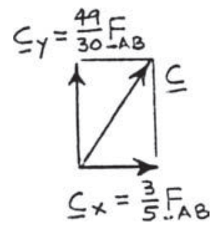
See solution to Problem 4.15 for F. B. D. and derivation of Eq. (1)

$$F_{DE} = \frac{5}{6}F_{AB} \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}F_{AB} + C_x = 0 \quad C_x = \frac{3}{5}F_{AB}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}F_{AB} + C_y - F_{DE} = 0$$

$$-\frac{4}{5}F_{AB} + C_y - \frac{5}{6}F_{AB} = 0$$



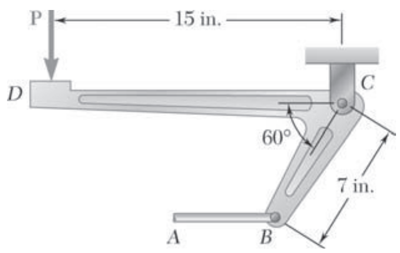
$$C_y = \frac{49}{30}F_{AB}$$

$$C = \sqrt{C_x^2 + C_y^2}$$

$$= \frac{1}{30}\sqrt{(49)^2 + (18)^2}F_{AB}$$

$$C = 1.74005F_{AB}$$

For $\quad C = 1600 \text{ N}, \quad 1600 \text{ N} = 1.74005F_{AB} \qquad F_{AB} = 920 \text{ N}$ ◀

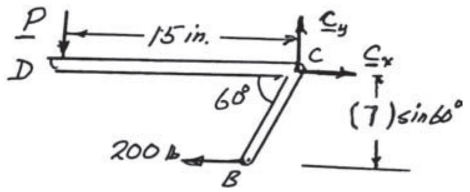


PROBLEM 4.17

The required tension in cable *AB* is 200 lb. Determine (*a*) the vertical force **P** that must be applied to the pedal, (*b*) the corresponding reaction at *C*.

SOLUTION

Free-Body Diagram:



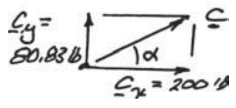
$BC = 7$ in.

(*a*) $+\circlearrowleft \Sigma M_C = 0{:}\quad P(15\text{ in.}) - (200\text{ lb})(6.062\text{ in.}) = 0$

$P = 80.83$ lb $\qquad$ $\mathbf{P} = 80.8$ lb $\downarrow$ ◀

(*b*) $\xrightarrow{+} \Sigma F_y = 0{:}\quad C_x - 200\text{ lb} = 0$ $\qquad$ $\mathbf{C}_x = 200$ lb $\longrightarrow$

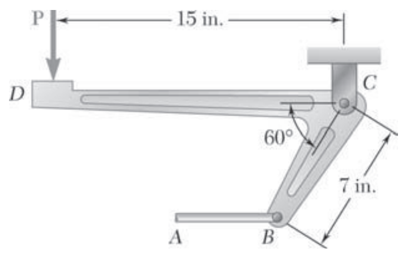
$+\uparrow \Sigma F_y = 0{:}\quad C_y - P = 0 \quad C_y - 80.83\text{ lb} = 0$ $\qquad$ $\mathbf{C}_y = 80.83$ lb $\uparrow$



$\alpha = 22.0°$

$C = 215.7$ lb

$\mathbf{C} = 216$ lb $\measuredangle$ 22.0° ◀

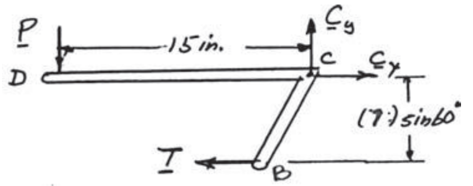


PROBLEM 4.18

Determine the maximum tension that can be developed in cable AB if the maximum allowable value of the reaction at C is 250 lb.

SOLUTION

Free-Body Diagram:



$BC = 7$ in.

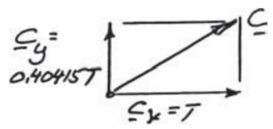
$+\circlearrowleft \Sigma M_C = 0\!: \quad P(15\text{ in.}) - T(6.062\text{ in.}) = 0 \qquad P = 0.40415T$

$+\uparrow \Sigma F_y = 0\!: \quad -P + C_y = 0 \quad -0.40415P + C_y = 0$

$$C_y = 0.40415T$$

$\xrightarrow{+} \Sigma F_x = 0\!: \quad -T + C_x = 0 \qquad C_x = T$

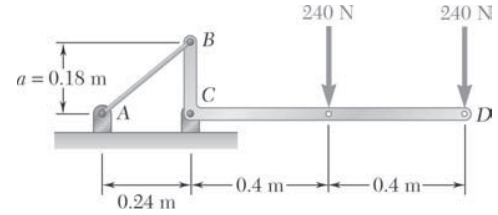
$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{T^2 + (0.40415T)^2}$$



$$C = 1.0786T$$

For $\qquad C = 250$ lb

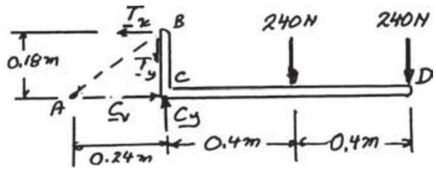
$$250\text{ lb} = 1.0786T \qquad T = 232\text{ lb} \blacktriangleleft$$



PROBLEM 4.19

The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



At *B*:
$$\frac{T_y}{T_x} = \frac{0.18\text{ m}}{0.24\text{ m}}$$

$$T_y = \frac{3}{4}T_x \qquad (1)$$

(*a*) $+\circlearrowleft \Sigma M_C = 0\text{:}\quad T_x(0.18\text{ m}) - (240\text{ N})(0.4\text{ m}) - (240\text{ N})(0.8\text{ m}) = 0$

$$T_x = +1600\text{ N}$$

Eq. (1)
$$T_y = \frac{3}{4}(1600\text{ N}) = 1200\text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{1600^2 + 1200^2} = 2000\text{ N}$$

$T = 2.00\text{ kN}$ ◀

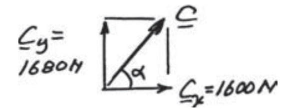
(*b*) $\xrightarrow{+} \Sigma F_x = 0\text{:}\quad C_x - T_x = 0$

$$C_x - 1600\text{ N} = 0 \quad C_x = +1600\text{ N}$$

$\mathbf{C}_x = 1600\text{ N} \longrightarrow$

$+\uparrow \Sigma F_y = 0\text{:}\quad C_y - T_y - 240\text{ N} - 240\text{ N} = 0$

$$C_y - 1200\text{ N} - 480\text{ N} = 0$$

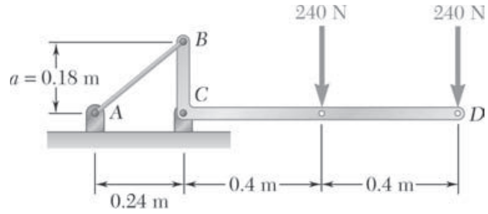


$$C_y = +1680\text{ N}$$

$\mathbf{C}_y = 1680\text{ N}\uparrow$

$$\alpha = 46.4^\circ$$
$$C = 2320\text{ N}$$

$\mathbf{C} = 2.32\text{ kN} \measuredangle 46.4^\circ$ ◀

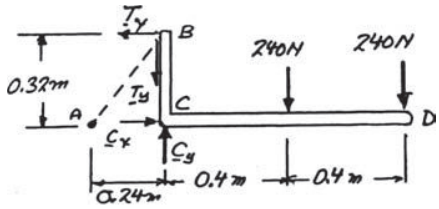


PROBLEM 4.20

Solve Problem 4.19, assuming that $a = 0.32$ m.

PROBLEM 4.19 The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



At *B*:

$$\frac{T_y}{T_x} = \frac{0.32 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{4}{3} T_x$$

$$+\circlearrowleft \Sigma M_C = 0: \quad T_x(0.32 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$$

$$T_x = 900 \text{ N}$$

Eq. (1)

$$T_y = \frac{4}{3}(900 \text{ N}) = 1200 \text{ N}$$

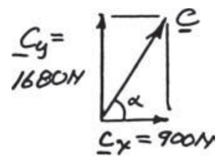
$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{900^2 + 1200^2} = 1500 \text{ N} \qquad T = 1.500 \text{ kN} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_x = 0$$

$$C_x - 900 \text{ N} = 0 \quad C_x = +900 \text{ N} \qquad \mathbf{C}_x = 900 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$

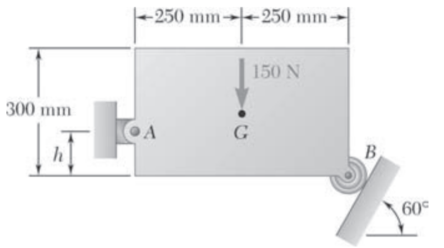


$$C_y = +1680 \text{ N} \qquad \mathbf{C}_y = 1680 \text{ N} \uparrow$$

$$\alpha = 61.8°$$

$$C = 1906 \text{ N} \qquad \mathbf{C} = 1.906 \text{ kN} \measuredangle 61.8° \blacktriangleleft$$

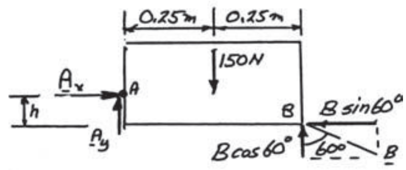
PROBLEM 4.21



Determine the reactions at A and B when (a) $h = 0$, (b) $h = 200$ mm.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos 60°)(0.5 \text{ m}) - (B\sin 60°)h - (150 \text{ N})(0.25 \text{ m}) = 0$$

$$B = \frac{37.5}{0.25 - 0.866h} \qquad (1)$$

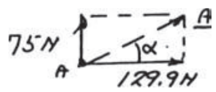
(a) When $h = 0$:

Eq. (1): $$B = \frac{37.5}{0.25} = 150 \text{ N}$$ $\mathbf{B} = 150.0 \text{ N} \searrow 30.0°$ ◀

$$\xrightarrow{+} \Sigma F_y = 0: \quad A_x - B\sin 60° = 0$$

$$A_x = (150)\sin 60° = 129.9 \text{ N}$$ $\mathbf{A}_x = 129.9 \text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$$



$$A_y = 150 - (150)\cos 60° = 75 \text{ N}$$ $\mathbf{A}_y = 75 \text{ N} \uparrow$

$$\alpha = 30°$$

$$A = 150.0 \text{ N}$$ $\mathbf{A} = 150.0 \text{ N} \measuredangle 30.0°$ ◀

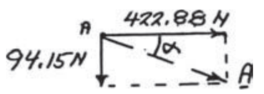
(b) When $h = 200 \text{ mm} = 0.2 \text{ m}$

Eq. (1): $$B = \frac{37.5}{0.25 - 0.866(0.2)} = 488.3 \text{ N}$$ $\mathbf{B} = 488 \text{ N} \searrow 30.0°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B\sin 60° = 0$$

$$A_x = (488.3)\sin 60° = 422.88 \text{ N}$$ $\mathbf{A}_x = 422.88 \text{ N} \longrightarrow$

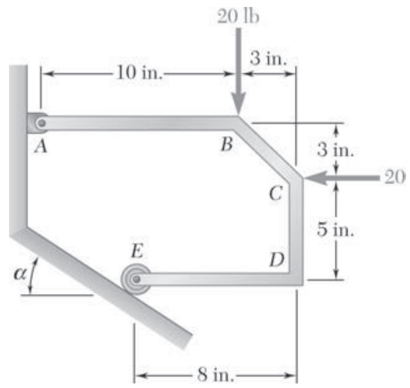
$$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$$



$$A_y = 150 - (488.3)\cos 60° = -94.15 \text{ N}$$ $\mathbf{A}_y = 94.15 \text{ N} \downarrow$

$$\alpha = 12.55°$$

$$A = 433.2 \text{ N}$$ $\mathbf{A} = 433 \text{ N} \searrow 12.6°$ ◀

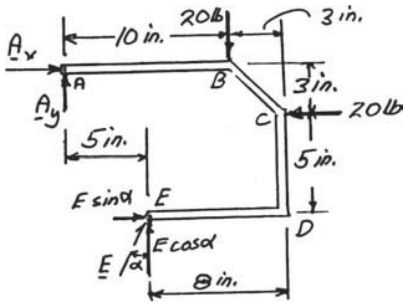


PROBLEM 4.22

For the frame and loading shown, determine the reactions at A and E when (a) $\alpha = 30°$, (b) $\alpha = 45°$.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (E\sin\alpha)(8 \text{ in.}) + (E\cos\alpha)(5 \text{ in.}) - (20 \text{ lb})(10 \text{ in.}) - (20 \text{ lb})(3 \text{ in.}) = 0$$

$$E = \frac{260}{8\sin\alpha + 5\cos\alpha}$$

(a) When $\alpha = 30°$:

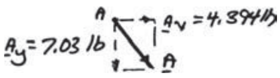
$$E = \frac{260}{8\sin 30° + 5\cos 30°} = 31.212 \text{ lb}$$

E $= 31.2$ lb $\measuredangle$ 60.0° ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 20 \text{ lb} + (31.212 \text{ lb})\sin 30° = 0$$

$$A_x = +4.394 \text{ lb}$$

$\mathbf{A}_x = 4.394$ lb $\longrightarrow$



$$+\uparrow \Sigma F_y = 0: \quad A_y - 20° + (31.212 \text{ lb})\cos 30° = 0$$

$$A_y = -7.03 \text{ lb}$$

$\mathbf{A}_y = 7.03$ lb $\downarrow$

A $= 8.29$ lb ⦪ 58.0° ◀

PROBLEM 4.22 (Continued)

(b) When $\alpha = 45°$:

$$E = \frac{260}{8\sin 45° + 5\cos\alpha} = 28.28 \text{ lb}$$

$\mathbf{E} = 28.3 \text{ lb} \measuredangle 45.0°$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - 20 \text{ lb} + (28.28 \text{ lb})\sin 45° = 0$

$A_x = 0$ $\qquad \mathbf{A}_x = 0$

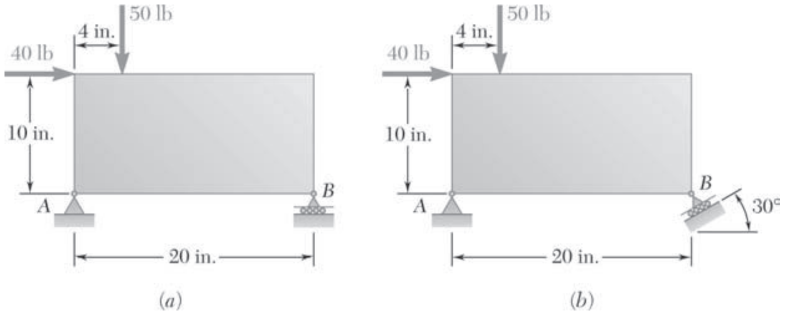
$+\uparrow \Sigma F_y = 0$: $A_y - 20 \text{ lb} + (28.28 \text{ lb})\cos 45° = 0$

$A_y = 0$ $\qquad A_y = 0$

$\mathbf{A} = 0$ ◀

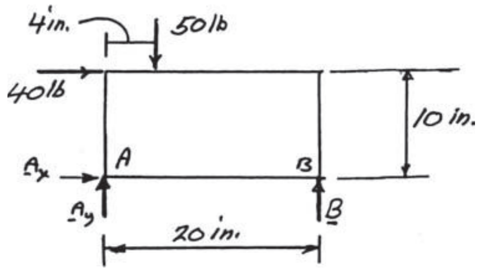
PROBLEM 4.23

For each of the plates and loadings shown, determine the reactions at A and B.



SOLUTION

(*a*) **Free-Body Diagram:**



$$+\circlearrowleft \Sigma M_A = 0: \quad B(20 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

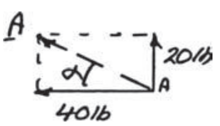
$B = +30$ lb $\qquad$ $\mathbf{B} = 30.0 \text{ lb} \uparrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 40 \text{ lb} = 0$$

$A_x = -40$ lb $\qquad$ $\mathbf{A}_x = 40.0 \text{ lb} \longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B - 50 \text{ lb} = 0$$

$$A_y + 30 \text{ lb} - 50 \text{ lb} = 0$$



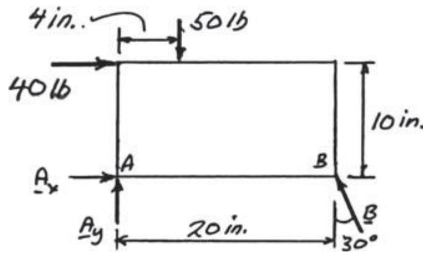
$A_y = +20$ lb $\qquad$ $\mathbf{A}_y = 20.0 \text{ lb} \uparrow$

$\alpha = 26.56°$

$A = 44.72$ lb $\qquad$ $\mathbf{A} = 44.7 \text{ lb} \nwarrow 26.6°$ ◀

PROBLEM 4.23 (Continued)

(*b*) **Free-Body Diagram:**



$+\circlearrowleft \Sigma M_A = 0$: $(B\cos 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) = 0$

$B = 34.64 \text{ lb}$ $\quad$ $\mathbf{B} = 34.6 \text{ lb} \measuredangle 60.0°$ ◀

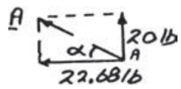
$\xrightarrow{+} \Sigma F_x = 0$: $A_x - B\sin 30° + 40 \text{ lb}$

$A_x - (34.64 \text{ lb})\sin 30° + 40 \text{ lb} = 0$

$A_x = -22.68 \text{ lb}$ $\quad$ $\mathbf{A}_x = 22.68 \text{ lb} \longleftarrow$

$+\uparrow \Sigma F_y = 0$: $A_y + B\cos 30° - 50 \text{ lb} = 0$

$A_y + (34.64 \text{ lb})\cos 30° - 50 \text{ lb} = 0$



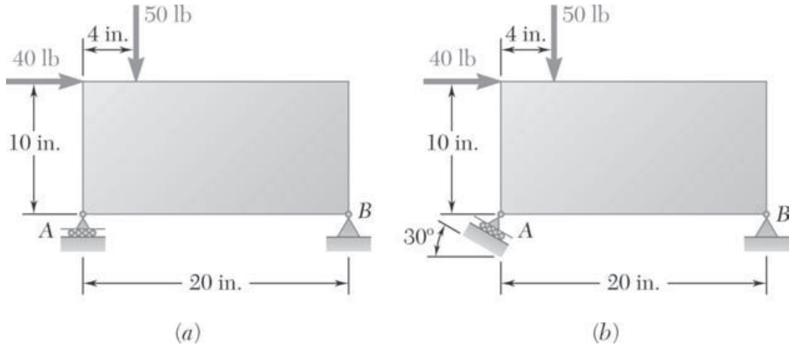
$A_y = +20 \text{ lb}$ $\quad$ $\mathbf{A}_y = 20.0 \text{ lb} \uparrow$

$\alpha = 41.4°$

$A = 30.24 \text{ lb}$ $\quad$ $\mathbf{A} = 30.2 \text{ lb} \measuredangle 41.4°$ ◀

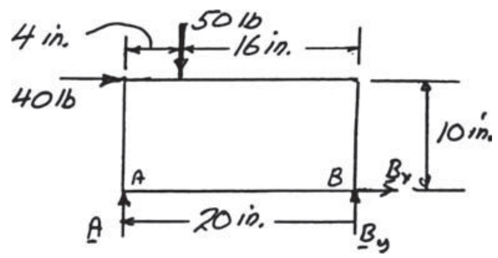
PROBLEM 4.24

For each of the plates and loadings shown, determine the reactions at *A* and *B*.



SOLUTION

(*a*) **Free-Body Diagram:**



$$+\circlearrowleft \Sigma M_B = 0: \quad A(20 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$$A = +20 \text{ lb}$$

$\mathbf{A} = 20.0 \text{ lb} \uparrow$ ◀

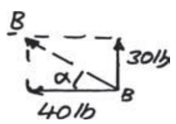
$$\xrightarrow{+} \Sigma F_x = 0: \quad 40 \text{ lb} + B_x = 0$$

$$B_x = -40 \text{ lb}$$

$\mathbf{B}_x = 40 \text{ lb} \longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad A + B_y - 50 \text{ lb} = 0$$

$$20 \text{ lb} + B_y - 50 \text{ lb} = 0$$



$$B_y = +30 \text{ lb}$$

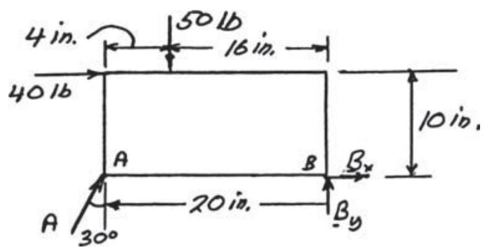
$\mathbf{B}_y = 30 \text{ lb} \uparrow$

$\alpha = 36.87°$ $\qquad B = 50 \text{ lb}$

$\mathbf{B} = 50.0 \text{ lb} \nwarrow 36.9°$ ◀

PROBLEM 4.24 (Continued)

(*b*)



$$+\circlearrowleft \Sigma M_A = 0: \quad -(A\cos 30°)(20\text{ in.}) - (40\text{ lb})(10\text{ in.}) + (50\text{ lb})(16\text{ in.}) = 0$$

$$A = 23.09\text{ lb} \qquad \mathbf{A} = 23.1\text{ lb} \measuredangle 60.0° \blacktriangleleft$$

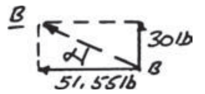
$$\xrightarrow{+} \Sigma F_x = 0: \quad A\sin 30° + 40\text{ lb} + B_x = 0$$

$$(23.09\text{ lb})\sin 30° + 40\text{ lb} + 8_x = 0$$

$$B_x = -51.55\text{ lb} \qquad \mathbf{B}_x = 51.55\text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A\cos 30° + B_y - 50\text{ lb} = 0$$

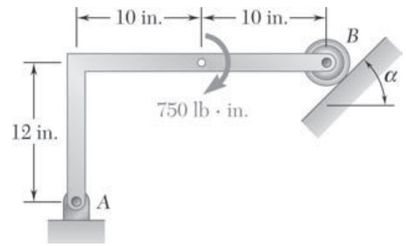
$$(23.09\text{ lb})\cos 30° + B_y - 50\text{ lb} = 0$$



$$B_y = +30\text{ lb} \qquad \mathbf{B}_y = 30\text{ lb} \uparrow$$

$$\alpha = 30.2°$$

$$B = 59.64\text{ lb} \qquad \mathbf{B} = 59.6\text{ lb} \searrow 30.2° \blacktriangleleft$$

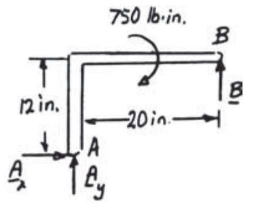


PROBLEM 4.25

Determine the reactions at A and B when (a) $\alpha = 0$, (b) $\alpha = 90°$, (c) $\alpha = 30°$.

SOLUTION

(a) $\underline{\alpha = 0}$



$+\circlearrowleft \Sigma M_A = 0: \quad B(20 \text{ in.}) - 750 \text{ lb} \cdot \text{in.} = 0$

$$B = 37.5 \text{ lb}$$

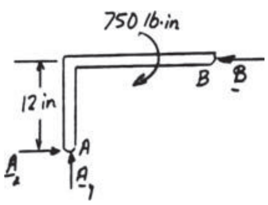
$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + 37.5 \text{ lb} = 0$

$$A_y = -37.5 \text{ lb}$$

$\mathbf{A} = 37.5 \text{ lb} \downarrow \qquad \mathbf{B} = 37.5 \text{ lb} \uparrow$ ◀

(b) $\underline{\alpha = 90°}$



$+\circlearrowleft \Sigma M_A = 0: \quad B(12 \text{ in.}) - 750 \text{ lb} \cdot \text{in.} = 0$

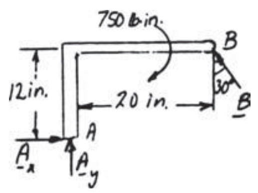
$$B = 62.5 \text{ lb}$$

$\xrightarrow{+} \Sigma F_A = 0: \quad A_x - 62.5 \text{ lb} = 0, \quad A_x = 62.5 \text{ lb}$

$+\uparrow \Sigma F_y = 0: \quad A_y = 0$

$\mathbf{A} = 62.5 \text{ lb} \longrightarrow \qquad \mathbf{B} = 62.5 \text{ lb} \longleftarrow$ ◀

(c) $\underline{\alpha = 30°}$



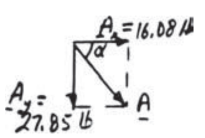
$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos 30°)(20 \text{ in.}) + (B \sin 30°)(12 \text{ in.}) - 750 \text{ lb} \cdot \text{in.} = 0$

$B = 32.16 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (32.16 \text{ lb}) \sin 30° = 0$

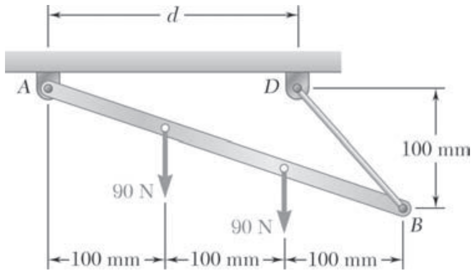
$$A_x = 16.08 \text{ lb}$$

$+\uparrow \Sigma F_y = 0: \quad A_y + (32.16 \text{ lb}) \cos 30° = 0 \quad A_y = -27.85 \text{ lb}$



$A = 32.16 \text{ lb} \quad \alpha = 60.0°$

$\mathbf{A} = 32.2 \text{ lb} \nwarrow 60.0° \qquad \mathbf{B} = 32.2 \text{ lb} \searrow 60.0°$ ◀

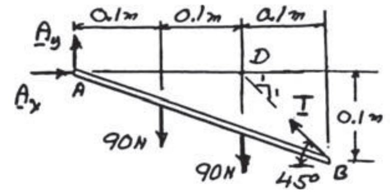


PROBLEM 4.26

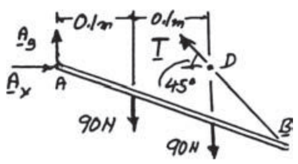
A rod AB hinged at A and attached at B to cable BD supports the loads shown. Knowing that $d = 200$ mm, determine (*a*) the tension in cable BD, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:



(*a*) Move **T** along BD until it acts at Point D.



$$+\circlearrowleft \Sigma M_A = 0: \quad (T\sin 45°)(0.2 \text{ m}) + (90 \text{ N})(0.1 \text{ m}) + (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 190.92 \text{ N} \qquad\qquad T = 190.9 \text{ N} \blacktriangleleft$$

(*b*)

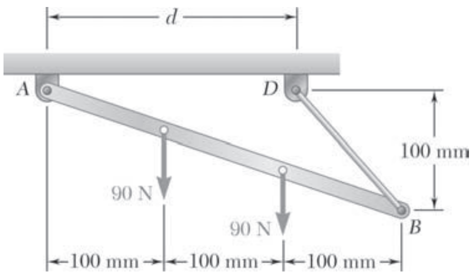
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (190.92 \text{ N})\cos 45° = 0$$

$$A_x = +135 \text{ N} \qquad\qquad \mathbf{A}_x = 135.0 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (190.92 \text{ N})\sin 45° = 0$$

$$A_y = +45 \text{ N} \qquad\qquad \mathbf{A}_y = 45.0 \text{ N}\uparrow$$

$$\mathbf{A} = 142.3 \text{ N} \measuredangle 18.43° \blacktriangleleft$$

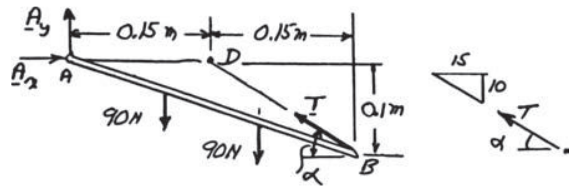


PROBLEM 4.27

A rod AB hinged at A and attached at B to cable BD supports the loads shown. Knowing that $d = 150$ mm, determine (a) the tension in cable BD, (b) the reaction at A.

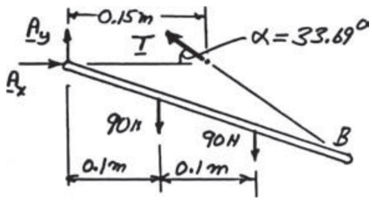
SOLUTION

Free-Body Diagram:



$$\tan\alpha = \frac{10}{15}; \quad \alpha = 33.69°$$

(a) Move **T** along BD until it acts at Point D.



$$+\circlearrowleft \Sigma M_A = 0: \quad (T\sin 33.69°)(0.15 \text{ m}) - (90 \text{ N})(0.1 \text{ m}) - (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 324.5 \text{ N} \qquad\qquad T = 324 \text{ N} \blacktriangleleft$$

(b)

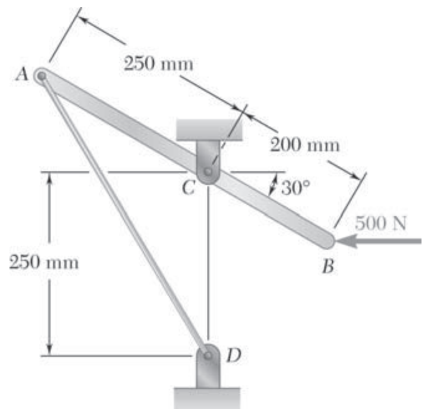
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (324.99 \text{ N})\cos 33.69° = 0$$

$$A_x = +270 \text{ N} \qquad\qquad \mathbf{A}_x = 270 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (324.5 \text{ N})\sin 33.69° = 0$$

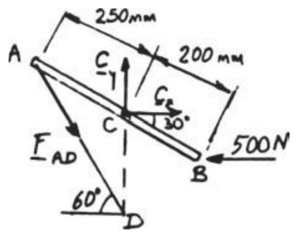
$$A_y = 0 \qquad\qquad \mathbf{A}_y = 0 \qquad\qquad \mathbf{A} = 270 \text{ N} \longrightarrow \blacktriangleleft$$

PROBLEM 4.28



A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 500-N horizontal force at B, determine (*a*) the tension in the cable, (*b*) the reaction at C.

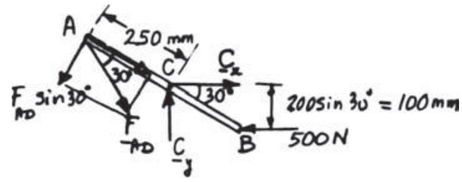
SOLUTION



Triangle ACD is isosceles with $\measuredangle C = 90° + 30° = 120°$ $\measuredangle A = \measuredangle D = \frac{1}{2}(180° - 120°) = 30°$

Thus DA forms angle of 60° with horizontal.

(*a*) We resolve $\mathbf{F}_{AD}$ into components along AB and perpendicular to AB.



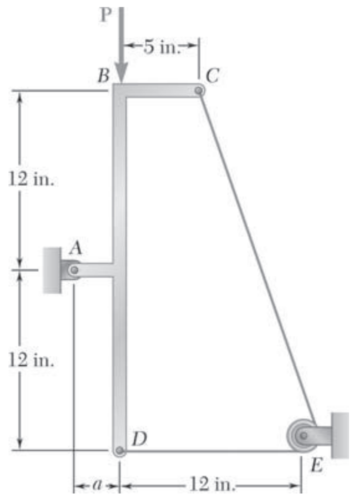
$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{AD} \sin 30°)(250 \text{ mm}) - (500 \text{ N})(100 \text{ mm}) = 0 \qquad F_{AD} = 400 \text{ N} \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(400 \text{ N})\cos 60° + C_x - 500 \text{ N} = 0 \qquad C_x = +300 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -(400 \text{ N})\sin 60° + C_y = 0 \qquad C_y = +346.4 \text{ N}$$

$$\mathbf{C} = 458 \text{ N} \measuredangle 49.1° \blacktriangleleft$$

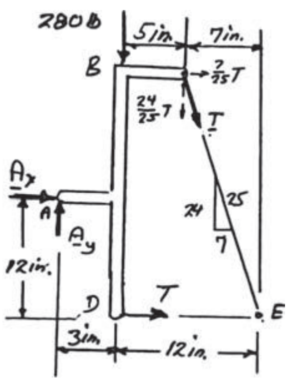


PROBLEM 4.29

A force **P** of magnitude 280 lb is applied to member *ABCD*, which is supported by a frictionless pin at *A* and by the cable *CED*. Since the cable passes over a small pulley at *E*, the tension may be assumed to be the same in portions *CE* and *ED* of the cable. For the case when $a = 3$ in., determine (*a*) the tension in the cable, (*b*) the reaction at *A*.

SOLUTION

Free-Body Diagram:



(*a*) $+\circlearrowleft \Sigma M_A = 0: \quad -(280 \text{ lb})(8 \text{ in.})$

$$T(12 \text{ in.}) - \frac{7}{25}T(12 \text{ in.})$$

$$-\frac{24}{25}T(8 \text{ in.}) = 0$$

$$(12 - 11.04)T = 840$$

$T = 875$ lb ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad \frac{7}{25}(875 \text{ lb}) + 875 \text{ lb} + A_x = 0$

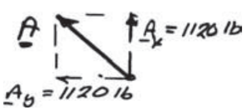
$$A_x = -1120$$

$\mathbf{A}_x = 1120$ lb ⟵

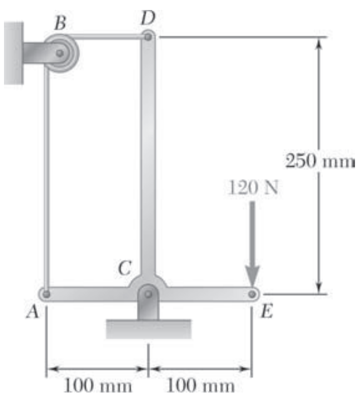
$+\uparrow \Sigma F_y = 0: \quad A_y - 280 \text{ lb} - \frac{24}{25}(875 \text{ lb}) = 0$

$$A_y = +1120$$

$\mathbf{A}_y = 1120$ lb ↑



$\mathbf{A} = 1584$ lb ◸ 45.0° ◀

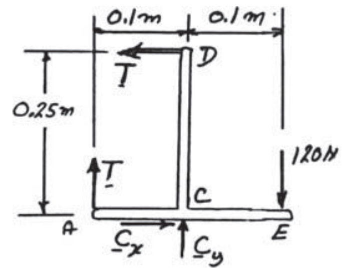


PROBLEM 4.30

Neglecting friction, determine the tension in cable *ABD* and the reaction at support *C*.

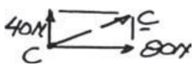
SOLUTION

Free-Body Diagram:



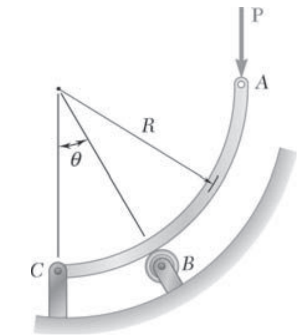
$$+\circlearrowleft \Sigma M_C = 0: \quad T(0.25 \text{ m}) - T(0.1 \text{ m}) - (120 \text{ N})(0.1 \text{ m}) = 0 \qquad T = 80.0 \text{ N} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 80 \text{ N} = 0 \quad C_x = +80 \text{ N} \qquad \mathbf{C}_x = 80.0 \text{ N} \longrightarrow$$



$$+\uparrow \Sigma F_y = 0: \quad C_y - 120 \text{ N} + 80 \text{ N} = 0 \quad C_y = +40 \text{ N} \qquad \mathbf{C}_y = 40.0 \text{ N} \uparrow$$

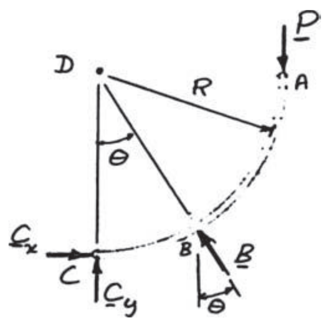
$$\mathbf{C} = 89.4 \text{ N} \measuredangle 26.6° \blacktriangleleft$$



PROBLEM 4.31

Rod ABC is bent in the shape of an arc of circle of radius R. Knowing the $\theta = 30°$, determine the reaction (a) at B, (b) at C.

SOLUTION



Free-Body Diagram: $+\circlearrowleft \Sigma M_D = 0\text{:}\quad C_x(R) - P(R) = 0$

$$C_x = +P$$

$$\xrightarrow{+} \Sigma F_x = 0\text{:}\quad C_x - B\sin\theta = 0$$

$$P - B\sin\theta = 0$$

$$B = P/\sin\theta$$

$$\mathbf{B} = \frac{P}{\sin\theta} \measuredangle \theta$$

$$+\!\uparrow \Sigma F_y = 0\text{:}\quad C_y + B\cos\theta - P = 0$$

$$C_y + (P/\sin\theta)\cos\theta - P = 0$$

$$C_y = P\left(1 - \frac{1}{\tan\theta}\right)$$

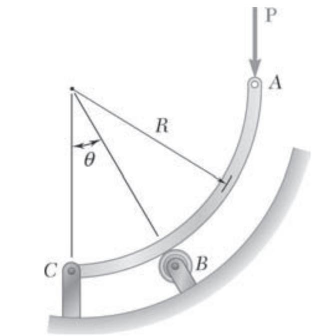
For $\theta = 30°$:

(a) $\quad B = P/\sin 30° = 2P \qquad \mathbf{B} = 2P \measuredangle 60.0°$ ◀

(b) $\quad C_x = +P \quad C_x = P \longrightarrow$

$C_y = P(1 - 1/\tan 30°) = -0.732/P \qquad \mathbf{C}_y = 0.7321P \downarrow$

$\mathbf{C} = 1.239P \measuredangle 36.2°$ ◀



PROBLEM 4.32

Rod ABC is bent in the shape of an arc of circle of radius R. Knowing the $\theta = 60°$, determine the reaction (a) at B, (b) at C.

SOLUTION

See the solution to Problem 4.31 for the free-body diagram and analysis leading to the following expressions:

$$C_x = +P$$

$$C_y = P\left(1 - \frac{1}{\tan\theta}\right)$$

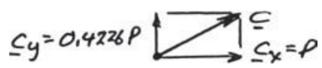
$$B = \frac{P}{\sin\theta}$$

For $\theta = 60°$:

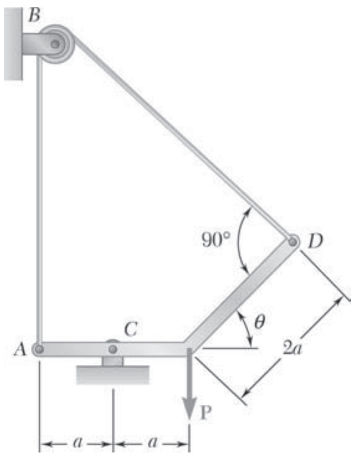
(a) $B = P/\sin 60° = 1.1547P$ $\quad$ $\mathbf{B} = 1.155P$ ⦨ $30.0°$ ◀

(b) $C_x = +P \quad C_x = P \longrightarrow$

$C_y = P(1 - 1/\tan 60°) = +0.4226P$ $\quad$ $\mathbf{C}_y = 0.4226P \downarrow$



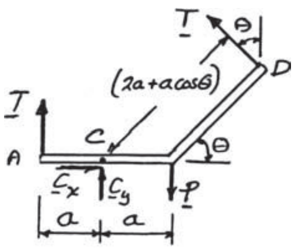
$\mathbf{C} = 1.086P$ ⦨ $22.9°$ ◀



PROBLEM 4.33

Neglecting friction, determine the tension in cable *ABD* and the reaction at *C* when $\theta = 60°$.

SOLUTION



$+\circlearrowleft \Sigma M_C = 0: \quad T(2a + a\cos\theta) - Ta + Pa = 0$

$$T = \frac{P}{1+\cos\theta} \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T\sin\theta = 0$

$$C_x = T\sin\theta = \frac{P\sin\theta}{1+\cos\theta}$$

$+\circlearrowleft \Sigma F_y = 0: \quad C_y + T + T\cos\theta - P = 0$

$$C_y = P - T(1+\cos\theta) = P - P\frac{1+\cos\theta}{1+\cos\theta}$$

$$C_y = 0$$

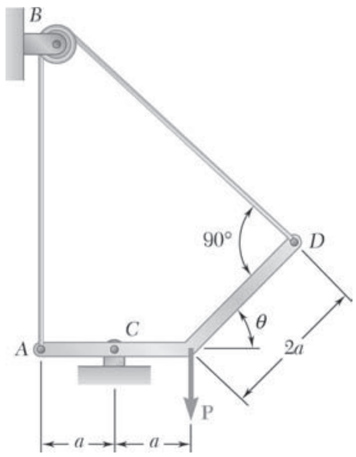
Since $\quad C_y = 0, \quad C = C_x \qquad \mathbf{C} = P\dfrac{\sin\theta}{1+\cos\theta} \longrightarrow \qquad (2)$

For $\theta = 60°$:

Eq. (1): $\quad T = \dfrac{P}{1+\cos 60°} = \dfrac{P}{1+\frac{1}{2}} \qquad T = \dfrac{2}{3}P$ ◀

Eq. (2): $\quad C = P\dfrac{\sin 60°}{1+\cos 60°} = P\dfrac{0.866}{1+\frac{1}{2}} \qquad \mathbf{C} = 0.577P \longrightarrow$ ◀

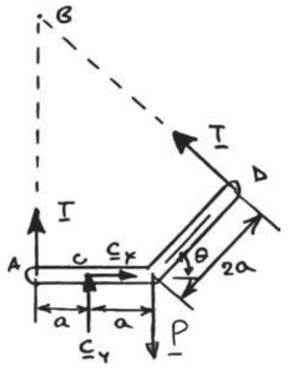
PROBLEM 4.34



Neglecting friction, determine the tension in cable *ABD* and the reaction at *C* when $\theta = 45°$.

SOLUTION

Free-Body Diagram:



Equilibrium for bracket:

$$+\circlearrowleft \Sigma M_C = 0: \quad -T(a) - P(a) + (T\sin 45°)(2a\sin 45°) + (T\cos 45°)(a + 2a\cos 45°) = 0$$

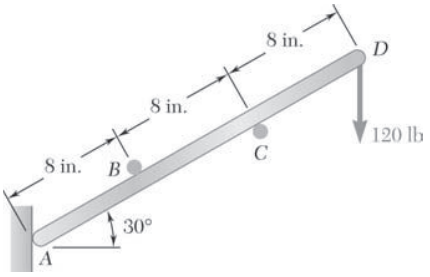
$$T = 0.58579 \qquad \text{or } T = 0.586P \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (0.58579P)\sin 45° = 0$$

$$C_x = 0.41422P$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 0.58579P - P + (0.58579P)\cos 45° = 0$$

$$C_y = 0 \qquad \text{or } \mathbf{C} = 0.414P \longrightarrow \blacktriangleleft$$

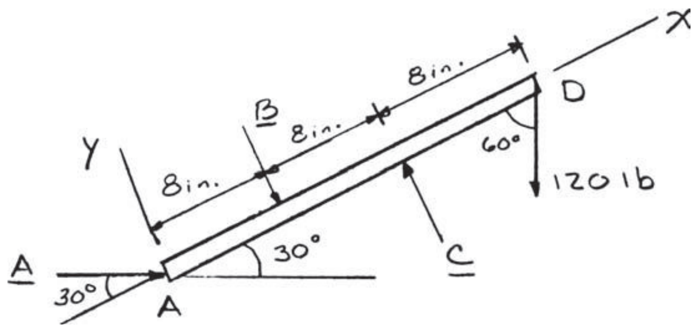


PROBLEM 4.35

A light rod *AD* is supported by frictionless pegs at *B* and *C* and rests against a frictionless wall at *A*. A vertical 120-lb force is applied at *D*. Determine the reactions at *A*, *B*, and *C*.

SOLUTION

Free-Body Diagram:



$\nearrow \Sigma F_x = 0$: $A\cos 30° - (120\text{ lb})\cos 60° = 0$

$A = 69.28\text{ lb}$ $\quad$ $\mathbf{A} = 69.3\text{ lb} \longrightarrow$ ◀

$+\circlearrowleft \Sigma M_B = 0$: $C(8\text{ in.}) - (120\text{ lb})(16\text{ in.})\cos 30° + (69.28\text{ lb})(8\text{ in.})\sin 30° = 0$

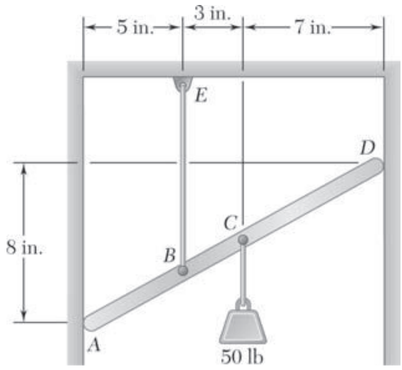
$C = 173.2\text{ lb}$ $\quad$ $\mathbf{C} = 173.2\text{ lb} \searrow 60.0°$ ◀

$+\circlearrowleft \Sigma M_C = 0$: $B(8\text{ in.}) - (120\text{ lb})(8\text{ in.})\cos 30° + (69.28\text{ lb})(16\text{ in.})\sin 30° = 0$

$B = 34.6\text{ lb}$ $\quad$ $\mathbf{B} = 34.6\text{ lb} \searrow 60.0°$ ◀

Check: $\nwarrow \Sigma F_y = 0$: $173.2 - 34.6 - (69.28)\sin 30° - (120)\sin 60° = 0$

$0 = 0\,(\text{check})$

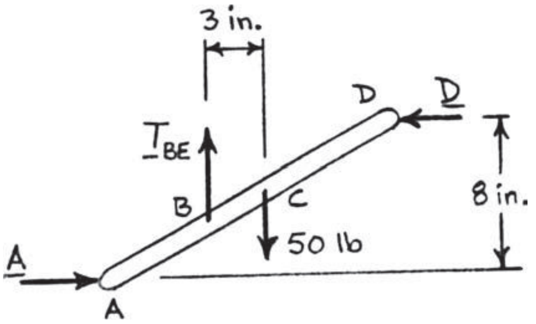


PROBLEM 4.36

A light bar AD is suspended from a cable BE and supports a 50-lb block at C. The ends A and D of the bar are in contact with frictionless vertical walls. Determine the tension in cable BE and the reactions at A and D.

SOLUTION

Free-Body Diagram:



$\Sigma F_x = 0: \quad A = D$

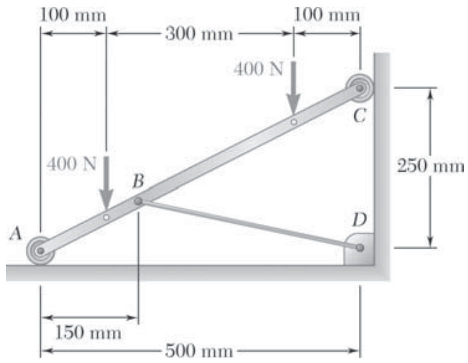
$\Sigma F_y = 0: \quad T_{BE} = 50.0 \text{ lb}$ ◀

We note that the forces shown form two couples.

$+\circlearrowleft \Sigma M = 0: \quad A(8 \text{ in.}) - (50 \text{ lb})(3 \text{ in.}) = 0$

$A = 18.75 \text{ lb}$

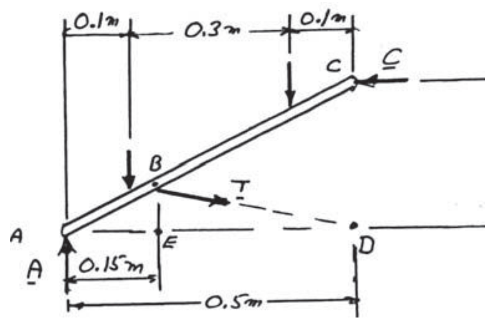
$\mathbf{A} = 18.75 \text{ lb} \longrightarrow \qquad \mathbf{D} = 18.75 \text{ lb} \longleftarrow$ ◀



PROBLEM 4.37

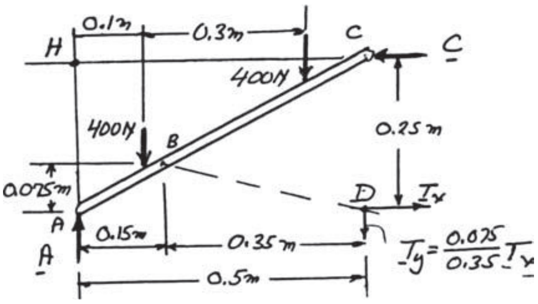
Bar AC supports two 400-N loads as shown. Rollers at A and C rest against frictionless surfaces and a cable BD is attached at B. Determine (*a*) the tension in cable BD, (*b*) the reaction at A, (*c*) the reaction at C.

SOLUTION



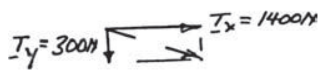
Similar triangles: ABE and ACD

$$\frac{AE}{AD}=\frac{BE}{CD};\quad \frac{0.15\text{ m}}{0.5\text{ m}}=\frac{BE}{0.25\text{ m}};\quad BE=0.075\text{ m}$$



(*a*) $\quad +\circlearrowleft \Sigma M_A=0{:}\quad T_x(0.25\text{ m})-\left(\dfrac{0.075}{0.35}T_x\right)(0.5\text{ m})-(400\text{ N})(0.1\text{ m})-(400\text{ N})(0.4\text{ m})=0$

$$T_x=1400\text{ N}$$



$$T_y=\frac{0.075}{0.35}(1400\text{ N})$$
$$=300\text{ N}$$

$$T=1432\text{ lb} \blacktriangleleft$$

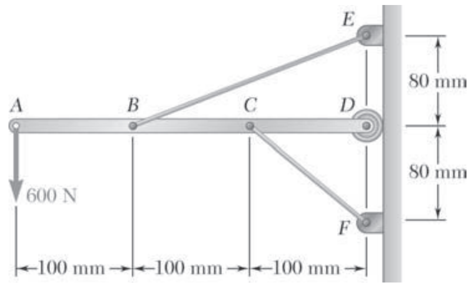
PROBLEM 4.37 (Continued)

(*b*) $+\uparrow \Sigma F_y = 0$: $A - 300 \text{ N} - 400 \text{ N} - 400 \text{ N} = 0$

$A = +1100 \text{ N}$ $\mathbf{A} = 1100 \text{ N} \uparrow$ ◀

(*c*) $\xrightarrow{+} \Sigma F_x = 0$: $-C + 1400 \text{ N} = 0$

$C = +1400 \text{ N}$ $\mathbf{C} = 1400 \text{ N} \longleftarrow$ ◀



PROBLEM 4.38

Determine the tension in each cable and the reaction at *D*.

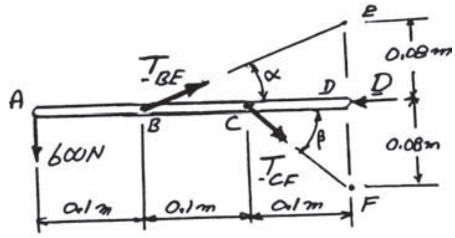
SOLUTION

$$\tan\alpha = \frac{0.08\text{ m}}{0.2\text{ m}}$$

$$\alpha = 21.80°$$

$$\tan\beta = \frac{0.08\text{ m}}{0.1\text{ m}}$$

$$\beta = 38.66°$$



$$+\circlearrowleft \Sigma M_B = 0: \quad (600\text{ N})(0.1\text{ m}) - (T_{CF}\sin 38.66°)(0.1\text{ m}) = 0$$

$$T_{CF} = 960.47\text{ N} \qquad T_{CF} = 96.0\text{ N} \blacktriangleleft$$

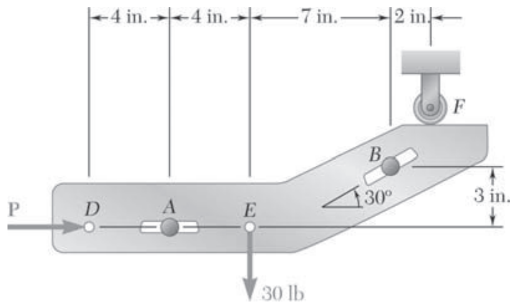
$$+\circlearrowleft \Sigma M_C = 0: \quad (600\text{ N})(0.2\text{ m}) - (T_{BE}\sin 21.80°)(0.1\text{ m}) = 0$$

$$T_{BE} = 3231.1\text{ N} \qquad T_{BE} = 3230\text{ N} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_y = 0: \quad T_{BE}\cos\alpha + T_{CF}\cos\beta - D = 0$$

$$(3231.1\text{ N})\cos 21.80° + (960.47\text{ N})\cos 38.66° - D = 0$$

$$D = 3750.03\text{ N} \qquad \mathbf{D} = 3750\text{ N} \longleftarrow \blacktriangleleft$$

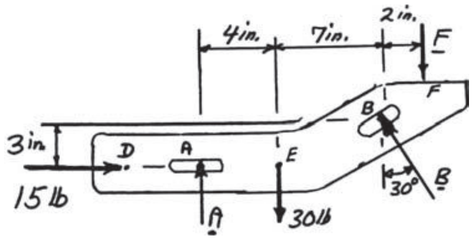


PROBLEM 4.39

Two slots have been cut in plate *DEF*, and the plate has been placed so that the slots fit two fixed, frictionless pins *A* and *B*. Knowing that $P = 15$ lb, determine (*a*) the force each pin exerts on the plate, (*b*) the reaction at *F*.

SOLUTION

Free-Body Diagram:



$\xrightarrow{+} \Sigma F_x = 0\text{:}\quad 15\text{ lb} - B\sin 30^\circ = 0$ $\qquad\qquad \mathbf{B} = 30.0\text{ lb} \nwarrow 60.0^\circ$ ◀

$+\circlearrowleft \Sigma M_A = 0\text{:}\quad -(30\text{ lb})(4\text{ in.}) + B\sin 30^\circ(3\text{ in.}) + B\cos 30^\circ(11\text{ in.}) - F(13\text{ in.}) = 0$

$$-120\text{ lb}\cdot\text{in.} + (30\text{ lb})\sin 30^\circ(3\text{ in.}) + (30\text{ lb})\cos 30^\circ(11\text{ in.}) - F(13\text{ in.}) = 0$$

$$F = +16.2145\text{ lb}$$

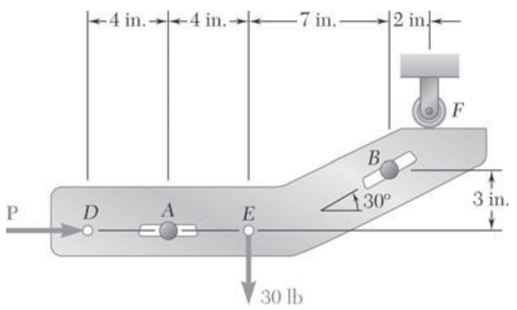
$\mathbf{F} = 16.21\text{ lb}\downarrow$ ◀

$+\uparrow \Sigma F_y = 0\text{:}\quad A - 30\text{ lb} + B\cos 30^\circ - F = 0$

$$A - 30\text{ lb} + (30\text{ lb})\cos 30^\circ - 16.2145\text{ lb} = 0$$

$$A = +20.23\text{ lb}$$

$\mathbf{A} = 20.2\text{ lb}\uparrow$ ◀



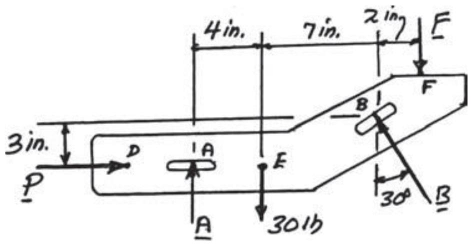
PROBLEM 4.40

For the plate of Problem 4.39 the reaction at F must be directed downward, and its maximum allowable value is 20 lb. Neglecting friction at the pins, determine the required range of values of P.

PROBLEM 4.39 Two slots have been cut in plate DEF, and the plate has been placed so that the slots fit two fixed, frictionless pins A and B. Knowing that $P = 15$ lb, determine (a) the force each pin exerts on the plate, (b) the reaction at F.

SOLUTION

Free-Body Diagram:



$\xrightarrow{+} \Sigma F_x = 0$: $\quad P - B\sin 30° = 0 \qquad\qquad \mathbf{B} = 2P \measuredangle 60°$

$+\circlearrowleft \Sigma M_A = 0$: $\quad -(30 \text{ lb})(4 \text{ in.}) + B\sin 30°(3 \text{ in.}) + B\cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$

$$-120 \text{ lb}\cdot\text{in.} + 2P\sin 30°(3 \text{ in.}) + 2P\cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$$

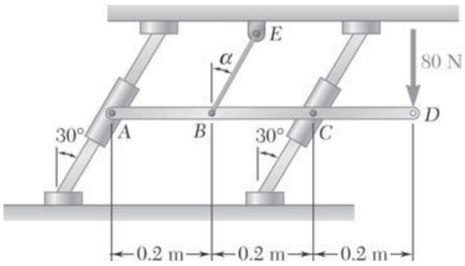
$$-120 + 3P + 19.0525P - 13F = 0$$

$$P = \frac{13E + 120}{22.0525} \tag{1}$$

For $F = 0$: $\quad P = \dfrac{13(0) + 120}{22.0525} = 5.442 \text{ lb}$

For $P = 20$ lb: $\quad P = \dfrac{13(20) + 120}{22.0525} = 17.232 \text{ lb}$

For $\quad 0 \le F \le 20$ lb: $\qquad 5.44 \text{ lb} \le P \le 17.231 \text{ lb}$ ◀

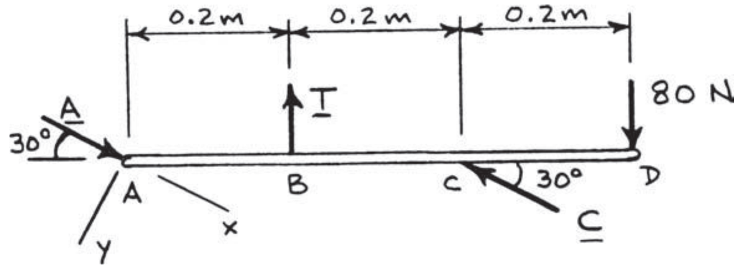


PROBLEM 4.41

Bar AD is attached at A and C to collars that can move freely on the rods shown. If the cord BE is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at A and C.

SOLUTION

Free-Body Diagram:



$$+\swarrow \Sigma F_y = 0: \quad -T\cos 30° + (80 \text{ N})\cos 30° = 0$$

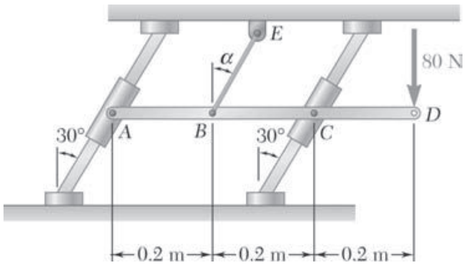
$$T = 80 \text{ N} \qquad T = 80.0 \text{ N} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (A\sin 30°)(0.4 \text{ m}) - (80 \text{ N})(0.2 \text{ m}) - (80 \text{ N})(0.2 \text{ m}) = 0$$

$$A = +160 \text{ N} \qquad \mathbf{A} = 160.0 \text{ N} \nwarrow 30.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (80 \text{ N})(0.2 \text{ m}) - (80 \text{ N})(0.6 \text{ m}) + (C\sin 30°)(0.4 \text{ m}) = 0$$

$$C = +160 \text{ N} \qquad \mathbf{C} = 160.0 \text{ N} \searrow 30.0° \blacktriangleleft$$



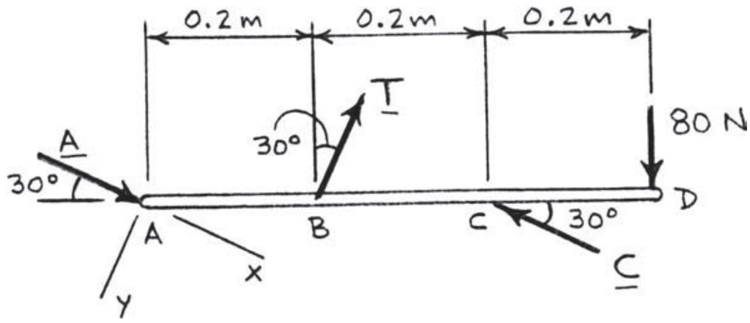
PROBLEM 4.42

Solve Problem 4.41 if the cord BE is parallel to the rods ($\alpha = 30°$).

PROBLEM 4.41 Bar *AD* is attached at *A* and *C* to collars that can move freely on the rods shown. If the cord *BE* is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at *A* and *C*.

SOLUTION

Free-Body Diagram:



$$+\swarrow \Sigma F_y = 0: \quad -T + (80\text{ N})\cos 30° = 0$$

$$T = 69.282\text{ N} \qquad\qquad T = 69.3\text{ N} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -(69.282\text{ N})\cos 30°(0.2\text{ m})$$
$$-(80\text{ N})(0.2\text{ m}) + (A\sin 30°)(0.4\text{ m}) = 0$$

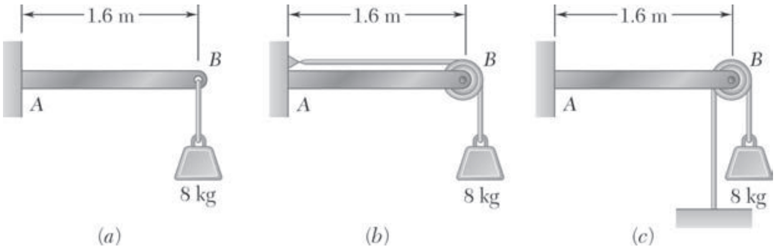
$$A = +140.000\text{ N} \qquad \mathbf{A} = 140.0\text{ N} \;\searrow\; 30.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad +(69.282\text{ N})\cos 30°(0.2\text{ m})$$
$$-(80\text{ N})(0.6\text{ m}) + (C\sin 30°)(0.4\text{ m}) = 0$$

$$C = +180.000\text{ N} \qquad \mathbf{C} = 180.0\text{ N} \;\nwarrow\; 30.0° \blacktriangleleft$$

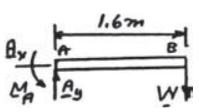
PROBLEM 4.43

An 8-kg mass can be supported in the three different ways shown. Knowing that the pulleys have a 100-mm radius, determine the reaction at A in each case.



SOLUTION

$$W = mg = (8\text{ kg})(9.81\,\text{m/s}^2) = 78.48\text{ N}$$



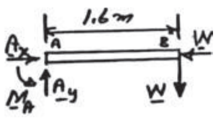
(a) $\quad \Sigma F_x = 0{:}\quad A_x = 0$

$+\uparrow \Sigma F_y = 0{:}\quad A_y - W = 0 \qquad \mathbf{A}_y = 78.48\text{ N}\uparrow$

$+\circlearrowleft \Sigma M_A = 0{:}\quad M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.48\text{ N})(1.6\text{ m}) \qquad \mathbf{M}_A = 125.56\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 78.5\text{ N}\uparrow \qquad \mathbf{M}_A = 125.6\text{ N}\cdot\text{m}\circlearrowleft$ ◀



(b) $\quad \xrightarrow{+} \Sigma F_x = 0{:}\quad A_x - W = 0 \qquad \mathbf{A}_x = 78.48\uparrow$

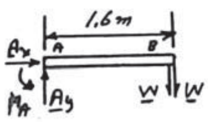
$+\uparrow \Sigma F_y = 0{:}\quad A_y - W = 0 \qquad \mathbf{A}_y = 78.48\longleftarrow$

$$\mathbf{A} = (78.48\text{ N})\sqrt{2} = 110.99\text{ N} \measuredangle 45^\circ$$

$+\circlearrowleft \Sigma M_A = 0{:}\quad M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.48\text{ N})(1.6\text{ m}) \qquad \mathbf{M}_A = 125.56\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 111.0\text{ N} \measuredangle 45^\circ \qquad \mathbf{M}_A = 125.6\text{ N}\cdot\text{m}\circlearrowleft$ ◀



(c) $\quad \Sigma F_x = 0{:}\quad A_x = 0$

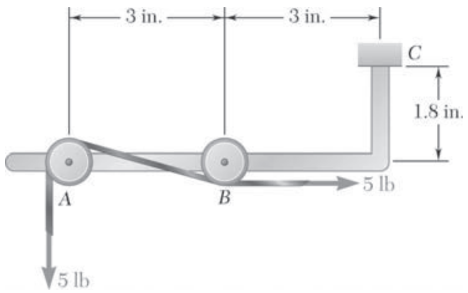
$+\uparrow \Sigma F_y = 0{:}\quad A_y - 2W = 0$

$$A_y = 2W = 2(78.48\text{ N}) = 156.96\text{ N}\uparrow$$

$+\circlearrowleft \Sigma M_A = 0{:}\quad M_A - 2W(1.6\text{ m}) = 0$

$M_A = +2(78.48\text{ N})(1.6\text{ m}) \qquad \mathbf{M}_A = 125.1\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 157.0\text{ N}\uparrow \qquad \mathbf{M}_A = 125\text{ N}\cdot\text{m}\circlearrowleft$ ◀

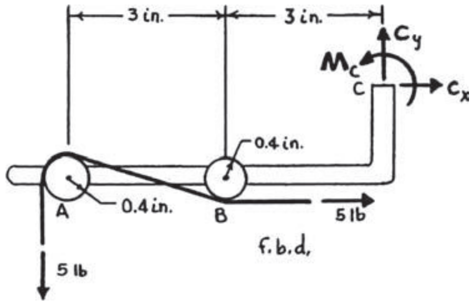


PROBLEM 4.44

A tension of 5 lb is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at C.

SOLUTION

From f.b.d. of system



$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5 \text{ lb}) = 0$$

$$C_x = -5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5 \text{ lb}) = 0$$

$$C_y = 5 \text{ lb}$$

Then

$$C = \sqrt{(C_x)^2 + (C_y)^2}$$

$$= \sqrt{(5)^2 + (5)^2}$$

$$= 7.0711 \text{ lb}$$

and

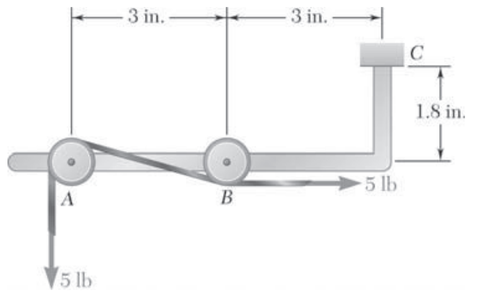
$$\theta = \tan^{-1}\left(\frac{+5}{-5}\right) = -45°$$

or $\mathbf{C} = 7.07 \text{ lb} \searrow 45.0°$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5 \text{ lb})(6.4 \text{ in.}) + (5 \text{ lb})(2.2 \text{ in.}) = 0$$

$$M_C = -43.0 \text{ lb} \cdot \text{in}$$

or $\mathbf{M}_C = 43.0 \text{ lb} \cdot \text{in.} \circlearrowright$ ◀



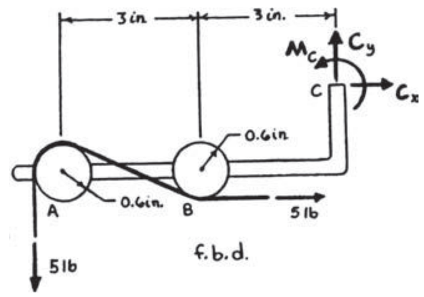
PROBLEM 4.45

Solve Problem 4.44, assuming that 0.6-in.-radius pulleys are used.

PROBLEM 4.44 A tension of 5 lb is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at *C*.

SOLUTION

From f.b.d. of system



$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5\text{ lb}) = 0$$

$$C_x = -5\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5\text{ lb}) = 0$$

$$C_y = 5\text{ lb}$$

Then

$$C = \sqrt{(C_x)^2 + (C_y)^2}$$
$$= \sqrt{(5)^2 + (5)^2}$$
$$= 7.0711\text{ lb}$$

and

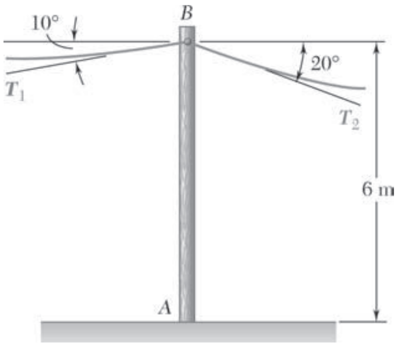
$$\theta = \tan^{-1}\left(\frac{5}{-5}\right) = -45.0°$$

or $\mathbf{C} = 7.07\text{ lb} \measuredangle 45.0°$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5\text{ lb})(6.6\text{ in.}) + (5\text{ lb})(2.4\text{ in.}) = 0$$

$$M_C = -45.0\text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_C = 45.0\text{ lb}\cdot\text{in.} \circlearrowright$ ◀

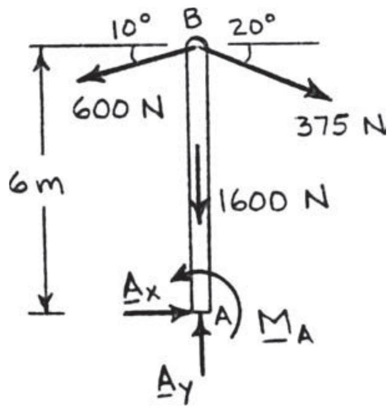


PROBLEM 4.46

A 6-m telephone pole weighing 1600 N is used to support the ends of two wires. The wires form the angles shown with the horizontal and the tensions in the wires are, respectively, $T_1 = 600$ N and $T_2 = 375$ N. Determine the reaction at the fixed end A.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (375\text{ N})\cos 20° - (600\text{ N})\cos 10° = 0$$

$$A_x = +238.50\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1600\text{ N} - (600\text{ N})\sin 10° - (375\text{ N})\sin 20° = 0$$

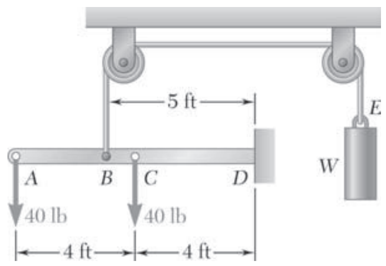
$$A_y = +1832.45\text{ N}$$

$$A = \sqrt{238.50^2 + 1832.45^2}$$

$$\theta = \tan^{-1}\frac{1832.45}{238.50} \qquad \mathbf{A} = 1848\text{ N} \measuredangle 82.6° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A + (600\text{ N})\cos 10°(6\text{ m}) - (375\text{ N})\cos 20°(6\text{ m}) = 0$$

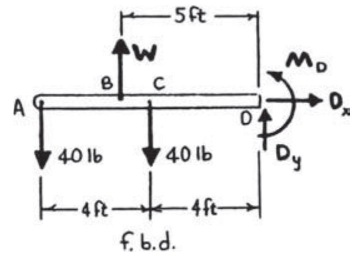
$$M_A = -1431.00\text{ N}\cdot\text{m} \qquad \mathbf{M}_A = 1431\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 4.47

Beam AD carries the two 40-lb loads shown. The beam is held by a fixed support at D and by the cable BE that is attached to the counterweight W. Determine the reaction at D when (*a*) $W = 100$ lb, (*b*) $W = 90$ lb.

SOLUTION



(*a*) $W = 100$ lb

From f.b.d. of beam AD

$\xrightarrow{+} \Sigma F_x = 0$: $D_x = 0$

$+\uparrow \Sigma F_y = 0$: $D_y - 40\text{ lb} - 40\text{ lb} + 100\text{ lb} = 0$

$D_y = -20.0$ lb or $\mathbf{D} = 20.0\text{ lb}\downarrow$ ◀

$+\circlearrowleft \Sigma M_D = 0$: $M_D - (100\text{ lb})(5\text{ ft}) + (40\text{ lb})(8\text{ ft}) + (40\text{ lb})(4\text{ ft}) = 0$

$M_D = 20.0\text{ lb}\cdot\text{ft}$ or $\mathbf{M}_D = 20.0\text{ lb}\cdot\text{ft}\circlearrowleft$ ◀

(*b*) $W = 90$ lb

From f.b.d. of beam AD

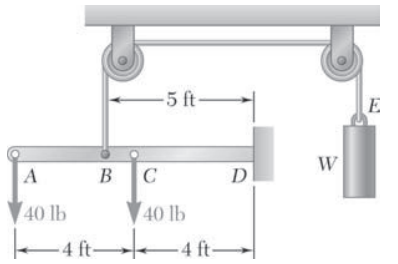
$\xrightarrow{+} \Sigma F_x = 0$: $D_x = 0$

$+\uparrow \Sigma F_y = 0$: $D_y + 90\text{ lb} - 40\text{ lb} - 40\text{ lb} = 0$

$D_y = -10.00$ lb or $\mathbf{D} = 10.00\text{ lb}\downarrow$ ◀

$+\circlearrowleft \Sigma M_D = 0$: $M_D - (90\text{ lb})(5\text{ ft}) + (40\text{ lb})(8\text{ ft}) + (40\text{ lb})(4\text{ ft}) = 0$

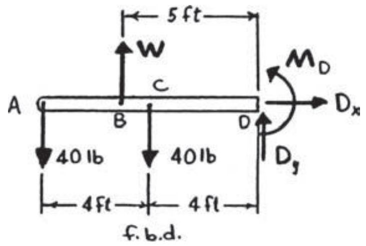
$M_D = -30.0\text{ lb}\cdot\text{ft}$ or $\mathbf{M}_D = -30.0\text{ lb}\cdot\text{ft}\circlearrowright$ ◀



PROBLEM 4.48

For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed 40 lb · ft.

SOLUTION



For $W_{\min}$, $\quad M_D = -40 \text{ lb} \cdot \text{ft}$

From f.b.d. of beam AD $\quad +\circlearrowleft \Sigma M_D = 0$: $\quad (40 \text{ lb})(8 \text{ ft}) - W_{\min}(5 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) - 40 \text{ lb} \cdot \text{ft} = 0$

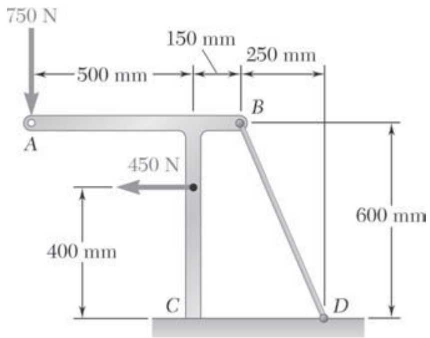
$$W_{\min} = 88.0 \text{ lb}$$

For $W_{\max}$, $\quad M_D = 40 \text{ lb} \cdot \text{ft}$

From f.b.d. of beam AD $\quad +\circlearrowleft \Sigma M_D = 0$: $\quad (40 \text{ lb})(8 \text{ ft}) - W_{\max}(5 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) + 40 \text{ lb} \cdot \text{ft} = 0$

$$W_{\max} = 104.0 \text{ lb}$$

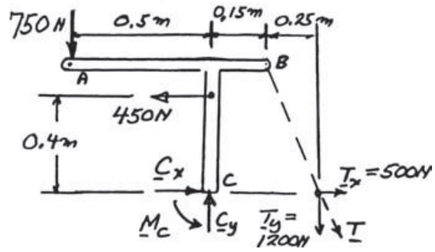
or $88.0 \text{ lb} \le W \le 104.0 \text{ lb}$ ◀



PROBLEM 4.49

Knowing that the tension in wire *BD* is 1300 N, determine the reaction at the fixed support *C* of the frame shown.

SOLUTION



$$T = 1300 \text{ N}$$

$$T_x = \frac{5}{13}T$$

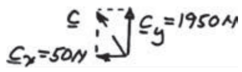
$$= 500 \text{ N}$$

$$T_y = \frac{12}{13}T$$

$$= 1200 \text{ N}$$

$\xrightarrow{+} \Sigma M_x = 0$: $C_x - 450 \text{ N} + 500 \text{ N} = 0 \quad C_x = -50 \text{ N}$ $\qquad \mathbf{C}_x = 50 \text{ N} \longleftarrow$

$+\uparrow \Sigma F_y = 0$: $C_y - 750 \text{ N} - 1200 \text{ N} = 0 \quad C_y = +1950 \text{ N}$ $\qquad \mathbf{C}_y = 1950 \text{ N} \uparrow$

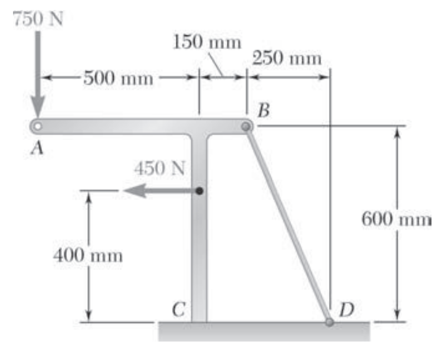


$$\mathbf{C} = 1951 \text{ N} \measuredangle 88.5° \blacktriangleleft$$

$+\circlearrowleft \Sigma M_C = 0$: $M_C + (750 \text{ N})(0.5 \text{ m}) + (4.50 \text{ N})(0.4 \text{ m})$

$$-(1200 \text{ N})(0.4 \text{ m}) = 0$$

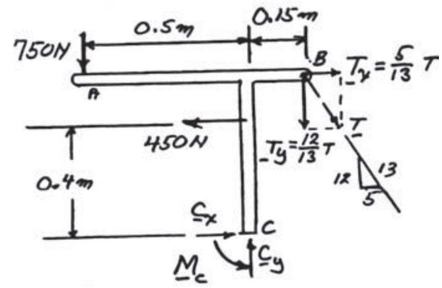
$M_C = -75.0 \text{ N} \cdot \text{m}$ $\qquad \mathbf{M}_C = 75.0 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$



PROBLEM 4.50

Determine the range of allowable values of the tension in wire *BD* if the magnitude of the couple at the fixed support *C* is not to exceed 100 N · m.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad (750\text{ N})(0.5\text{ m}) + (450\text{ N})(0.4\text{ m}) - \left(\frac{5}{13}T\right)(0.6\text{ m})$$

$$-\left(\frac{12}{13}T\right)(0.15\text{ m}) + M_C = 0$$

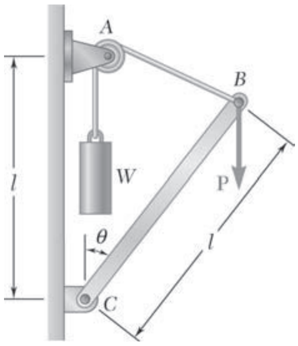
$$375\text{ N}\cdot\text{m} + 180\text{ N}\cdot\text{m} - \left(\frac{4.8}{13}\text{m}\right)T + M_C = 0$$

$$T = \frac{13}{4.8}(555 + M_C)$$

For $\quad M_C = -100\text{ N}\cdot\text{m}: \quad T = \frac{13}{4.8}(555 - 100) = 1232\text{ N}$

For $\quad M_C = +100\text{ N}\cdot\text{m}: \quad T = \frac{13}{4.8}(555 + 100) = 1774\text{ N}$

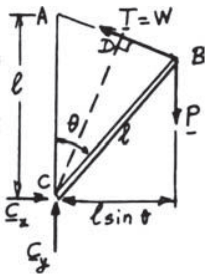
For $\quad |M_C| \le 100\text{ N}\cdot\text{m}: \qquad 1.232\text{ kN} \le T \le 1.774\text{ kN}$ ◀



PROBLEM 4.51

A vertical load **P** is applied at end B of rod BC. (a) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of P, l, and the counterweight W. (b) Determine the value of θ corresponding to equilibrium if $P = 2W$.

SOLUTION



(a) Triangle ABC is isosceles.

We have $$CD = (BC)\cos\frac{\theta}{2} = l\cos\frac{\theta}{2}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad W\left(l\cos\frac{\theta}{2}\right) - P(l\sin\theta) = 0$$

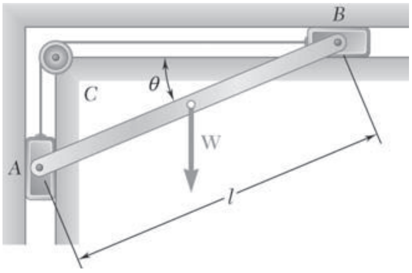
Setting $$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}: \quad Wl\cos\frac{\theta}{2} - 2Pl\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0$$

$$W - 2P\sin\frac{\theta}{2} = 0 \qquad \theta = 2\sin^{-1}\left(\frac{W}{2P}\right) \blacktriangleleft$$

(b) For $P = 2W$: $$\sin\frac{\theta}{2} = \frac{W}{2P} = \frac{W}{4W} = 0.25$$

$$\frac{\theta}{2} = 14.5° \qquad \theta = 29.0° \blacktriangleleft$$

or $$\frac{\theta}{2} = 165.5° \quad \theta = 331°\text{(discard)}$$

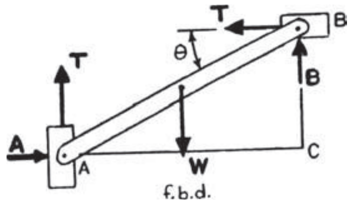


PROBLEM 4.52

A slender rod AB, of weight W, is attached to blocks A and B, which move freely in the guides shown. The blocks are connected by an elastic cord that passes over a pulley at C. (*a*) Express the tension in the cord in terms of W and θ. (*b*) Determine the value of θ for which the tension in the cord is equal to $3W$.

SOLUTION

(*a*) From f.b.d. of rod AB



$$+\circlearrowleft \Sigma M_C = 0: \quad T(l\sin\theta) + W\left[\left(\frac{1}{2}\right)\cos\theta\right] - T(l\cos\theta) = 0$$

$$T = \frac{W\cos\theta}{2(\cos\theta - \sin\theta)}$$

Dividing both numerator and denominator by cos θ,

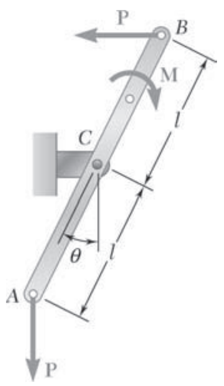
$$T = \frac{W}{2}\left(\frac{1}{1-\tan\theta}\right) \qquad \text{or } T = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)} \blacktriangleleft$$

(*b*) For $T = 3W$,

$$3W = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)}$$

$$1-\tan\theta = \frac{1}{6}$$

or $$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.806°$$ or $\theta = 39.8° \blacktriangleleft$

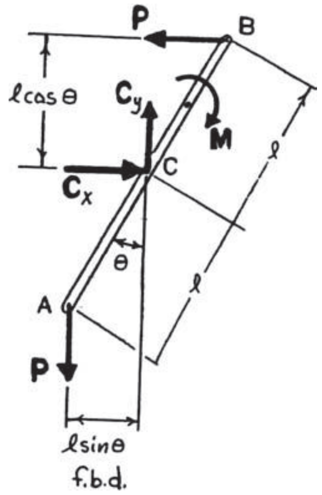


PROBLEM 4.53

Rod AB is acted upon by a couple $\mathbf{M}$ and two forces, each of magnitude P. (a) Derive an equation in θ, P, M, and l that must be satisfied when the rod is in equilibrium. (b) Determine the value of θ corresponding to equilibrium when $M = 150 \text{ N} \cdot \text{m}$, $P = 200 \text{ N}$, and $l = 600 \text{ mm}$.

SOLUTION

Free-Body Diagram:



(a) From free-body diagram of rod AB

$$+\circlearrowleft \Sigma M_C = 0: \quad P(l\cos\theta) + P(l\sin\theta) - M = 0$$

$$\text{or } \sin\theta + \cos\theta = \frac{M}{Pl} \blacktriangleleft$$

(b) For $M = 150 \text{ lb} \cdot \text{in.}$, $P = 20 \text{ lb}$, and $l = 6 \text{ in.}$

$$\sin\theta + \cos\theta = \frac{150 \text{ lb} \cdot \text{in.}}{(20 \text{ lb})(6 \text{ in.})} = \frac{5}{4} = 1.25$$

Using identity $\sin^2\theta + \cos^2\theta = 1$

$$\sin\theta + (1 - \sin^2\theta)^{1/2} = 1.25$$

$$(1 - \sin^2\theta)^{1/2} = 1.25 - \sin\theta$$

$$1 - \sin^2\theta = 1.5625 - 2.5\sin\theta + \sin^2\theta$$

$$2\sin^2\theta - 2.5\sin\theta + 0.5625 = 0$$

Using quadratic formula

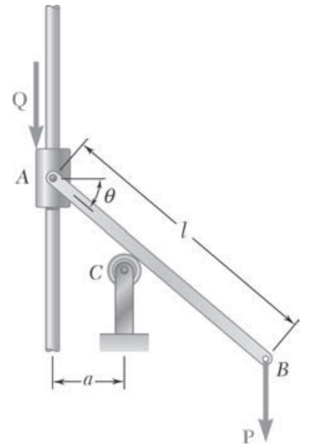
$$\sin\theta = \frac{-(-2.5) \pm \sqrt{(625) - 4(2)(0.5625)}}{2(2)}$$

$$= \frac{2.5 \pm \sqrt{1.75}}{4}$$

or $\sin\theta = 0.95572$ and $\sin\theta = 0.29428$

$$\theta = 72.886° \quad \text{and} \quad \theta = 17.1144°$$

$$\text{or } \theta = 17.11° \quad \text{and} \quad \theta = 72.9° \blacktriangleleft$$

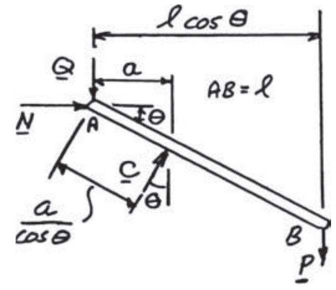


PROBLEM 4.54

Rod AB is attached to a collar at A and rests against a small roller at C. (a) Neglecting the weight of rod AB, derive an equation in P, Q, a, l, and θ that must be satisfied when the rod is in equilibrium. (b) Determine the value of θ corresponding to equilibrium when $P = 16$ lb, $Q = 12$ lb, $l = 20$ in., and $a = 5$ in.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad C\cos\theta - P - Q = 0$$

$$C = \frac{P+Q}{\cos\theta}$$

(a) $$+\circlearrowleft \Sigma M_A = 0: \quad C\frac{a}{\cos\theta} - Pl\cos\theta = 0$$

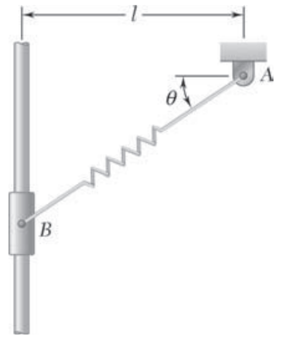
$$\frac{P+Q}{\cos\theta}\cdot\frac{a}{\cos\theta} - Pl\cos\theta = 0 \qquad \cos^3\theta = \frac{a(P+Q)}{Pl} \blacktriangleleft$$

(b) For $P = 16$ lb, $Q = 12$ lb, $l = 20$ in., and $a = 5$ in.

$$\cos^3\theta = \frac{(5\text{ in.})(16\text{ lb} + 12\text{ lb})}{(16\text{ lb})(20\text{ in.})}$$

$$= 0.4375$$

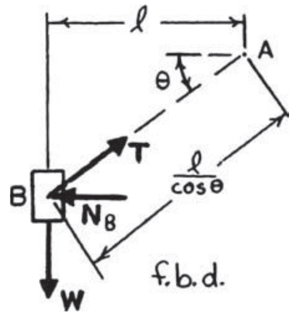
$$\cos\theta = 0.75915 \qquad \theta = 40.6° \blacktriangleleft$$



PROBLEM 4.55

A collar B of weight W can move freely along the vertical rod shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (*a*) Derive an equation in θ, W, k, and l that must be satisfied when the collar is in equilibrium. (*b*) Knowing that $W = 300$ N, $l = 500$ mm, and $k = 800$ N/m, determine the value of θ corresponding to equilibrium.

SOLUTION



First note $\qquad T = ks$

Where $\qquad k =$ spring constant

$s =$ elongation of spring

$$= \frac{l}{\cos\theta} - l$$

$$= \frac{l}{\cos\theta}(1 - \cos\theta)$$

$$T = \frac{kl}{\cos\theta}(1 - \cos\theta)$$

(*a*) From f.b.d. of collar B $\qquad +\uparrow \Sigma F_y = 0: \quad T\sin\theta - W = 0$

or $\qquad \dfrac{kl}{\cos\theta}(1 - \cos\theta)\sin\theta - W = 0 \qquad$ or $\quad \tan\theta - \sin\theta = \dfrac{W}{kl}$ ◀

(*b*) For

$$W = 3 \text{ lb}$$

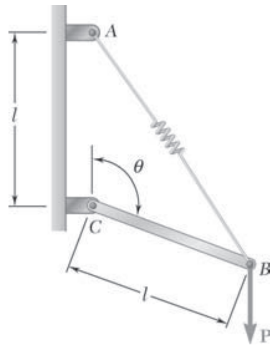
$$l = 6 \text{ in.}$$

$$k = 8 \text{ lb/ft}$$

$$l = \frac{6 \text{ in.}}{12 \text{ in./ft}} = 0.5 \text{ ft}$$

$$\tan\theta - \sin\theta = \frac{3 \text{ lb}}{(8 \text{ lb/ft})(0.5 \text{ ft})} = 0.75$$

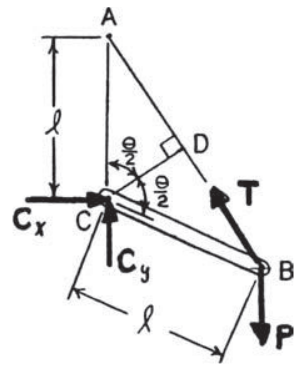
Solving numerically, $\qquad \theta = 57.957° \qquad$ or $\qquad \theta = 58.0°$ ◀



PROBLEM 4.56

A vertical load **P** is applied at end B of rod BC. The constant of the spring is k, and the spring is unstretched when $\theta = 90°$. (a) Neglecting the weight of the rod, express the angle θ corresponding to equilibrium in terms of P, k, and l. (b) Determine the value of θ corresponding to equilibrium when $P = \frac{1}{4}kl$.

SOLUTION



First note $\quad T = \text{tension in spring} = ks$

where $\quad s = \text{elongation of spring}$

$$= (\overline{AB})_\theta - (\overline{AB})_{\theta=90°}$$

$$= 2l\sin\left(\frac{\theta}{2}\right) - 2l\sin\left(\frac{90°}{2}\right)$$

$$= 2l\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]$$

$$T = 2kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right] \qquad (1)$$

(a) From f.b.d. of rod BC $\quad +\circlearrowleft \Sigma M_C = 0: \quad T\left[l\cos\left(\frac{\theta}{2}\right)\right] - P(l\sin\theta) = 0$

Substituting T from Equation (1)

$$2kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\left[l\cos\left(\frac{\theta}{2}\right)\right] - P(l\sin\theta) = 0$$

$$2kl^2\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\cos\left(\frac{\theta}{2}\right) - Pl\left[2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] = 0$$

Factoring out $\quad 2l\cos\left(\frac{\theta}{2}\right)$, leaves

$$kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right] - P\sin\left(\frac{\theta}{2}\right) = 0$$

or $\quad \sin\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\left(\frac{kl}{kl - P}\right) \qquad \theta = 2\sin^{-1}\left[\frac{kl}{\sqrt{2}(kl - P)}\right]$ ◀

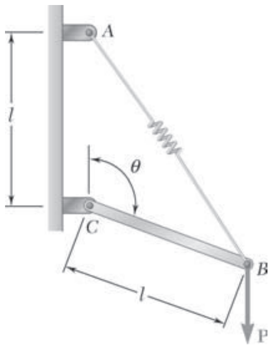
PROBLEM 4.56 (Continued)

(*b*)

$$P = \frac{kl}{4}$$

$$\begin{aligned}\theta &= 2\sin^{-1}\left[\frac{kl}{\sqrt{2}\left(kl - \frac{kl}{4}\right)}\right] \\ &= 2\sin^{-1}\left[\frac{kl}{\sqrt{2}}\left(\frac{4}{3kl}\right)\right] \\ &= 2\sin^{-1}\left(\frac{4}{3\sqrt{2}}\right) \\ &= 2\sin^{-1}(0.94281) \\ &= 141.058^\circ\end{aligned}$$

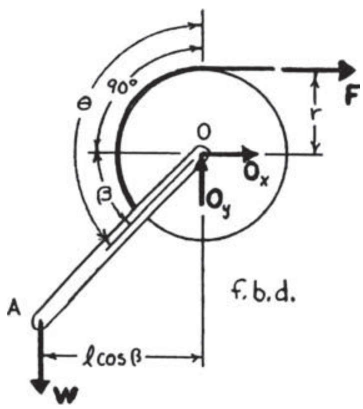
or $\theta = 141.1^\circ$ ◀



PROBLEM 4.57

Solve Sample Problem 4.56, assuming that the spring is unstretched when $\theta = 90°$.

SOLUTION



First note $\quad T = \text{tension in spring} = ks$

where $\quad s = \text{deformation of spring}$

$$= r\beta$$

$$F = kr\beta$$

From f.b.d. of assembly $\quad +\circlearrowleft \Sigma M_0 = 0: \quad W(l\cos\beta) - F(r) = 0$

or $\quad Wl\cos\beta - kr^2\beta = 0$

$$\cos\beta = \frac{kr^2}{Wl}\beta$$

For $\quad k = 250 \text{ lb/in.}$

$$r = 3 \text{ in.}$$

$$l = 8 \text{ in.}$$

$$W = 400 \text{ lb}$$

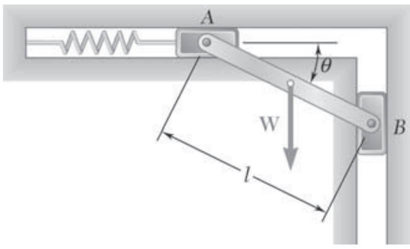
$$\cos\beta = \frac{(250 \text{ lb/in.})(3 \text{ in.})^2}{(400 \text{ lb})(8 \text{ in.})}\beta$$

or $\quad \cos\beta = 0.703125\beta$

Solving numerically, $\quad \beta = 0.89245 \text{ rad}$

or $\quad \beta = 51.134°$

Then $\quad \theta = 90° + 51.134° = 141.134° \qquad$ or $\theta = 141.1°$ ◀

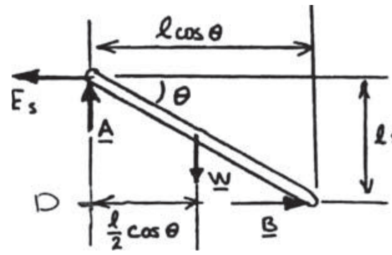


PROBLEM 4.58

A slender rod AB, of weight W, is attached to blocks A and B that move freely in the guides shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (a) Neglecting the weight of the blocks, derive an equation in W, k, l, and θ that must be satisfied when the rod is in equilibrium. (b) Determine the value of θ when $W = 75$ lb, $l = 30$ in., and $k = 3$ lb/in.

SOLUTION

Free-Body Diagram:



Spring force: $F_s = ks = k(l - l\cos\theta) = kl(1-\cos\theta)$

(a) $+\circlearrowleft \Sigma M_D = 0\text{:}\quad F_s(l\sin\theta) - W\left(\dfrac{l}{2}\cos\theta\right) = 0$

$$kl(1-\cos\theta)l\sin\theta - \frac{W}{2}l\cos\theta = 0$$

$$kl(1-\cos\theta)\tan\theta - \frac{W}{2} = 0 \qquad \text{or} \quad (1-\cos\theta)\tan\theta = \frac{W}{2kl} \blacktriangleleft$$

(b) For given values of $W = 75$ lb

$l = 30$ in.

$k = 3$ lb/in.

$$(1-\cos\theta)\tan\theta = \tan\theta - \sin\theta$$

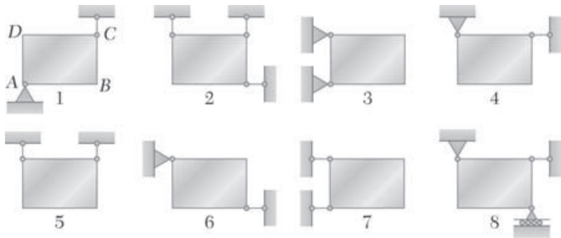
$$= \frac{75\text{ lb}}{2(3\text{ lb/in.})(30\text{ in.})}$$

$$= 0.41667$$

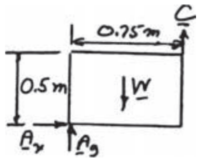
Solving numerically: $\theta = 49.710°$ or $\theta = 49.7° \blacktriangleleft$

PROBLEM 4.59

Eight identical 500×750-mm rectangular plates, each of mass $m = 40$ kg, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions.

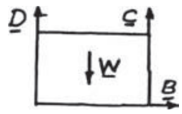


SOLUTION



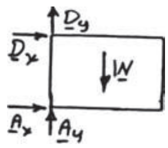
1. Three non-concurrent, non-parallel reactions
   - (*a*) Plate: completely constrained
   - (*b*) Reactions: determinate
   - (*c*) Equilibrium maintained

$$\mathbf{A} = \mathbf{C} = 196.2 \text{ N} \uparrow$$



2. Three non-concurrent, non-parallel reactions
   - (*a*) Plate: completely constrained
   - (*b*) Reactions: determinate
   - (*c*) Equilibrium maintained

$$\mathbf{B} = 0, \quad \mathbf{C} = \mathbf{D} = 196.2 \text{ N} \uparrow$$

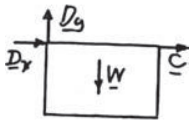


3. Four non-concurrent, non-parallel reactions
   - (*a*) Plate: completely constrained
   - (*b*) Reactions: indeterminate
   - (*c*) Equilibrium maintained

$$\mathbf{A}_x = 294 \text{ N} \longrightarrow, \quad \mathbf{D}_x = 294 \text{ N} \longleftarrow$$

$$(\mathbf{A}_y + \mathbf{D}_y = 392 \text{ N} \uparrow)$$

4. Three concurrent reactions (through D)
   - (*a*) Plate: improperly constrained
   - (*b*) Reactions: indeterminate
   - (*c*) No equilibrium $\qquad (\Sigma M_D \neq 0)$



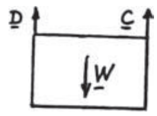
PROBLEM 4.59 (Continued)

5. Two reactions

   (*a*) Plate: partial constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



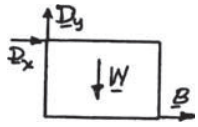
$$\mathbf{C} = \mathbf{R} = 196.2\text{ N}\uparrow$$

6. Three non-concurrent, non-parallel reactions

   (*a*) Plate: completely constrained

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



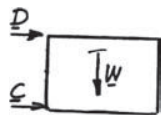
$$\mathbf{B} = 294\text{ N}\longrightarrow, \quad \mathbf{D} = 491\text{ N} \measuredangle 53.1°$$

7. Two reactions

   (*a*) Plate: improperly constrained

   (*b*) Reactions determined by dynamics

   (*c*) No equilibrium $(\Sigma F_y \neq 0)$

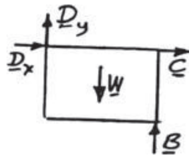


8. For non-concurrent, non-parallel reactions

   (*a*) Plate: completely constrained

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained

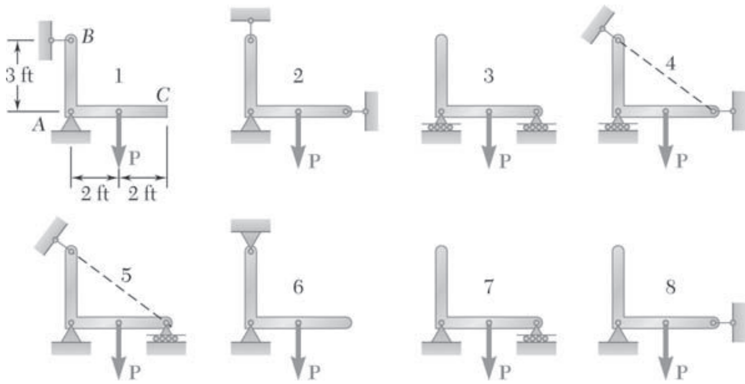


$$\mathbf{B} = \mathbf{D}_y = 196.2\text{ N}\uparrow$$

$$(\mathbf{C} + \mathbf{D}_x = 0)$$

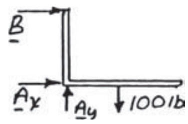
PROBLEM 4.60

The bracket ABC can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. For each case, answer the questions listed in Problem 4.59, and, wherever possible, compute the reactions, assuming that the magnitude of the force **P** is 100 lb.



SOLUTION

1. Three non-concurrent, non-parallel reactions



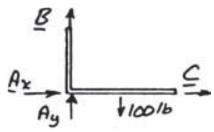
   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$\mathbf{A} = 120.2\text{ lb} \measuredangle 56.3°, \quad \mathbf{B} = 66.7\text{ lb} \longleftarrow$

2. Four concurrent, reactions (through A)

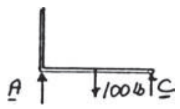


   (*a*) Bracket: improper constraint

   (*b*) Reactions: indeterminate

   (*c*) No equilibrium $(\Sigma M_A \neq 0)$

3. Two reactions



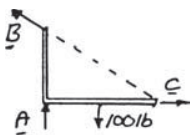
   (*a*) Bracket: partial constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$\mathbf{A} = 50\text{ lb}\uparrow, \quad \mathbf{C} = 50\text{ lb}\uparrow$

4. Three non-concurrent, non-parallel reactions



   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

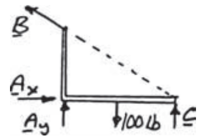
$\mathbf{A} = 50\text{ lb}\uparrow, \quad \mathbf{B} = 83.3\text{ lb} \searrow 36.9°, \quad \mathbf{C} = 66.7\text{ lb} \longrightarrow$

5. Four non-concurrent, non-parallel reactions

   (*a*) Bracket: complete constraint

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained $(\Sigma M_C = 0)$ $\mathbf{A}_y = 50 \text{ lb} \uparrow$

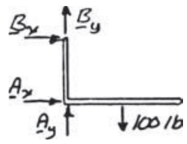


6. Four non-concurrent, non-parallel reactions

   (*a*) Bracket: complete constraint

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained



$$\mathbf{A}_x = 66.7 \text{ lb} \uparrow, \quad \mathbf{B} = 66.7 \text{ lb} \longleftarrow$$

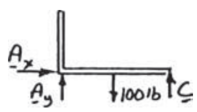
$$(\mathbf{A}_y + \mathbf{B}_y = 100 \text{ lb} \uparrow)$$

7. Three non-concurrent, non-parallel reactions

   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained



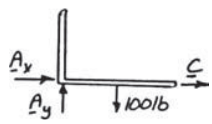
$$\mathbf{A} = \mathbf{C} = 50 \text{ lb} \uparrow$$

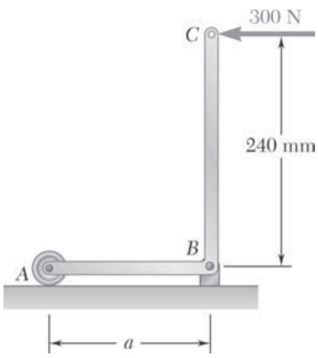
8. Three concurrent, reactions (through *A*)

   (*a*) Bracket: improper constraint

   (*b*) Reactions: indeterminate

   (*c*) No equilibrium $(\Sigma M_A \neq 0)$





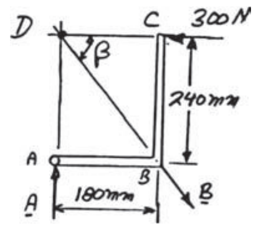
PROBLEM 4.61

Determine the reactions at A and B when $a = 180$ mm.

SOLUTION

Reaction at B must pass through D where A and 300-N load intersect.

Free-Body Diagram:
(Three-force member)

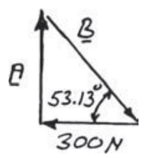


$\underline{\Delta BCD}$:

$$\tan \beta = \frac{240}{180}$$

$$\beta = 53.13^\circ$$

Force triangle



$$A = (300 \text{ N}) \tan 53.13^\circ$$

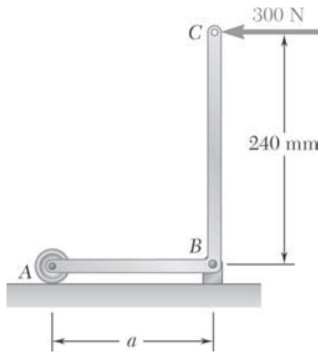
$$= 400 \text{ N}$$

$\mathbf{A} = 400 \text{ N} \uparrow$ ◀

$$B = \frac{300 \text{ N}}{\cos 53.13^\circ}$$

$$= 500 \text{ N}$$

$\mathbf{B} = 500 \text{ N} \searrow 53.1^\circ$ ◀



PROBLEM 4.62

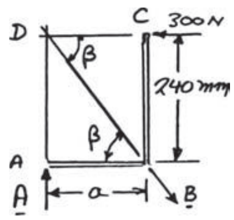
For the bracket and loading shown, determine the range of values of the distance a for which the magnitude of the reaction at B does not exceed 600 N.

SOLUTION

Reaction at B must pass through D where A and 300-N load intersect.

Free-Body Diagram:

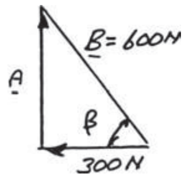
(Three-force member)



$$a = \frac{240 \text{ mm}}{\tan \beta} \tag{1}$$

Force Triangle

(with $B = 600$ N)



$$\cos \beta = \frac{300 \text{ N}}{600 \text{ N}} = 0.5$$

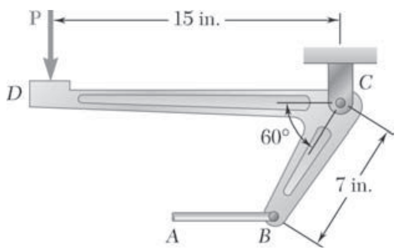
$$\beta = 60.0°$$

Eq. (1)

$$a = \frac{240 \text{ mm}}{\tan 60.0°}$$

$$= 138.56 \text{ mm}$$

For $B \leq 600$ N $\quad a \geq 138.6$ mm ◀



PROBLEM 4.63

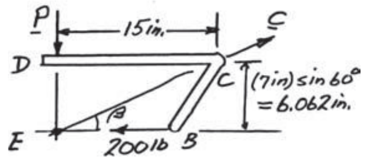
Using the method of Section 4.7, solve Problem 4.17.

PROBLEM 4.17 The required tension in cable *AB* is 200 lb. Determine (*a*) the vertical force **P** that must be applied to the pedal, (*b*) the corresponding reaction at *C*.

SOLUTION

Free-Body Diagram:

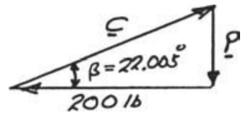
(Three-Force body)



Reaction at *C* must pass through *E*, where *D* and 200-lb force intersect.

$$\tan\beta = \frac{6.062 \text{ in.}}{15 \text{ in.}}$$

$$\beta = 22.005°$$



Force triangle

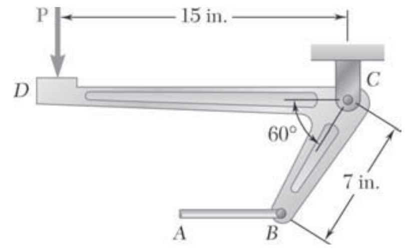
(*a*) $$P = (200 \text{ lb})\tan 22.005°$$

$$P = 80.83 \text{ lb}$$

$\mathbf{P} = 80.8 \text{ lb}\downarrow$ ◀

(*b*) $$C = \frac{200 \text{ lb}}{\cos 22.005°} = 215.7 \text{ lb}$$

$\mathbf{C} = 216 \text{ lb} \measuredangle 22.0°$ ◀



PROBLEM 4.64

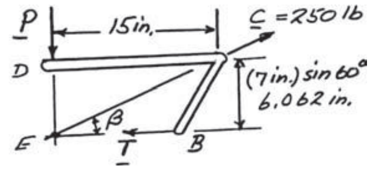
Using the method of Section 4.7, solve Problem 4.18.

PROBLEM 4.18 Determine the maximum tension that can be developed in cable *AB* if the maximum allowable value of the reaction at *C* is 250 lb.

SOLUTION

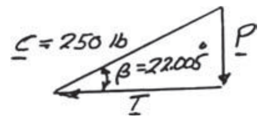
Free-Body Diagram:

(Three -Force body)



Reaction at *C* must pass through *E,* where *D* and the force **T** intersect.

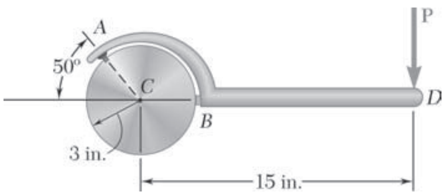
$$\tan\beta = \frac{6.062 \text{ in.}}{15 \text{ in.}}$$
$$\beta = 22.005°$$



Force triangle

$$T = (250 \text{ lb})\cos 22.005°$$
$$T = 231.8 \text{ lb}$$

$T = 232$ lb ◀



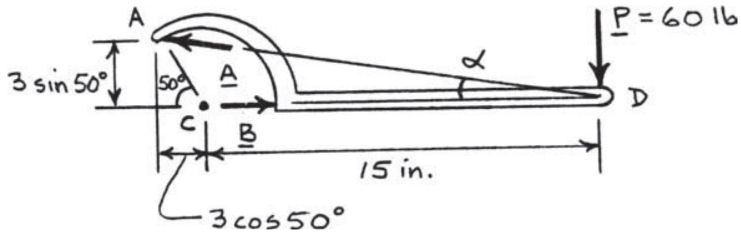
PROBLEM 4.65

The spanner shown is used to rotate a shaft. A pin fits in a hole at *A*, while a flat, frictionless surface rests against the shaft at *B*. If a 60-lb force **P** is exerted on the spanner at *D*, find the reactions at *A* and *B*.

SOLUTION

Free-Body Diagram:

(Three-Force body)

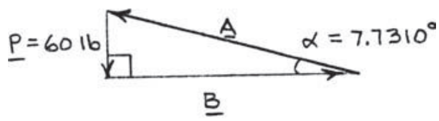


The line of action of **A** must pass through *D*, where **B** and **P** intersect.

$$\tan\alpha = \frac{3\sin 50°}{3\cos 50° + 15}$$
$$= 0.135756$$
$$\alpha = 7.7310°$$

Force triangle

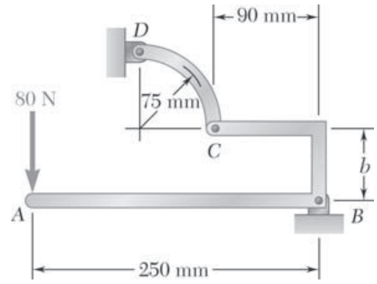
$$A = \frac{60 \text{ lb}}{\sin 7.7310°}$$
$$= 446.02 \text{ lb}$$



$$B = \frac{60 \text{ lb}}{\tan 7.7310°}$$
$$= 441.97 \text{ lb}$$

A = 446 lb ⦫ 7.73° ◀

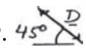
B = 442 lb ⟶ ◀



PROBLEM 4.66

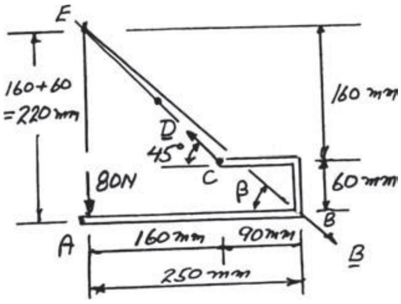
Determine the reactions at *B* and *D* when $b = 60$ mm.

SOLUTION

Since *CD* is a two-force member, the line of action of reaction at *D* must pass through Points *C* and *D*. 

Free-Body Diagram:

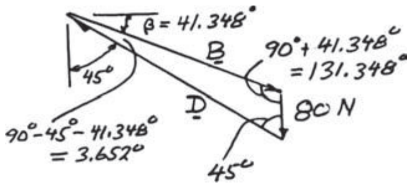
(Three-Force body)



Reaction at *B* must pass through *E*, where the reaction at *D* and 80-N force intersect.

$$\tan \beta = \frac{220 \text{ mm}}{250 \text{ mm}}$$
$$\beta = 41.348°$$

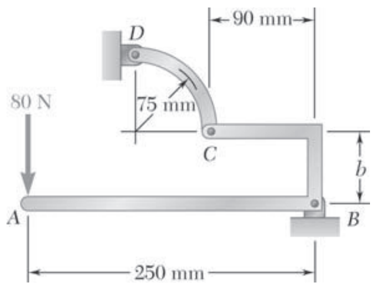
<u>Force triangle</u>



<u>Law of sines</u>

$$\frac{80 \text{ N}}{\sin 3.652°} = \frac{B}{\sin 45°} = \frac{D}{\sin 131.348°}$$
$$B = 888.0 \text{ N}$$
$$D = 942.8 \text{ N}$$

B $= 888$ N ↖ 41.3° **D** $= 943$ N ↘ 45.0° ◀



PROBLEM 4.67

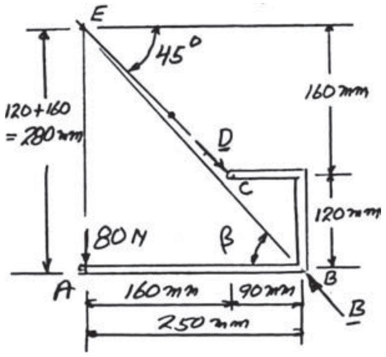
Determine the reactions at B and D when $b = 120$ mm.

SOLUTION

Since CD is a two-force member, line of action of reaction at D must pass through C and D .

Free-Body Diagram:

(Three-Force body)

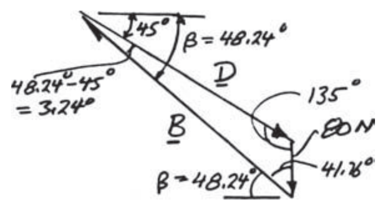


Reaction at B must pass through E, where the reaction at D and 80-N force intersect.

$$\tan\beta = \frac{280 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 48.24°$$

Force triangle



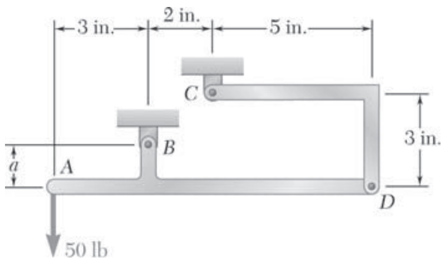
Law of sines

$$\frac{80 \text{ N}}{\sin 3.24°} = \frac{B}{\sin 135°} = \frac{D}{\sin 41.76°}$$

$$B = 1000.9 \text{ N}$$

$$D = 942.8 \text{ N}$$

$\mathbf{B} = 1001 \text{ N} \measuredangle 48.2°$ $\quad \mathbf{D} = 943 \text{ N} \measuredangle 45.0°$ ◀



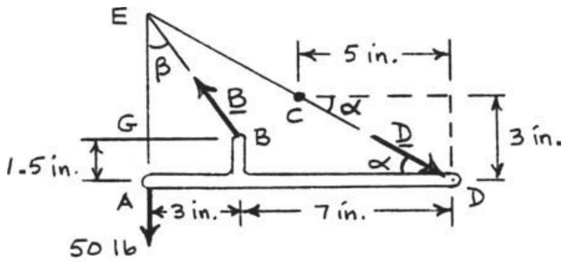
PROBLEM 4.68

Determine the reactions at B and C when $a = 1.5$ in.

SOLUTION

Since CD is a two-force member, the force it exerts on member ABD is directed along DC.

Free-Body Diagram of ABD: (Three-Force member)

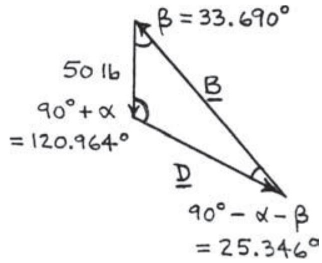


The reaction at B must pass through E, where **D** and the 50-lb load intersect.

Triangle CFD:
$$\tan\alpha = \frac{3}{5} = 0.6$$
$$\alpha = 30.964°$$

Triangle EAD:
$$AE = 10\tan\alpha = 6 \text{ in.}$$
$$GE = AE - AG = 6 - 1.5 = 4.5 \text{ in.}$$

Triangle EGB:
$$\tan\beta = \frac{GB}{GE} = \frac{3}{4.5}$$
$$\beta = 33.690°$$

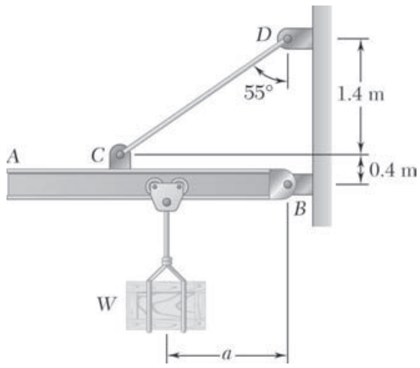


<u>Force triangle</u>
$$\frac{B}{\sin 120.964°} = \frac{D}{\sin 33.690°} = \frac{50 \text{ lb}}{\sin 25.346°}$$

$$B = 100.155 \text{ lb}$$
$$D = 64.789 \text{ lb}$$

$\mathbf{B} = 100.2$ lb ⦪ 56.3° ◀

$\mathbf{C} = \mathbf{D} = 64.8$ lb ⦫ 31.0° ◀

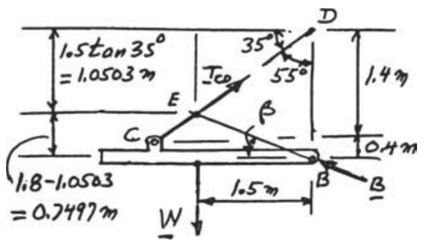


PROBLEM 4.69

A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

Free-Body Diagram:



Three-Force body: **W** and $\mathbf{T}_{CD}$ intersect at *E*.

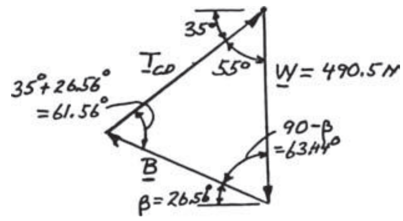
$$\tan\beta = \frac{0.7497 \text{ m}}{1.5 \text{ m}}$$

$$\beta = 26.56°$$

Force triangle 3 forces intersect at *E*.

$$W = (50 \text{ kg})\ 9.81 \text{ m/s}^2$$

$$= 490.5 \text{ N}$$



Law of sines

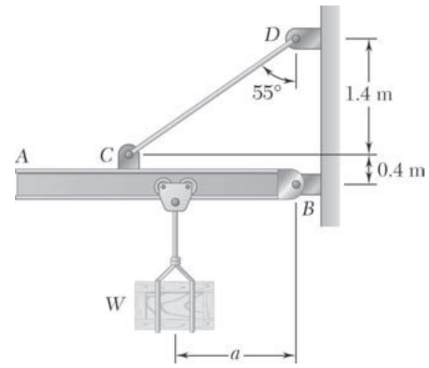
$$\frac{490.5 \text{ N}}{\sin 61.56°} = \frac{T_{CD}}{\sin 63.44°} = \frac{B}{\sin 55°}$$

$$T_{CD} = 498.9 \text{ N}$$

$$B = 456.9 \text{ N}$$

(*a*) $T_{CD} = 499 \text{ N}$ ◀

(*b*) $\mathbf{B} = 457 \text{ N} \searrow 26.6°$ ◀



PROBLEM 4.70

Solve Problem 4.69, assuming that $a = 3$ m.

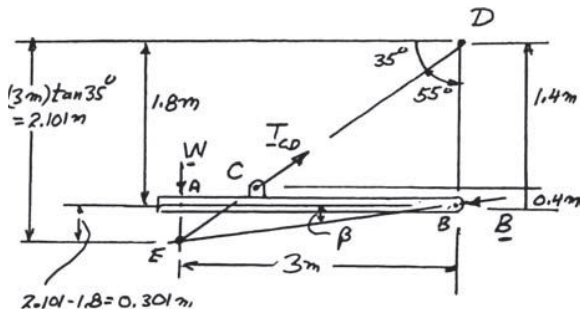
PROBLEM 4.69 A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

W and $\mathbf{T}_{CD}$ intersect at *E*

Free-Body Diagram:

Three-Force body:

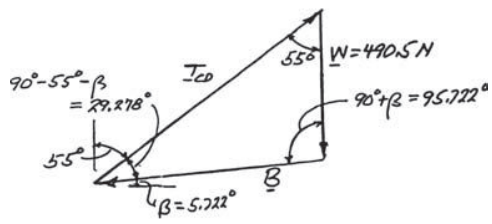


$$\tan\beta = \frac{AE}{AB} = \frac{0.301 \text{ m}}{3 \text{ m}}$$

$$\beta = 5.722°$$

Force Triangle (Three forces intersect at *E*.)

$$W = (50 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$= 490.5 \text{ N}$$



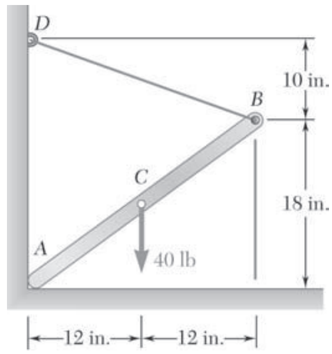
Law of sines
$$\frac{490.5 \text{ N}}{\sin 29.278°} = \frac{T_{CD}}{\sin 95.722°} = \frac{B}{\sin 55°}$$

$$T_{CD} = 997.99 \text{ N}$$

$$B = 821.59 \text{ N}$$

(*a*) $T_{CD} = 998$ N ◀

(*b*) $\mathbf{B} = 822$ N ↙ 5.72° ◀

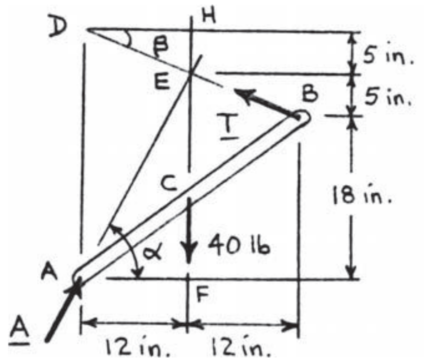


PROBLEM 4.71

One end of rod *AB* rests in the corner *A* and the other end is attached to cord *BD*. If the rod supports a 40-lb load at its midpoint *C*, find the reaction at *A* and the tension in the cord.

SOLUTION

Free-Body Diagram: (Three-Force body)



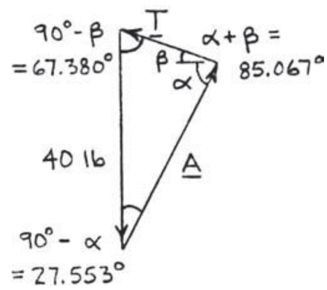
The line of action of reaction at *A* must pass through *E*, where **T** and the 40-lb load intersect.

$$\tan\alpha = \frac{EF}{AF} = \frac{23}{12}$$

$$\alpha = 62.447°$$

$$\tan\beta = \frac{EH}{DH} = \frac{5}{12}$$

$$\beta = 22.620°$$

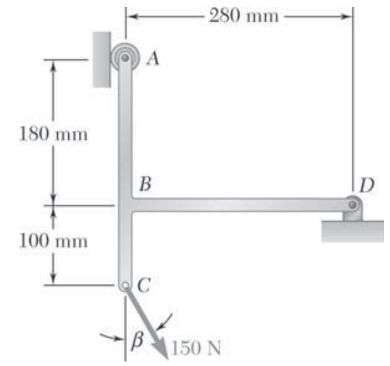


Force triangle

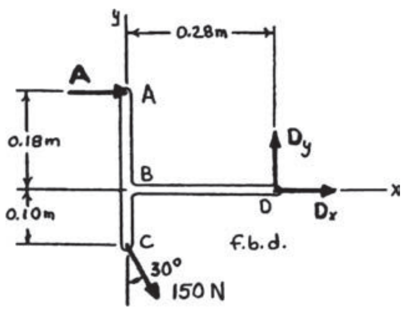
$$\frac{A}{\sin 67.380°} = \frac{T}{\sin 27.553°} = \frac{40 \text{ lb}}{\sin 85.067°}$$

A = 37.1 lb ∠ 62.4° ◀

$T = 18.57$ lb ◀

	**PROBLEM 4.72** Determine the reactions at A and D when $\beta = 30°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18 \text{ m}) + [(150 \text{ N})\sin 30°](0.10 \text{ m}) + [(150 \text{ N})\cos 30°](0.28 \text{ m}) = 0$$

$$A = 243.74 \text{ N}$$

or $\mathbf{A} = 244 \text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (243.74 \text{ N}) + (150 \text{ N})\sin 30° + D_x = 0$$

$$D_x = -318.74 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150 \text{ N})\cos 30° = 0$$

$$D_y = 129.904 \text{ N}$$

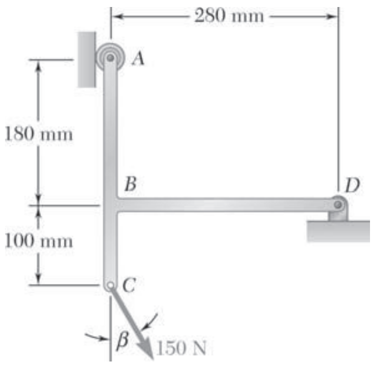
Then

$$D = \sqrt{(D_x)^2 + D_x^2}$$
$$= \sqrt{(318.74)^2 + (129.904)^2}$$
$$= 344.19 \text{ N}$$

and

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right)$$
$$= \tan^{-1}\left(\frac{129.904}{-318.74}\right)$$
$$= -22.174°$$

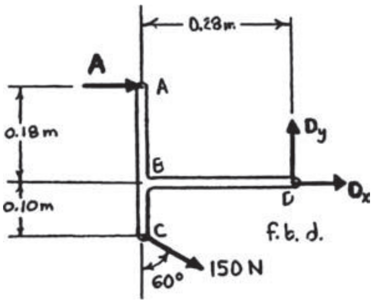
or $\mathbf{D} = 344 \text{ N} \searrow 22.2°$ ◀



PROBLEM 4.73

Determine the reactions at A and D when $\beta = 60°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18 \text{ m}) + [(150 \text{ N})\sin 60°](0.10 \text{ m})$$
$$+[(150 \text{ N})\cos 60°](0.28 \text{ m}) = 0$$

$$A = 188.835 \text{ N}$$

or $\mathbf{A} = 188.8 \text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (188.835 \text{ N}) + (150 \text{ N})\sin 60° + D_x = 0$$

$$D_x = -318.74 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150 \text{ N})\cos 60° = 0$$

$$D_y = 75.0 \text{ N}$$

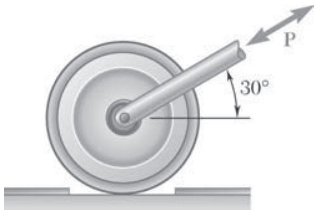
Then

$$D = \sqrt{(D_x)^2 + (D_y)^2}$$
$$= \sqrt{(318.74)^2 + (75.0)^2}$$
$$= 327.44 \text{ N}$$

and

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right)$$
$$= \tan^{-1}\left(\frac{75.0}{-318.74}\right)$$
$$= -13.2409°$$

or $\mathbf{D} = 327 \text{ N} \measuredangle 13.24°$ ◀



PROBLEM 4.74

A 40-lb roller, of diameter 8 in., which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 0.3 in., determine the force **P** required to move the roller onto the tiles if the roller is (*a*) pushed to the left, (*b*) pulled to the right.

SOLUTION

See solution to Problem 4.73 for free-body diagram and analysis leading to the following equations:

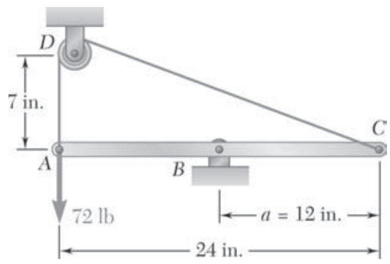
$$T = \frac{P}{1+\cos\theta} \qquad (1)$$

$$\mathbf{C} = P\frac{\sin\theta}{1+\cos\theta} \qquad (2)$$

For $\theta = 45°$

Eq. (1): $$T = \frac{P}{1+\cos 45°} = \frac{P}{1.7071}$$ $T = 0.586P$ ◀

Eq. (2): $$C = P\frac{\sin 45°}{1+\cos 45°} = P\frac{0.7071}{1.7071}$$ $\mathbf{C} = 0.444P \longrightarrow$ ◀

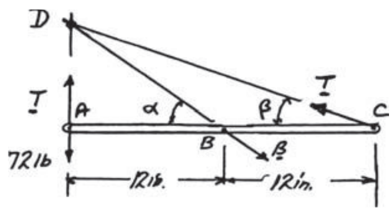


PROBLEM 4.75

Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord attached at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portions *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION

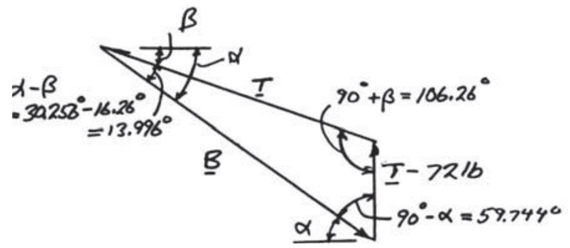
Free-Body Diagram:



Reaction at *B* must pass through *D*.

$$\tan\alpha = \frac{7 \text{ in.}}{12 \text{ in.}}$$
$$\alpha = 30.256^\circ$$
$$\tan\beta = \frac{7 \text{ in.}}{24 \text{ in.}}$$
$$\beta = 16.26^\circ$$

Force triangle



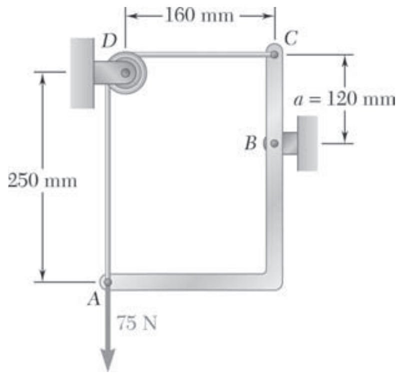
Law of sines

$$\frac{T}{\sin 59.744^\circ} = \frac{T - 72 \text{ lb}}{\sin 13.996^\circ} = \frac{B}{\sin 106.26}$$
$$T(\sin 13.996^\circ) = (T - 72 \text{ lb})(\sin 59.744^\circ)$$
$$T(0.24185) = (T - 72)(0.86378)$$
$$T = 100.00 \text{ lb}$$

$T = 100.0$ lb ◀

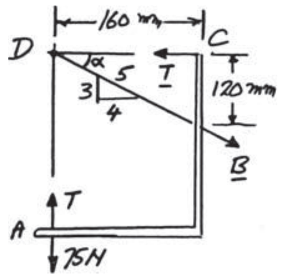
$$B = (100 \text{ lb})\frac{\sin 106.26^\circ}{\sin 59.744}$$
$$= 111.14 \text{ lb}$$

$\mathbf{B} = 111.1 \text{ lb} \measuredangle 30.3^\circ$ ◀

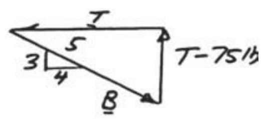
	**PROBLEM 4.76** Member ABC is supported by a pin and bracket at B and by an inextensible cord attached at A and C and passing over a frictionless pulley at D. The tension may be assumed to be the same in portions AD and CD of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at B.

SOLUTION

Free-Body Diagram:



Force triangle

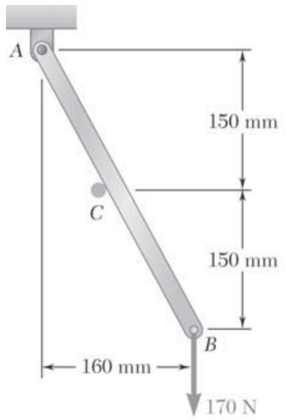


Reaction at B must pass through D.

$$\tan\alpha = \frac{120}{160}; \quad \alpha = 36.9^\circ \qquad \frac{T}{4} = \frac{T - 75\text{ lb}}{3} = \frac{B}{5}$$

$$3T = 4T - 300; \quad T = 300\text{ lb}$$

$$B = \frac{5}{4}T = \frac{5}{4}(300\text{ lb}) = 375\text{ lb} \qquad \mathbf{B} = 375\text{ lb} \searrow 36.9^\circ$$

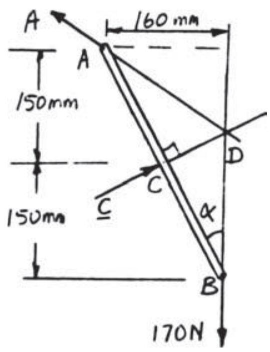


PROBLEM 4.77

Rod AB is supported by a pin and bracket at A and rests against a frictionless peg at C. Determine the reactions at A and C when a 170-N vertical force is applied at B.

SOLUTION

Free-Body Diagram:
(Three-Force body)



The reaction at A must pass through D where **C** and 170-N force intersect.

$$\tan\alpha = \frac{160 \text{ mm}}{300 \text{ mm}}$$

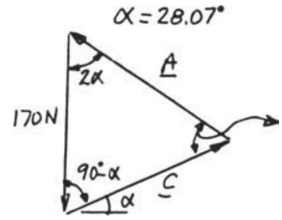
$$\alpha = 28.07°$$

We note that triangle ABD is isosceles (since $AC = BC$) and, therefore

$$\sphericalangle CAD = \alpha = 28.07°$$

Also, since $CD \perp CB$, reaction **C** forms angle $\alpha = 28.07°$ with horizontal.

<u>Force triangle</u>



We note that **A** forms angle 2α with vertical. Thus **A** and **C** form angle

$$180° - (90° - \alpha) - 2\alpha = 90° - \alpha$$

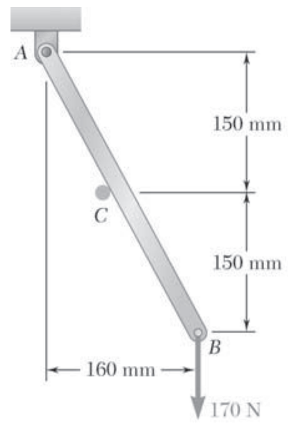
Force triangle is isosceles and we have

$$A = 170 \text{ N}$$

$$C = 2(170 \text{ N})\sin\alpha$$

$$= 160.0 \text{ N}$$

$\mathbf{A} = 170.0 \text{ N} \searrow 33.9°$ $\quad \mathbf{C} = 160.0 \text{ N} \measuredangle 28.1°$ ◀



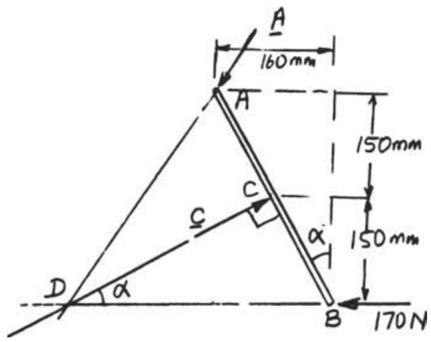
PROBLEM 4.78

Solve Problem 4.77, assuming that the 170-N force applied at *B* is horizontal and directed to the left.

PROBLEM 4.77 Rod *AB* is supported by a pin and bracket at *A* and rests against a frictionless peg at *C*. Determine the reactions at *A* and *C* when a 170-N vertical force is applied at *B*.

SOLUTION

Free-Body Diagram: (Three-Force body)



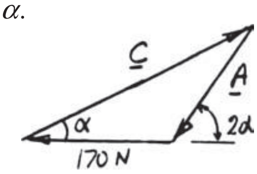
The reaction at *A* must pass through *D*, where **C** and the 170-N force intersect.

$$\tan\alpha = \frac{160 \text{ mm}}{300 \text{ mm}}$$

$$\alpha = 28.07°$$

We note that triangle *ADB* is isosceles (since $AC = BC$). Therefore $\measuredangle A = \measuredangle B = 90° - \alpha$.

Also $\measuredangle ADB = 2\alpha$



Force triangle

The angle between **A** and **C** must be $2\alpha - \alpha = \alpha$

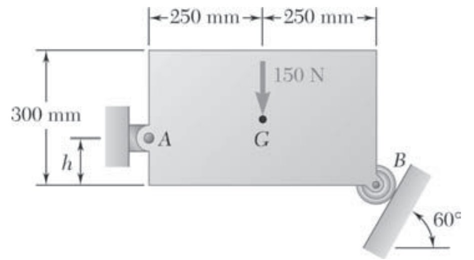
$\alpha = 28.07°$

Thus, force triangle is isosceles and

$$A = 170.0 \text{ N}$$

$$C = 2(170 \text{ N})\cos\alpha = 300 \text{ N}$$

A $= 170.0$ N ↗ 56.1° **C** $= 300$ N ↗ 28.1° ◀



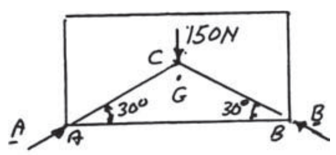
PROBLEM 4.79

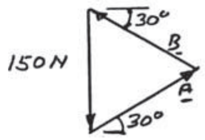
Using the method of Section 4.7, solve Problem 4.21.

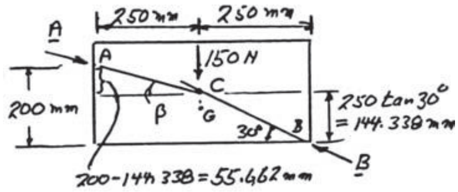
PROBLEM 4.21 Determine the reactions at A and B when (a) $h = 0$, (b) $h = 200$ mm.

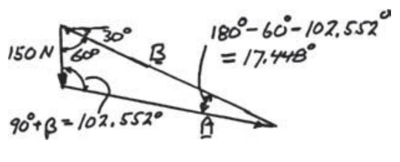
SOLUTION

Free-Body Diagram:









(a) <u>$h = 0$</u>

Reaction **A** must pass through C where 150-N weight and **B** interect.

<u>Force triangle</u> is equilateral

$$\mathbf{A} = 150.0 \text{ N} \measuredangle 30.0° \blacktriangleleft$$

$$\mathbf{B} = 150.0 \text{ N} \searrow 30.0° \blacktriangleleft$$

(b) <u>$h = 200$ mm</u>

$$\tan\beta = \frac{55.662}{250}$$

$$\beta = 12.552°$$

<u>Force triangle</u>

<u>Law of sines</u>

$$\frac{150 \text{ N}}{\sin 17.448°} = \frac{A}{\sin 60°} = \frac{B}{\sin 102.552°}$$

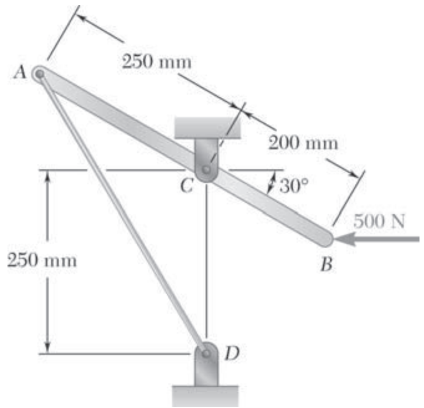
$$A = 433.247 \text{ N}$$

$$B = 488.31 \text{ N}$$

$$\mathbf{A} = 433 \text{ N} \searrow 12.55° \blacktriangleleft$$

$$\mathbf{B} = 488 \text{ N} \searrow 30.0° \blacktriangleleft$$

PROBLEM 4.80

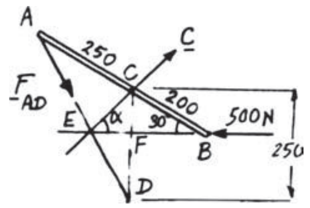


Using the method of Section 4.7, solve Problem 4.28.

PROBLEM 4.28 A lever *AB* is hinged at *C* and attached to a control cable at *A*. If the lever is subjected to a 500-N horizontal force at *B*, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:
(Three-Force body)



Dimensions in mm

Reaction at *C* must pass through *E*, where $\mathbf{F}_{AD}$ and 500-N force intersect.

Since $AC = CD = 250$ mm, triangle *ACD* is isosceles.

We have $\qquad \sphericalangle C = 90° + 30° = 120°$

and $\qquad \sphericalangle A = \sphericalangle D = \frac{1}{2}(180° - 120°) = 30°$

On the other hand, from triangle *BCF*:

$$CF = (BC)\sin 30° = 200 \sin 30° = 100 \text{ mm}$$
$$FD = CD - CF = 250 - 100 = 150 \text{ mm}$$

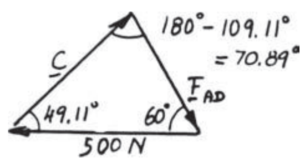
From triangle *EFD*, and since $\sphericalangle D = 30°$:

$$EF = (FD)\tan 30° = 150 \tan 30° = 86.60 \text{ mm}$$

From triangle *EFC*: $\qquad \tan\alpha = \dfrac{CF}{EF} = \dfrac{100 \text{ mm}}{86.60 \text{ mm}}$

$$\alpha = 49.11°$$

Force triangle

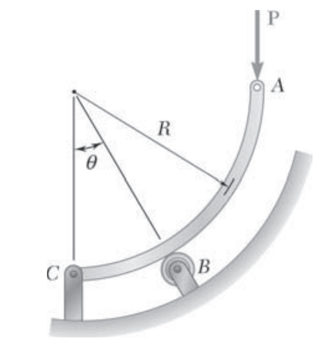


Law of sines $\qquad \dfrac{F_{AD}}{\sin 49.11°} = \dfrac{C}{\sin 60°} = \dfrac{500 \text{ N}}{\sin 70.89°}$

$$F_{AD} = 400 \text{ N}, \quad C = 458 \text{ N}$$

(*a*) $\qquad F_{AD} = 400 \text{ N}$ ◀

(*b*) $\qquad \mathbf{C} = 458 \text{ N} \measuredangle 49.1°$ ◀

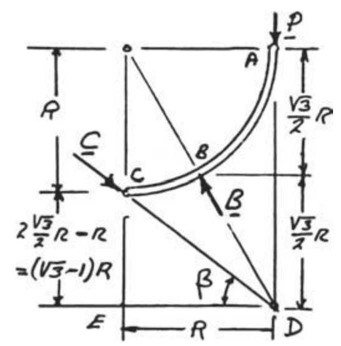


PROBLEM 4.81

Knowing that $\theta = 30°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Free-Body Diagram:
(Three-Force body)



Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

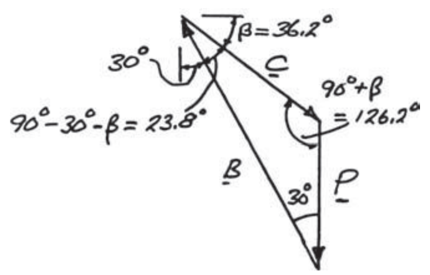
In Δ *CDE*:

$$\tan\beta = \frac{(\sqrt{3}-1)R}{R}$$

$$= \sqrt{3} - 1$$

$$\beta = 36.2°$$

Force triangle



Law of sines

$$\frac{P}{\sin 23.8°} = \frac{B}{\sin 126.2°} = \frac{C}{\sin 30°}$$

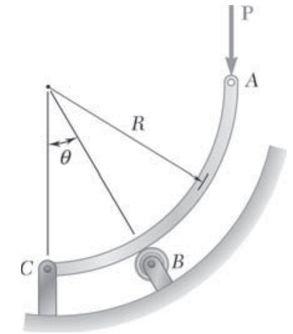
$$B = 2.00P$$

$$C = 1.239P$$

(*a*) **B** $= 2P$ ⦪ 60.0° ◀

(*b*) **C** $= 1.239P$ ⦪ 36.2° ◀

PROBLEM 4.82

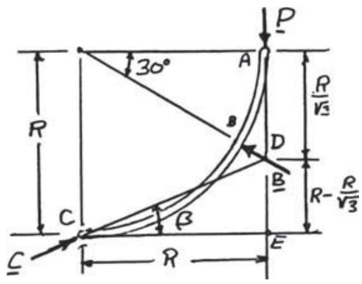


Knowing that $\theta = 60°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

Free-Body Diagram:
(Three-Force body)



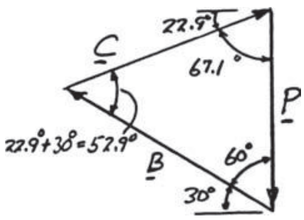
In ΔCDE:

$$\tan\beta = \frac{R - \frac{R}{\sqrt{3}}}{R}$$

$$= 1 - \frac{1}{\sqrt{3}}$$

$$\beta = 22.9°$$

Force triangle



Law of sines

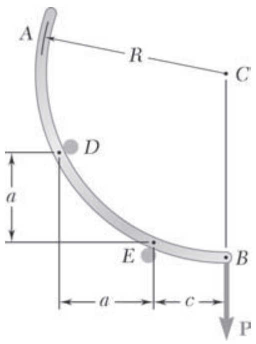
$$\frac{P}{\sin 52.9°} = \frac{B}{\sin 67.1°} = \frac{C}{\sin 60°}$$

$$B = 1.155P$$

$$C = 1.086P$$

(*a*) $\mathbf{B} = 1.155P$ ⦪ $30.0°$ ◀

(*b*) $\mathbf{C} = 1.086P$ ⦩ $22.9°$ ◀

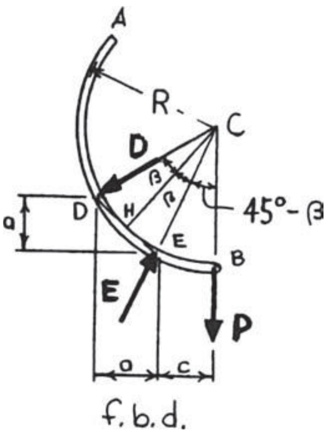


PROBLEM 4.83

Rod *AB* is bent into the shape of an arc of circle and is lodged between two pegs *D* and *E*. It supports a load **P** at end *B*. Neglecting friction and the weight of the rod, determine the distance *c* corresponding to equilibrium when $a = 20$ mm and $R = 100$ mm.

SOLUTION

Free-Body Diagram:



Since $\quad y_{ED} = x_{ED} = a,$

Slope of *ED* is $\searrow$ 45°

slope of *HC* is $\nearrow$ 45°

Also $\quad DE = \sqrt{2}a$

and $\quad DH = HE = \left(\frac{1}{2}\right)DE = \frac{a}{\sqrt{2}}$

For triangles *DHC* and *EHC* $\quad \sin\beta = \frac{\frac{a}{\sqrt{2}}}{R} = \frac{a}{\sqrt{2}R}$

Now $\quad c = R\sin(45° - \beta)$

For $\quad a = 20 \text{ mm} \quad \text{and} \quad R = 100 \text{ mm}$

$$\sin\beta = \frac{20 \text{ mm}}{\sqrt{2}(100 \text{ mm})}$$

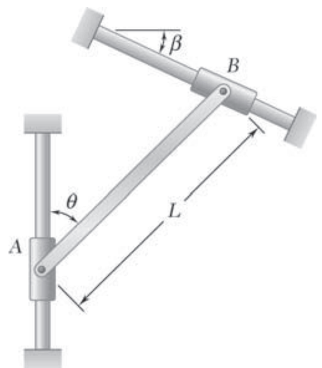
$$= 0.141421$$

$$\beta = 8.1301°$$

and $\quad c = (100 \text{ mm})\sin(45° - 8.1301°)$

$$= 60.00 \text{ mm}$$

or $c = 60.0$ mm ◀



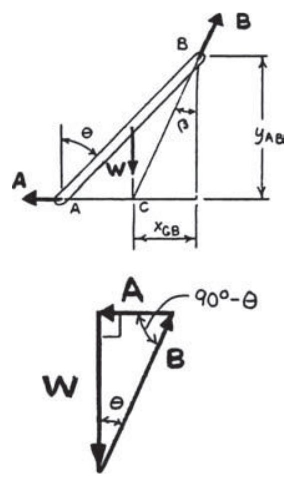
PROBLEM 4.84

A slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

SOLUTION

As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the force geometry

Free-Body Diagram:



$$\tan \beta = \frac{x_{GB}}{y_{AB}}$$

where

$$y_{AB} = L\cos\theta$$

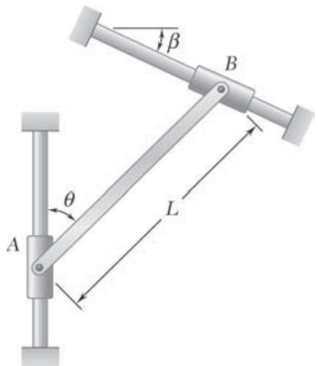
and

$$x_{GB} = \frac{1}{2}L\sin\theta$$

$$\tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta}$$

$$= \frac{1}{2}\tan\theta$$

or $\tan\theta = 2\tan\beta$ ◀



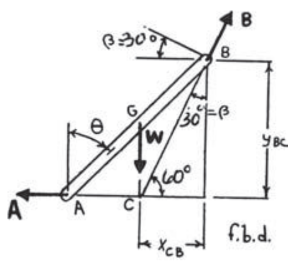
PROBLEM 4.85

An 8-kg slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 30°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

SOLUTION

(*a*) As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the geometry of the forces

Free-Body Diagram:



$$\tan\beta = \frac{x_{CB}}{y_{BC}}$$

where

$$x_{CB} = \frac{1}{2}L\sin\theta$$

and

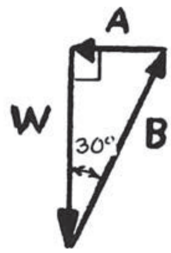
$$y_{BC} = L\cos\theta$$

$$\tan\beta = \frac{1}{2}\tan\theta$$

or $\quad \tan\theta = 2\tan\beta$

For $\quad \beta = 30°$

$$\tan\theta = 2\tan 30°$$
$$= 1.15470$$
$$\theta = 49.107°$$



or $\quad \theta = 49.1°$ ◀

(*b*) $\quad W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.480\text{ N}$

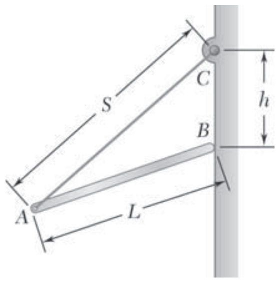
From force triangle $\quad A = W\tan\beta$

$$= (78.480\text{ N})\tan 30°$$
$$= 45.310\text{ N}$$

or $\quad \mathbf{A} = 45.3\text{ N} \longleftarrow$ ◀

and $\quad B = \dfrac{W}{\cos\beta} = \dfrac{78.480\text{ N}}{\cos 30°} = 90.621\text{ N}$

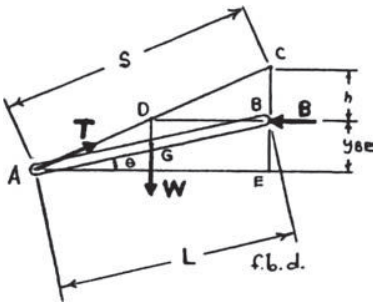
or $\quad \mathbf{B} = 90.6\text{ N} \measuredangle\ 60.0°$ ◀



PROBLEM 4.86

A slender uniform rod of length L is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length S. Derive an expression for the distance h in terms of L and S. Show that this position of equilibrium does not exist if $S > 2L$.

SOLUTION



From the f.b.d. of the three-force member AB, forces must intersect at D. Since the force T intersects Point D, directly above G,

$$y_{BE} = h$$

For triangle ACE: $\quad S^2 = (AE)^2 + (2h)^2 \quad (1)$

For triangle ABE: $\quad L^2 = (AE)^2 + (h)^2 \quad (2)$

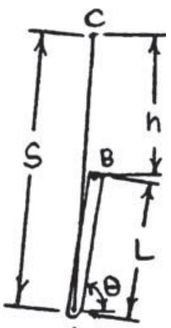
Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2 \quad (3)$$

or $$h = \sqrt{\frac{S^2 - L^2}{3}} \blacktriangleleft$$

As length S increases relative to length L, angle θ increases until rod AB is vertical. At this vertical position:

$$h + L = S \quad \text{or} \quad h = S - L$$



Therefore, for all positions of AB

$$h \geq S - L \quad (4)$$

or $$\sqrt{\frac{S^2 - L^2}{3}} \geq S - L$$

or
$$\begin{aligned} S^2 - L^2 &\geq 3(S - L)^2 \\ &= 3(S^2 - 2SL + L^2) \\ &= 3S^2 - 6SL + 3L^2 \end{aligned}$$

or $$0 \geq 2S^2 - 6SL + 4L^2$$

and $$0 \geq S^2 - 3SL + 2L^2 = (S - L)(S - 2L)$$

PROBLEM 4.86 (Continued)

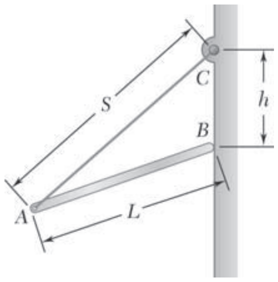
For $\quad S - L = 0 \quad S = L$

Minimum value of S is L

For $\quad S - 2L = 0 \quad S = 2L$

Maximum value of S is $2L$

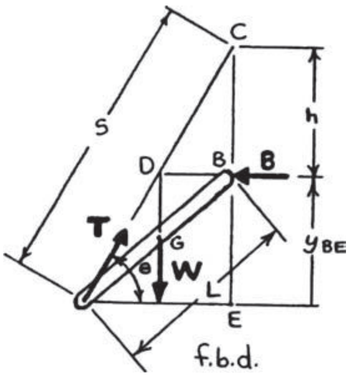
Therefore, equilibrium does not exist if $S > 2L$ ◀



PROBLEM 4.87

A slender uniform rod of length $L = 20$ in. is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length $S = 30$ in. Knowing that the weight of the rod is 10 lb, determine (*a*) the distance h, (*b*) the tension in the cord, (*c*) the reaction at B.

SOLUTION



From the f.b.d. of the three-force member AB, forces must intersect at D. Since the force T intersects Point D, directly above G,

$$y_{BE} = h$$

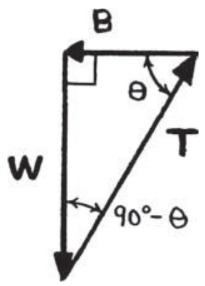
For triangle ACE: $$S^2 = (AE)^2 + (2h)^2 \tag{1}$$

For triangle ABE: $$L^2 = (AE)^2 + (h)^2 \tag{2}$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2$$

or $$h = \sqrt{\frac{S^2 - L^2}{3}}$$



(*a*) For $L = 20$ in. and $S = 30$ in.

$$h = \sqrt{\frac{(30)^2 - (20)^2}{3}}$$

$$= 12.9099 \text{ in.}$$ or $h = 12.91$ in. ◀

(*b*) We have $W = 10$ lb

and $$\theta = \sin^{-1}\left(\frac{2h}{s}\right)$$

$$= \sin^{-1}\left[\frac{2(12.9099)}{30}\right]$$

$$\theta = 59.391°$$

PROBLEM 4.87 (Continued)

From the force triangle

$$T = \frac{W}{\sin\theta}$$

$$= \frac{10 \text{ lb}}{\sin 59.391°}$$

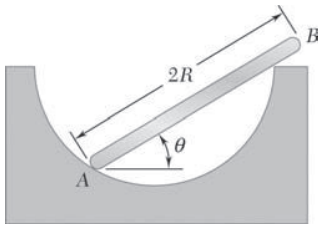
$$= 11.6190 \text{ lb} \qquad \text{or} \qquad T = 11.62 \text{ lb} \blacktriangleleft$$

(*c*)

$$B = \frac{W}{\tan\theta}$$

$$= \frac{10 \text{ lb}}{\tan 59.391°}$$

$$= 5.9161 \text{ lb} \qquad \text{or } \mathbf{B} = 5.92 \text{ lb} \longleftarrow \blacktriangleleft$$



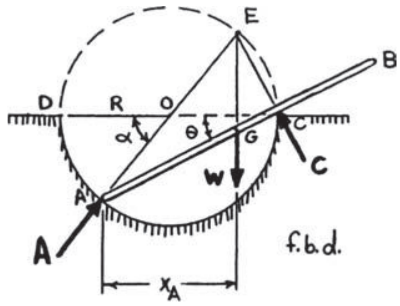
PROBLEM 4.88

A uniform rod AB of length $2R$ rests inside a hemispherical bowl of radius R as shown. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION

Based on the f.b.d., the uniform rod AB is a three-force body. Point E is the point of intersection of the three forces. Since force A passes through O, the center of the circle, and since force **C** is perpendicular to the rod, triangle ACE is a right triangle inscribed in the circle. Thus, E is a point on the circle.

Note that the angle α of triangle DOA is the central angle corresponding to the inscribed angle θ of triangle DCA.



$$\alpha = 2\theta$$

The horizontal projections of $AE, (x_{AE})$, and $AG, (x_{AG})$, are equal.

$$x_{AE} = x_{AG} = x_A$$

or $$(AE)\cos 2\theta = (AG)\cos\theta$$

and $$(2R)\cos 2\theta = R\cos\theta$$

Now $$\cos 2\theta = 2\cos^2\theta - 1$$

then $$4\cos^2\theta - 2 = \cos\theta$$

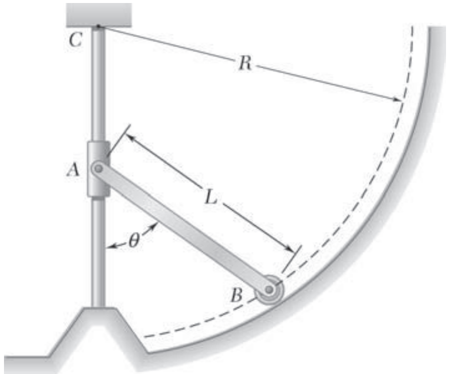
or $$4\cos^2\theta - \cos\theta - 2 = 0$$

Applying the quadratic equation

$$\cos\theta = 0.84307 \quad \text{and} \quad \cos\theta = -0.59307$$

$$\theta = 32.534° \quad \text{and} \quad \theta = 126.375°\text{(Discard)}$$

or $\theta = 32.5°$ ◀

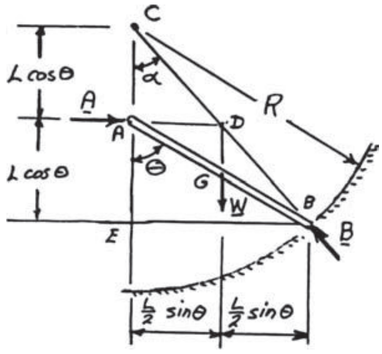


PROBLEM 4.89

A slender rod of length L and weight W is attached to a collar at A and is fitted with a small wheel at B. Knowing that the wheel rolls freely along a cylindrical surface of radius R, and neglecting friction, derive an equation in θ, L, and R that must be satisfied when the rod is in equilibrium.

SOLUTION

Free-Body Diagram (Three-Force body)



Reaction **B** must pass through D where **B** and **W** intersect.

Note that ΔABC and ΔBGD are similar.

$$AC = AE = L\cos\theta$$

In ΔABC:

$$(CE)^2 + (BE)^2 = (BC)^2$$

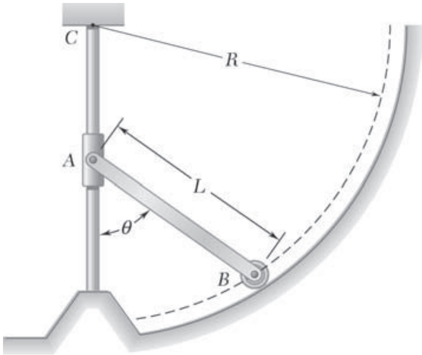
$$(2L\cos\theta)^2 + (L\sin\theta)^2 = R^2$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + \sin^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + 1 - \cos^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 3\cos^2\theta + 1$$

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right] \blacktriangleleft$$



PROBLEM 4.90

Knowing that for the rod of Problem 4.89, $L = 15$ in., $R = 20$ in., and $W = 10$ lb, determine (*a*) the angle θ corresponding to equilibrium, (*b*) the reactions at A and B.

SOLUTION

See the solution to Problem 4.89 for free-body diagram and analysis leading to the following equation

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right]$$

For $L = 15$ in., $R = 20$ in., and $W = 10$ lb.

(*a*)
$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{20 \text{ in.}}{15 \text{ in.}}\right)^2 - 1\right]; \quad \theta = 59.39°$$

$\theta = 59.4°$ ◀

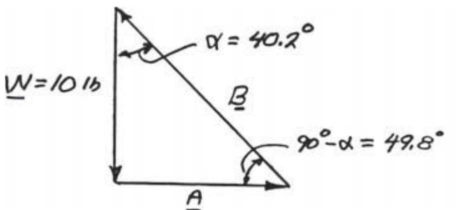
In $\underline{\Delta ABC}$:

$$\tan\alpha = \frac{BE}{CE} = \frac{L\sin\theta}{2L\cos\theta} = \frac{1}{2}\tan\theta$$

$$\tan\alpha = \frac{1}{2}\tan 59.39° = 0.8452$$

$$\alpha = 40.2°$$

Force triangle

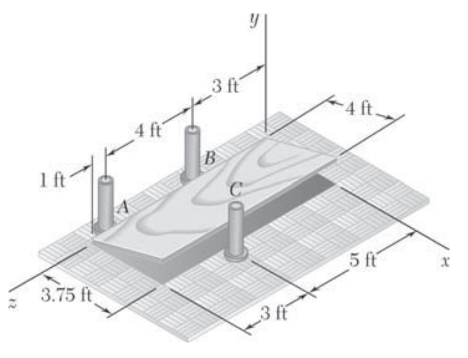


$$A = W\tan\alpha = (10 \text{ lb})\tan 40.2° = 8.45 \text{ lb}$$

$$B = \frac{W}{\cos\alpha} = \frac{(10 \text{ lb})}{\cos 40.2°} = 13.09 \text{ lb}$$

(*b*) **A** $= 8.45$ lb ⟶ ◀

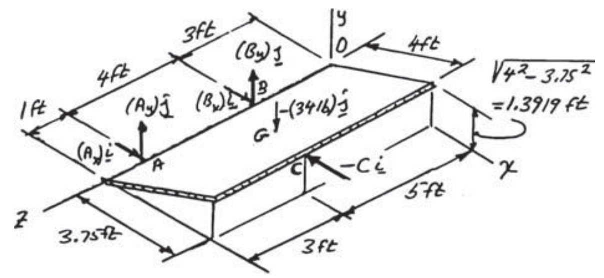
(*c*) **B** $= 13.09$ lb ⦨ 49.8° ◀



PROBLEM 4.91

A 4×8-ft sheet of plywood weighing 34 lb has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars at *A* and *B* and its upper edge leans against pipe *C*. Neglecting friction at all surfaces, determine the reactions at *A*, *B*, and *C*.

SOLUTION



$$\mathbf{r}_{G/B} = \frac{3.75}{2}\mathbf{i} + \frac{1.3919}{2}\mathbf{j} + \mathbf{k}$$

We have 5 unknowns and 6 Eqs. of equilibrium.

Plywood sheet is free to move in *z* direction, but equilibrium is maintained $(\Sigma F_z = 0)$.

$$\Sigma M_B = 0: \quad r_{A/B} \times (A_x\mathbf{i} + A_y\mathbf{j}) + r_{C/B} \times (-C\mathbf{i}) + r_{G/B} \times (-w\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ A_x & A_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.75 & 1.3919 & 2 \\ -C & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.875 & 0.696 & 1 \\ 0 & -34 & 0 \end{vmatrix} = 0$$

$$-4A_y\mathbf{i} + 4A_x\mathbf{j} - 2C\mathbf{j} + 1.3919C\mathbf{k} + 34\mathbf{i} - 63.75\mathbf{k} = 0$$

Equating coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -4A_y + 34 = 0 \qquad A_y = 8.5 \text{ lb}$$

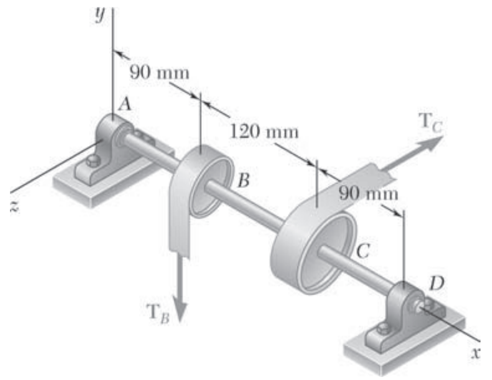
$$\mathbf{j}: \quad -2C + 4A_x = 0 \qquad A_x = \frac{1}{2}C = \frac{1}{2}(45.80) = 22.9 \text{ lb}$$

$$\mathbf{k}: \quad 1.3919C - 63.75 = 0 \qquad C = 45.80 \text{ lb} \qquad C = 45.8 \text{ lb}$$

$$\Sigma F_x = 0: \quad A_x + B_x - C = 0: \qquad B_x = 45.8 - 22.9 = 22.9 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y + B_y - W = 0: \qquad B_y = 34 - 8.5 = 25.5 \text{ lb}$$

$$\mathbf{A} = (22.9 \text{ lb})\mathbf{i} + (8.5 \text{ lb})\mathbf{j} \quad \mathbf{B} = (22.9 \text{ lb})\mathbf{i} + (25.5 \text{ lb})\mathbf{j} \quad \mathbf{C} = -(45.8 \text{ lb})\mathbf{i} \blacktriangleleft$$

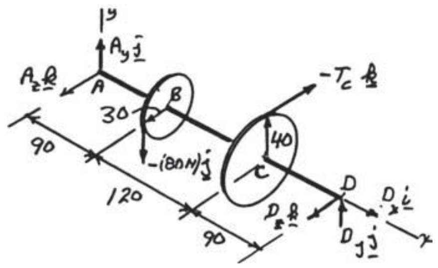


PROBLEM 4.92

Two tape spools are attached to an axle supported by bearings at A and D. The radius of spool B is 30 mm and the radius of spool C is 40 mm. Knowing that $T_B = 80$ N and that the system rotates at a constant rate, determine the reactions at A and D. Assume that the bearing at A does not exert any axial thrust and neglect the weights of the spools and axle.

SOLUTION

Dimensions in mm



We have six unknowns and six Eqs. of equilibrium.

$$\Sigma M_A = 0: \quad (90\mathbf{i} + 30\mathbf{k}) \times (-80\mathbf{j}) + (210\mathbf{i} + 40\mathbf{j}) \times (-T_C\mathbf{k}) + (300\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-7200\mathbf{k} + 2400\mathbf{i} + 210T_C\mathbf{j} - 40T_C\mathbf{i} + 300D_y\mathbf{k} - 300D_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad 2400 - 40T_C = 0 \qquad T_C = 60 \text{ N}$$

$$\mathbf{j}: \quad 210T_C - 300D_z = 0 \quad (210)(60) - 300D_z = 0 \qquad D_z = 42 \text{ N}$$

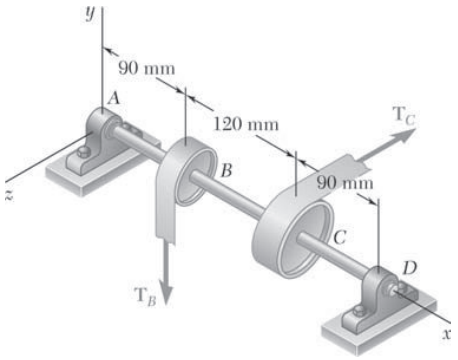
$$\mathbf{k}: \quad -7200 + 300D_y = 0 \qquad D_y = 24 \text{ N}$$

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 80 \text{ N} = 0 \qquad A_y = 80 - 24 = 56 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 60 \text{ N} = 0 \qquad A_z = 60 - 42 = 18 \text{ N}$$

$$\mathbf{A} = (56.0 \text{ N})\mathbf{j} + (18.00 \text{ N})\mathbf{k} \quad \mathbf{D} = (24.0 \text{ N})\mathbf{j} + (42.0 \text{ N})\mathbf{k} \blacktriangleleft$$



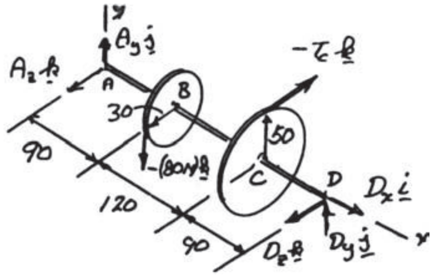
PROBLEM 4.93

Solve Problem 4.92, assuming that the spool C is replaced by a spool of radius 50 mm.

PROBLEM 4.92 Two tape spools are attached to an axle supported by bearings at A and D. The radius of spool B is 30 mm and the radius of spool C is 40 mm. Knowing that $T_B = 80$ N and that the system rotates at a constant rate, determine the reactions at A and D. Assume that the bearing at A does not exert any axial thrust and neglect the weights of the spools and axle.

SOLUTION

Dimensions in mm



We have six unknowns and six Eqs. of equilibrium.

$$\Sigma M_A = 0: \quad (90\mathbf{i} + 30\mathbf{k}) \times (-80\mathbf{j}) + (210\mathbf{i} + 50\mathbf{j}) \times (-T_C\mathbf{k}) + (300\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-7200\mathbf{k} + 2400\mathbf{i} + 210T_C\mathbf{j} - 50T_C\mathbf{i} + 300D_y\mathbf{k} - 300D_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad 2400 - 50T_C = 0 \qquad\qquad T_C = 48 \text{ N}$$

$$\mathbf{j}: \quad 210T_C - 300D_z = 0 \quad (210)(48) - 300D_z = 0 \qquad D_z = 33.6 \text{ N}$$

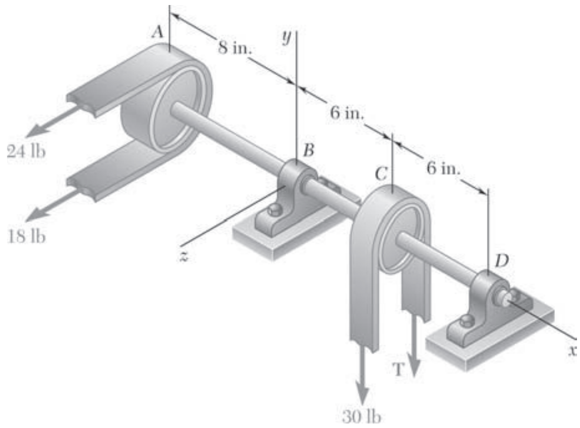
$$\mathbf{k}: \quad -7200 + 300D_y = 0 \qquad\qquad D_y = 24 \text{ N}$$

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 80 \text{ N} = 0 \qquad A_y = 80 - 24 = 56 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 48 = 0 \qquad A_z = 48 - 33.6 = 14.4 \text{ N}$$

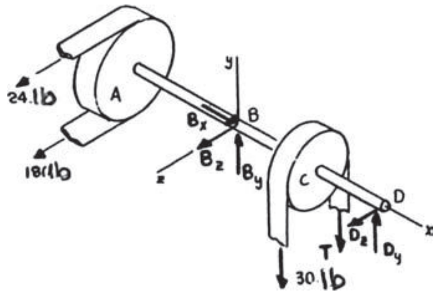
$$\mathbf{A} = (56.0 \text{ N})\mathbf{j} + (14.40 \text{ N})\mathbf{k} \quad \mathbf{D} = (24.0 \text{ N})\mathbf{j} + (33.6 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.94

Two transmission belts pass over sheaves welded to an axle supported by bearings at *B* and *D*. The sheave at *A* has a radius of 2.5 in., and the sheave at *C* has a radius of 2 in. Knowing that the system rotates at a constant rate, determine (*a*) the tension *T*, (*b*) the reactions at *B* and *D*. Assume that the bearing at *D* does not exert any axial thrust and neglect the weights of the sheaves and axle.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

(*a*) $\Sigma M_{x\text{-axis}} = 0$: $(24\text{ lb} - 18\text{ lb})(5\text{ in.}) + (30\text{ lb} - T)(4\text{ in.}) = 0$ $\quad T = 37.5\text{ lb}$ ◀

(*b*) $\Sigma F_x = 0$: $B_x = 0$

$\Sigma M_{D(z\text{-axis})} = 0$: $(30\text{ lb} + 37.5\text{ lb})(6\text{ in.}) - B_y(12\text{ in.}) = 0$

$B_y = 33.75\text{ lb}$

$\Sigma M_{D(y\text{-axis})} = 0$: $(24\text{ lb} + 18\text{ lb})(20\text{ in.}) + B_z(12\text{ in.}) = 0$

$B_z = -70.0\text{ lb}$

or $\mathbf{B} = (33.8\text{ lb})\mathbf{j} - (70.0\text{ lb})\mathbf{k}$ ◀

$\Sigma M_{B(z\text{-axis})} = 0$: $-(30\text{ lb} + 37.5\text{ lb})(6\text{ in.}) + D_y(12\text{ in.}) = 0$

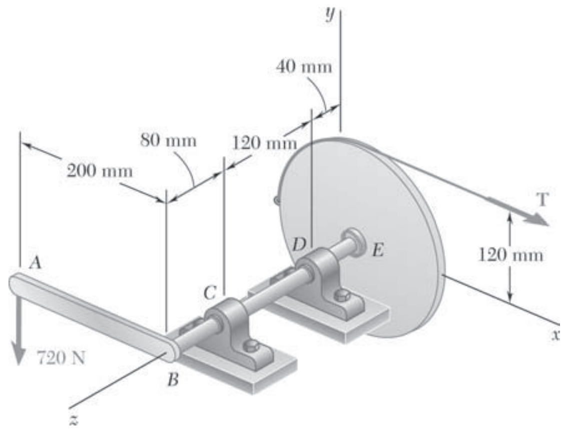
$D_y = 33.75\text{ lb}$

$\Sigma M_{B(y\text{-axis})} = 0$: $(24\text{ lb} + 18\text{ lb})(8\text{ in.}) + D_z(12\text{ in.}) = 0$

$D_z = -28.0\text{ lb}$

or $\mathbf{D} = (33.8\text{ lb})\mathbf{j} - (28.0\text{ lb})\mathbf{k}$ ◀

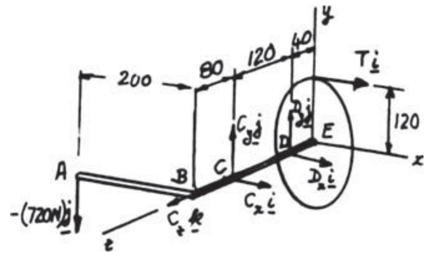
PROBLEM 4.95



A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six Eqs. of equilibrium–OK

$$\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k}) \times T\mathbf{i} + (80\mathbf{k} - 200\mathbf{i}) \times (-720\mathbf{j}) = 0$$

$$-120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\mathbf{k} - 160T\mathbf{j} + 57.6 \times 10^3\,\mathbf{i} + 144 \times 10^3\,\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors:

$\mathbf{k}$: $\quad -120T + 144 \times 10^3 = 0 \qquad$ (*a*) $T = 1200$ N ◀

$\mathbf{i}$: $\quad 120D_y + 57.6 \times 10^3 = 0 \qquad D_y = -480$ N

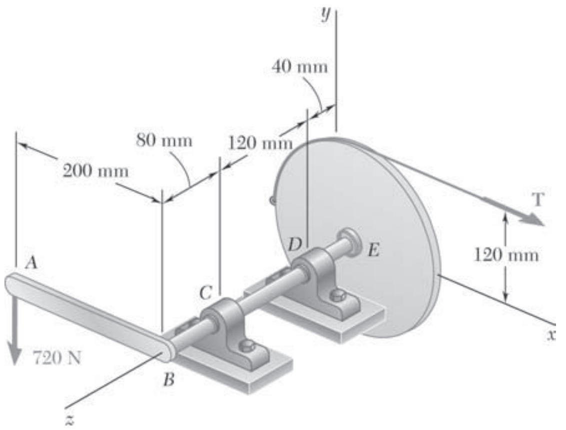
$\mathbf{j}$: $\quad -120D_x - 160(1200\text{ N}) = 0 \qquad D_x = -1600$ N

$\Sigma F_x = 0$: $\quad C_x + D_x + T = 0 \qquad C_x = 1600 - 1200 = 400$ N

$\Sigma F_y = 0$: $\quad C_y + D_y - 720 = 0 \qquad C_y = 480 + 720 = 1200$ N

$\Sigma F_z = 0$: $\quad C_z = 0$

(*b*) $\quad \mathbf{C} = (400\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j} \quad \mathbf{D} = -(1600\text{ N})\mathbf{i} - (480\text{ N})\mathbf{j}$ ◀



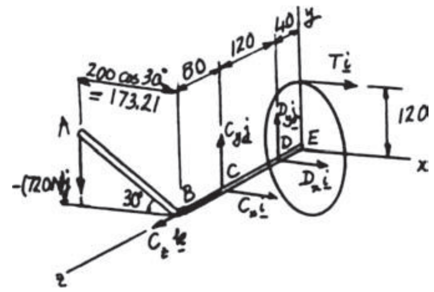
PROBLEM 4.96

Solve Problem 4.95, assuming that the axle has been rotated clockwise in its bearings by 30° and that the 720-N load remains vertical.

PROBLEM 4.95 A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six Eqs. of equilibrium.

$$\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k})\times(D_x\mathbf{i}+D_y\mathbf{j})+(120\mathbf{j}-160\mathbf{k})\times T\mathbf{i}+(80\mathbf{k}-173.21\mathbf{i})\times(-720\mathbf{j})=0$$

$$-120D_x\mathbf{j}+120D_y\mathbf{i}-120T\,\mathbf{k}-160T\mathbf{j}+57.6\times10^3\,\mathbf{i}+124.71\times10^3\,\mathbf{k}=0$$

Equating to zero the coefficients of the unit vectors:

$$\mathbf{k}: \quad -120T+124.71\times10^3=0 \qquad T=1039.2\text{ N} \qquad T=1039\text{ N} \blacktriangleleft$$

$$\mathbf{i}: \quad 120D_y+57.6\times10^3=0 \qquad D_y=-480\text{ N}$$

$$\mathbf{j}: \quad -120D_x-160(1039.2) \qquad D_x=-1385.6\text{ N}$$

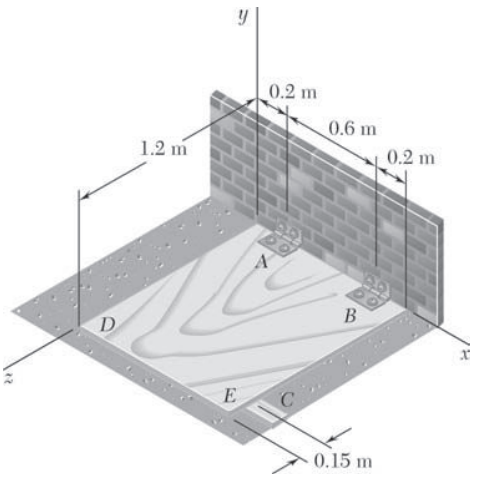
$$\Sigma F_x=0: \quad C_x+D_x+T=0 \qquad C_x=1385.6-1039.2=346.4$$

$$\Sigma F_y=0: \quad C_y+D_y-720=0 \qquad C_y=480+720=1200\text{ N}$$

$$\Sigma F_z=0: \quad C_z=0$$

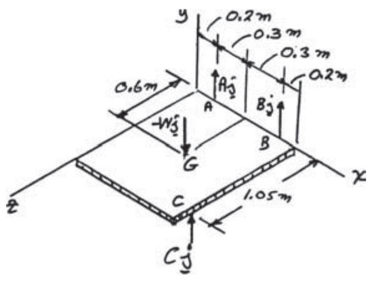
(*b*) $\mathbf{C}=(346\text{ N})\mathbf{i}+(1200\text{ N})\mathbf{j} \quad \mathbf{D}=-(1386\text{ N})\mathbf{i}-(480\text{ N})\mathbf{j} \blacktriangleleft$

PROBLEM 4.97



An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C. Determine the vertical component of the reaction (a) at A, (b) at B, (c) at C.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.8\mathbf{i} + 1.05\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18\text{ kg})9.81$$
$$W = 176.58\text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(0.6\mathbf{i}) \times B\mathbf{j} + (0.8\mathbf{i} + 1.05\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.8C\mathbf{k} - 1.05C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors of zero:

$$\mathbf{i}: \quad 1.05C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.05}\right)176.58\text{ N} = 100.90\text{ N}$$

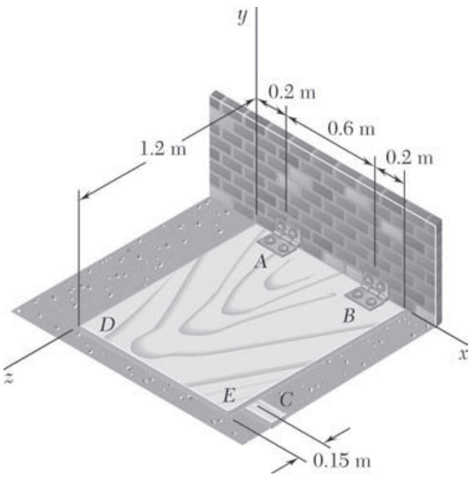
$$\mathbf{k}: \quad 0.6B + 0.8C - 0.3W = 0$$

$$0.6B + 0.8(100.90\text{ N}) - 0.3(176.58\text{ N}) = 0 \quad B = -46.24\text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 46.24\text{ N} + 100.90\text{ N} + 176.58\text{ N} = 0 \quad A = 121.92\text{ N}$$

(a) $A = 121.9$ N (b) $B = -46.2$ N (c) $C = 100.9$ N ◀

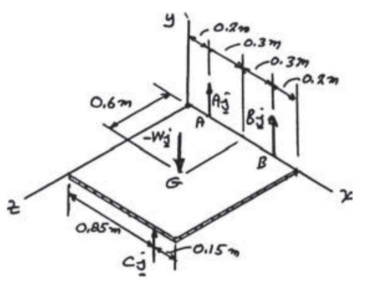


PROBLEM 4.98

Solve Problem 4.97, assuming that the small block C is moved and placed under edge DE at a point 0.15 m from corner E.

PROBLEM 4.97 An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C. Determine the vertical component of the reaction (*a*) at A, (*b*) at B, (*c*) at C.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.65\mathbf{i} + 1.2\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18\text{ kg})\,9.81\text{ m/s}^2$$
$$W = 176.58\text{ N}$$

$$\Sigma M_A = 0\text{:}\quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$0.6\mathbf{i} \times B\mathbf{j} + (0.65\mathbf{i} + 1.2\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.65C\mathbf{k} - 1.2C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}\text{:}\quad -1.2C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.2}\right)176.58\text{ N} = 88.29\text{ N}$$

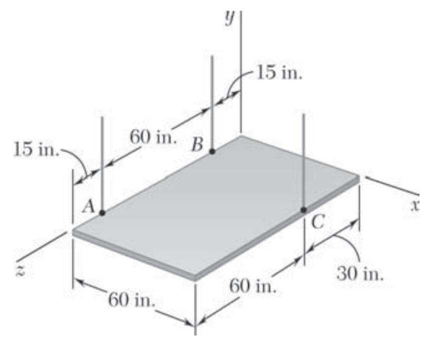
$$\mathbf{k}\text{:}\quad 0.6B + 0.65C - 0.3W = 0$$

$$0.6B + 0.65(88.29\text{ N}) - 0.3(176.58\text{ N}) = 0 \quad B = -7.36\text{ N}$$

$$\Sigma F_y = 0\text{:}\quad A + B + C - W = 0$$

$$A - 7.36\text{ N} + 88.29\text{ N} - 176.58\text{ N} = 0 \qquad A = 95.648\text{ N}$$

(*a*) $A = 95.6$ N (*b*) -7.36 N (*c*) 88.3 N ◀

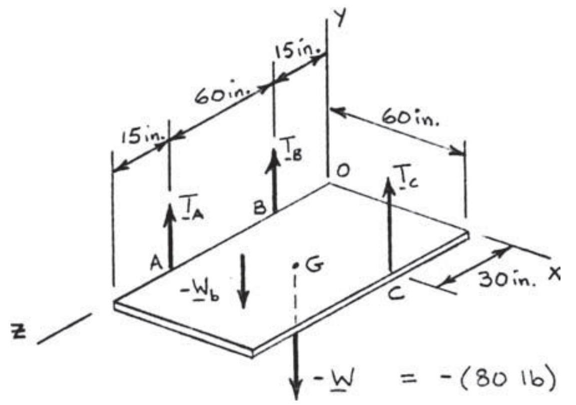


PROBLEM 4.99

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the tension in each wire.

SOLUTION

Free-Body Diagram:



$$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{A/B} \times T_A \mathbf{j} + \mathbf{r}_{C/B} \times T_C \mathbf{j} + r_{G/B} \times (-80 \text{ lb})\mathbf{j} = 0$$

$$(60 \text{ in.})\mathbf{k} \times T_A \mathbf{j} + [(60 \text{ in.})\mathbf{i} + (15 \text{ in.})\mathbf{k}] \times T_C \mathbf{j} + [(30 \text{ in.})\mathbf{i} + (30 \text{ in.})\mathbf{k}] \times (-80 \text{ lb})\mathbf{j} = 0$$

$$-60T_A \mathbf{i} + 60T_C \mathbf{k} - 15T_C \mathbf{i} - 2400\mathbf{k} + 2400\mathbf{i} = 0$$

Equating to zero the coefficients of the unit vectors:

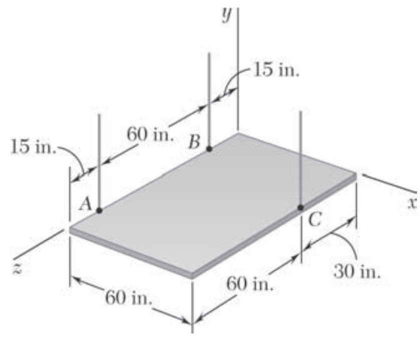
$$\mathbf{i}: \quad 60T_A - 15(40) + 2400 = 0 \qquad T_A = 30.0 \text{ lb} \blacktriangleleft$$

$$\mathbf{k}: \quad 60T_C - 2400 = 0 \qquad T_C = 40.0 \text{ lb} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 80 \text{ lb} = 0$$

$$30 \text{ lb} + T_B + 40 \text{ lb} - 80 \text{ lb} = 0 \qquad T_B = 10.00 \text{ lb} \blacktriangleleft$$

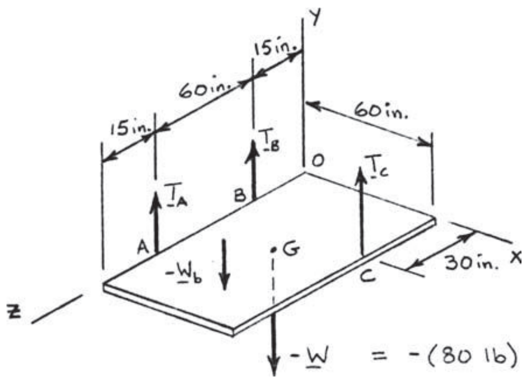
PROBLEM 4.100



The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the weight and location of the lightest block that should be placed on the plate if the tensions in the three wires are to be equal.

SOLUTION

Free-Body Diagram:



Let $-W_b\mathbf{j}$ be the weight of the block and x and z the block's coordinates.

Since tensions in wires are equal, let

$$T_A = T_B = T_C = T$$

$$\Sigma M_0 = 0: \quad (\mathbf{r}_A \times T\mathbf{j}) + (\mathbf{r}_B \times T\mathbf{j}) + (\mathbf{r}_C \times T\mathbf{j}) + \mathbf{r}_G \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or, $$(75\,\mathbf{k}) \times T\mathbf{j} + (15\,\mathbf{k}) \times T\mathbf{j} + (60\mathbf{i} + 30\mathbf{k}) \times T\mathbf{j} + (30\mathbf{i} + 45\mathbf{k}) \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or, $$-75T\mathbf{i} - 15T\mathbf{i} + 60T\mathbf{k} - 30T\mathbf{i} - 30W\mathbf{k} + 45W\mathbf{i} - W_b \times \mathbf{k} + W_b z\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -120T + 45W + W_b z = 0 \tag{1}$$

$$\mathbf{k}: \quad 60T - 30W - W_b x = 0 \tag{2}$$

Also, $$\Sigma F_y = 0: \quad 3T - W - W_b = 0 \tag{3}$$

Eq. (1) + 40 Eq. (3): $$5W + (z - 40)W_b = 0 \tag{4}$$

Eq. (2) – 20 Eq. (3): $$-10W - (x - 20)W_b = 0 \tag{5}$$

PROBLEM 4.100 (Continued)

Solving (4) and (5) for W_b/W and recalling of $0 \le x \le 60$ in., $0 \le z \le 90$ in.,

(4): $$\frac{W_b}{W} = \frac{5}{40 - z} \ge \frac{5}{40 - 0} = 0.125$$

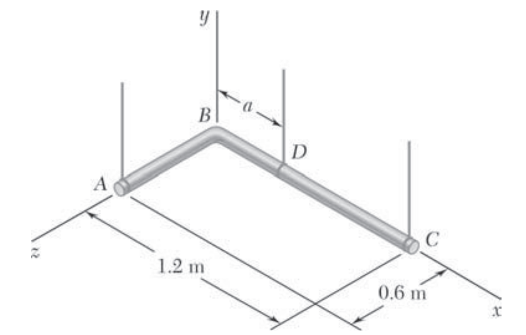
(5): $$\frac{W_b}{W} = \frac{10}{20 - x} \ge \frac{10}{20 - 0} = 0.5$$

Thus, $(W_b)_{\min} = 0.5W = 0.5(80) = 40$ lb $\qquad (W_b)_{\min} = 40.0 \text{ lb}$ ◀

Making $W_b = 0.5W$ in (4) and (5):

$$5W + (z - 40)(0.5W) = 0 \qquad z = 30.0 \text{ in.} \blacktriangleleft$$

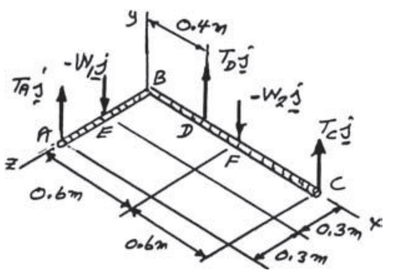
$$-10W - (x - 20)(0.5W) = 0 \qquad x = 0 \text{ in.} \blacktriangleleft$$



PROBLEM 4.101

Two steel pipes *AB* and *BC*, each having a mass per unit length of 8 kg/m, are welded together at *B* and supported by three wires. Knowing that $a = 0.4$ m, determine the tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-0.4\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-0.4\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + 0.2\mathbf{i} \times (-W_2\mathbf{j}) + 0.8\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-0.4T_A\mathbf{k} - 0.6T_A\mathbf{i} + 0.4W_1\mathbf{k} + 0.3W_1\mathbf{i} - 0.2W_2\mathbf{k} + 0.8T_C\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -0.4T_A + 0.4W_1 - 0.2W_2 + 0.8T_C = 0$$

$$-0.4(0.3m'g) + 0.4(0.6m'g) - 0.2(1.2m'g) + 0.8T_C = 0$$

$$T_C = \frac{(0.12 - 0.24 - 0.24)m'g}{0.8} = 0.15m'g$$

$$\Sigma F_y = 0: \quad T_A + T_C + T_D - W_1 - W_2 = 0$$

$$0.3m'g + 0.15m'g + T_D - 0.6m'g - 1.2m'g = 0$$

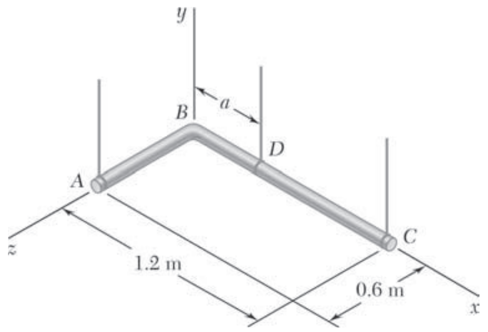
$$T_D = 1.35m'g$$

$$m'g = (8 \text{ kg/m})(9.81 \text{m/s}^2) = 78.48 \text{ N/m}$$

$T_A = 0.3m'g = 0.3 \times 78.45$ $\qquad T_A = 23.5$ N ◀

$T_B = 0.15m'g = 0.15 \times 78.45$ $\qquad T_B = 11.77$ N ◀

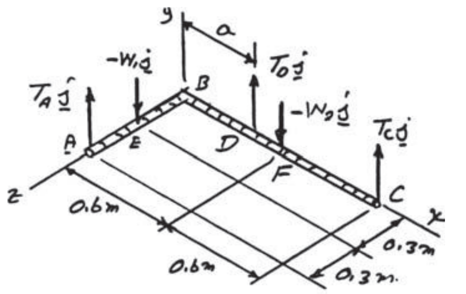
$T_C = 1.35m'g = 1.35 \times 78.45$ $\qquad T_C = 105.9$ N ◀



PROBLEM 4.102

For the pipe assembly of Problem 4.101, determine (*a*) the largest permissible value of *a* if the assembly is not to tip, (*b*) the corresponding tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-a\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-a\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + (0.6 - a)\mathbf{i} \times (-W_2\mathbf{j}) + (1.2 - a)\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-T_A a\mathbf{k} - 0.6T_A\mathbf{i} + W_1 a\mathbf{k} + 0.3W_1\mathbf{i} - W_2(0.6 - a)\mathbf{k} + T_C(1.2 - a)\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -T_A a + W_1 a - W_2(0.6 - a) + T_C(1.2 - a) = 0$$

$$-0.3m'ga + 0.6m'ga - 1.2m'g(0.6 - a) + T_C(1.2 - a) = 0$$

$$T_C = \frac{0.3a - 0.6a + 1.2(0.6 - a)}{1.2 - a} \qquad \text{For Max } a \text{ and no tipping, } T_C = 0$$

(*a*)

$$-0.3a + 1.2(0.6 - a) = 0$$
$$-0.3a + 0.72 - 1.2a = 0$$
$$1.5a = 0.72$$

$a = 0.480$ m ◀

PROBLEM 4.102 (Continued)

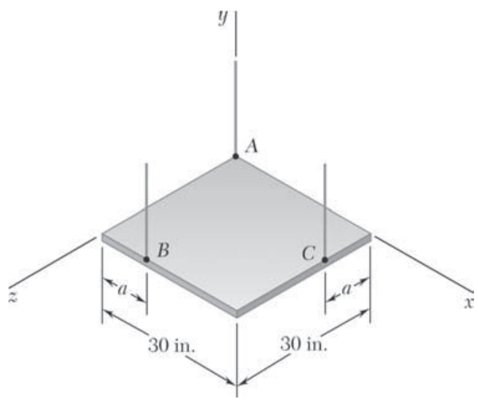
(*b*) Reactions: $m'g = (8 \text{ kg/m})\, 9.81 \text{ m/s}^2 = 78.48 \text{ N/m}$

$$T_A = 0.3m'g = 0.3 \times 78.48 = 23.544 \text{ N} \qquad T_A = 23.5 \text{ N} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_C + T_D - W_1 - W_2 = 0$$

$$T_A + 0 + T_D - 0.6m'g - 1.2m'g = 0$$

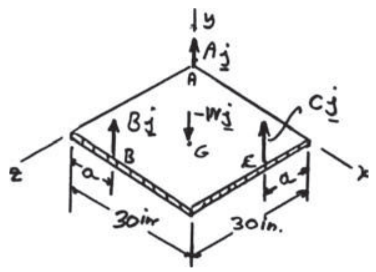
$$T_D = 1.8m'g - T_A = 1.8 \times 78.48 - 23.544 = 117.72 \qquad T_D = 117.7 \text{ N} \blacktriangleleft$$



PROBLEM 4.103

The 24-lb square plate shown is supported by three vertical wires. Determine (*a*) the tension in each wire when $a = 10$ in., (*b*) the value of *a* for which the tension in each wire is 8 lb.

SOLUTION



$$\mathbf{r}_{B/A} = a\mathbf{i} + 30\mathbf{k}$$
$$\mathbf{r}_{C/A} = 30\mathbf{i} + a\mathbf{k}$$
$$\mathbf{r}_{G/A} = 15\mathbf{i} + 15\mathbf{k}$$

By symmetry: $B = C$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_C \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(a\mathbf{i} + 30\mathbf{k}) \times B\mathbf{j} + (30\mathbf{i} + a\mathbf{k}) \times B\mathbf{j} + (15\mathbf{i} + 15\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$Ba\mathbf{k} - 30B\mathbf{i} + 30B\mathbf{k} - Ba\mathbf{i} - 15W\mathbf{k} + 15W\mathbf{i} = 0$$

Equate coefficient of unit vector *i* to zero:

$$\mathbf{i}: \quad -30B - Ba + 15W = 0$$

$$B = \frac{15W}{30 + a} \qquad C = B = \frac{15W}{30 + a} \tag{1}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A + 2\left[\frac{15W}{30 + a}\right] - W = 0; \quad A = \frac{aW}{30 + a} \tag{2}$$

(*a*) For $\quad a = 10$ in.

Eq. (1) $\quad C = B = \dfrac{15(24 \text{ lb})}{30 + 10} = 9.00 \text{ lb}$

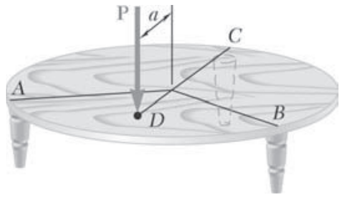
Eq. (2) $\quad A = \dfrac{10(24 \text{ lb})}{30 + 10} = 6.00 \text{ lb}$ $\qquad A = 6.00 \text{ lb} \quad B = C = 9.00 \text{ lb}$ ◀

PROBLEM 4.103 (Continued)

(*b*) For tension in each wire = 8 lb

Eq. (1) $$8 \text{ lb} = \frac{15(24 \text{ lb})}{30 + a}$$

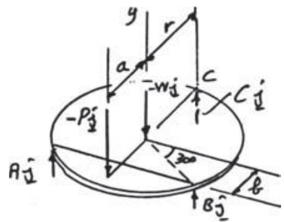
$$30 \text{ in.} + a = 45 \qquad a = 15.00 \text{ in.} \blacktriangleleft$$



PROBLEM 4.104

The table shown weighs 30 lb and has a diameter of 4 ft. It is supported by three legs equally spaced around the edge. A vertical load **P** of magnitude 100 lb is applied to the top of the table at D. Determine the maximum value of a if the table is not to tip over. Show, on a sketch, the area of the table over which **P** can act without tipping the table.

SOLUTION



$$r = 2 \text{ ft} \quad b = r\sin 30° = 1 \text{ ft}$$

We shall sum moments about AB.

$$(b+r)C + (a-b)P - bW = 0$$

$$(1+2)C + (a-1)100 - (1)30 = 0$$

$$C = \frac{1}{3}[30 - (a-1)100]$$

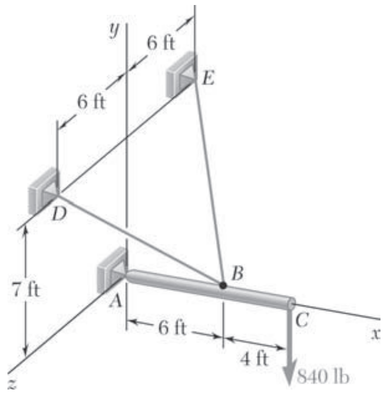
If table is not to tip, $C \geq 0$

$$[30 - (a-1)100] \geq 0$$

$$30 \geq (a-1)100$$

$$a - 1 \leq 0.3 \quad a \leq 1.3 \text{ ft} \quad a = 1.300 \text{ ft}$$

Only $\perp$ distance from P to AB matters. Same condition must be satisfied for each leg. P must be located in shaded area for no tipping



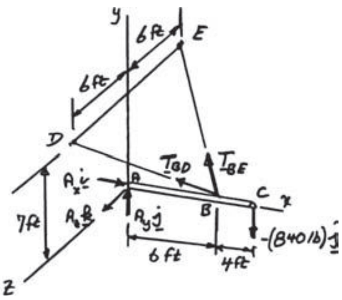
PROBLEM 4.105

A 10-ft boom is acted upon by the 840-lb force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at *A*.

SOLUTION

We have five unknowns and six Eqs. of equilibrium but equilibrium is maintained $(\Sigma M_x = 0)$.

Free-Body Diagram:



$$\overrightarrow{BD} = (-6 \text{ ft})\mathbf{i} + (7 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k} \quad BD = 11 \text{ ft}$$

$$BE = (-6 \text{ ft})\mathbf{i} + (7 \text{ ft})\mathbf{j} - (6 \text{ ft})\mathbf{k} \quad BE = 11 \text{ ft}$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times T_{BD} + \mathbf{r}_B \times T_{BE} + \mathbf{r}_C \times (-840\mathbf{j}) = 0$$

$$6\mathbf{i} \times \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}) + 6\mathbf{i} \times \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (-840\mathbf{j}) = 0$$

$$\frac{42}{11}T_{BD}\mathbf{k} - \frac{36}{11}T_{BD}\mathbf{j} + \frac{42}{11}T_{BE}\mathbf{k} + \frac{36}{11}T_{BE}\mathbf{j} - 8400\mathbf{k}$$

Equate coefficients of unit vectors to zero.

$$\mathbf{i}: \quad -\frac{36}{11}T_{BD} + \frac{36}{11}T_{BE} = 0 \quad T_{BE} = T_{BD}$$

$$\mathbf{k}: \quad \frac{42}{11}T_{BD} + \frac{42}{11}T_{BE} - 8400 = 0$$

$$2\left(\frac{42}{11}T_{BD}\right) = 8400 \qquad T_{BD} = 1100 \text{ lb} \blacktriangleleft$$

$$T_{BE} = 1100 \text{ lb} \blacktriangleleft$$

PROBLEM 4.105 (Continued)

$$\Sigma F_x = 0: \quad A_x - \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

$$A_x = 1200 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y + \frac{7}{11}(1100 \text{ lb}) + \frac{7}{11}(1100 \text{ lb}) - 840 \text{ lb} = 0$$

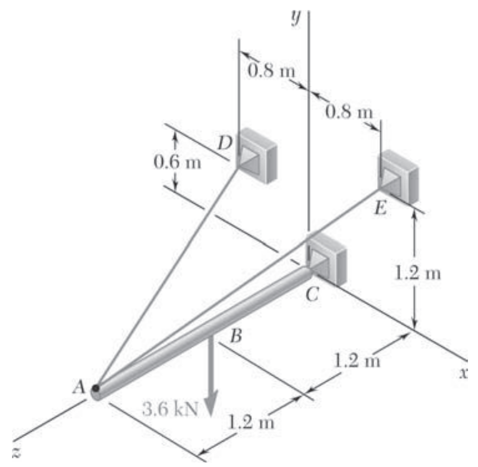
$$A_y = -560 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

$$A_z = 0$$

$$\mathbf{A} = (1200 \text{ lb})\mathbf{i} - (560 \text{ lb})\mathbf{j} \blacktriangleleft$$

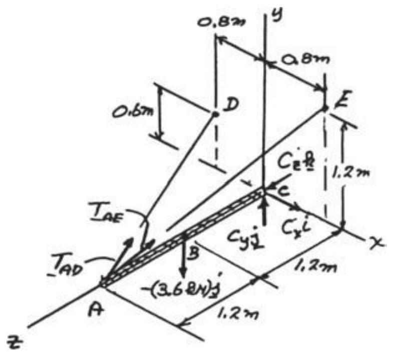
PROBLEM 4.106



A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five Unknowns and six Eqs. of equilibrium. Equilibrium is maintained $(\Sigma M_{AC} = 0)$.



$$\mathbf{r}_B = 1.2\mathbf{k}$$

$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$

$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero.

$$\mathbf{i}: \quad -0.55385\, T_{AD} - 1.02857\, T_{AE} + 4.32 = 0 \qquad (1)$$

$$\mathbf{j}: \quad -0.73846\, T_{AD} + 0.68671\, T_{AE} = 0$$

$$T_{AD} = 0.92857\, T_{AE} \qquad (2)$$

Eq. (1):

$$-0.55385(0.92857)\, T_{AE} - 1.02857\, T_{AE} + 4.32 = 0$$

$$1.54286\, T_{AE} = 4.32$$

$$T_{AE} = 2.800 \text{ kN} \qquad T_{AE} = 2.80 \text{ kN} \blacktriangleleft$$

PROBLEM 4.106 (Continued)

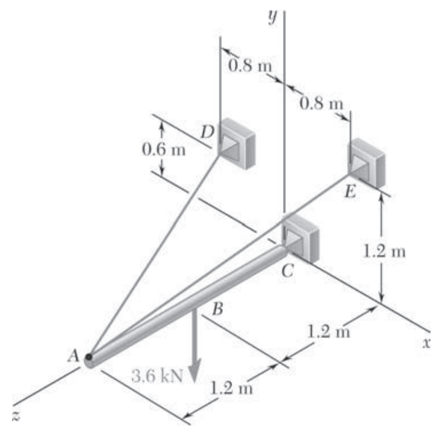
Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600 \text{ kN}$ $\quad T_{AD} = 2.60 \text{ kN}$ ◀

$$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(2.6 \text{ kN}) + \frac{0.8}{2.8}(2.8 \text{ kN}) = 0 \qquad C_x = 0$$

$$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(2.6 \text{ kN}) + \frac{1.2}{2.8}(2.8 \text{ kN}) - (3.6 \text{ kN}) = 0 \qquad C_y = 1.800 \text{ kN}$$

$$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(2.6 \text{ kN}) - \frac{2.4}{2.8}(2.8 \text{ kN}) = 0 \qquad C_z = 4.80 \text{ kN}$$

$\mathbf{C} = (1.800 \text{ kN})\mathbf{j} + (4.80 \text{ kN})\mathbf{k}$ ◀



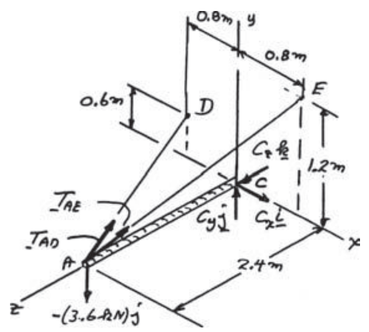
PROBLEM 4.107

Solve Problem 4.106, assuming that the 3.6-kN load is applied at Point *A*.

PROBLEM 4.106 A 2.4-m boom is held by a ball-and-socket joint at *C* and by two cables *AD* and *AE*. Determine the tension in each cable and the reaction at *C*.

SOLUTION

Free-Body Diagram: Five unknowns and six Eqs. of equilibrium. Equilibrium is maintained $(\Sigma M_{AC} = 0)$.



$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$

$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_A \times (-3.6 \text{ kN})\mathbf{j}$$

Factor r_A: $\qquad \mathbf{r}_A \times (\mathbf{T}_{AD} + \mathbf{T}_{AE} - (3.6 \text{ kN})\mathbf{j})$

or: $\qquad \mathbf{T}_{AD} + \mathbf{T}_{AE} - (3 \text{ kN})\mathbf{j} = 0 \quad$ (Forces concurrent at *A*)

<u>Coefficient of **i**</u>:

$$-\frac{T_{AD}}{2.6}(0.8) + \frac{T_{AE}}{2.8}(0.8) = 0$$

$$T_{AD} = \frac{2.6}{2.8}T_{AE} \tag{1}$$

<u>Coefficient of **j**</u>:

$$\frac{T_{AD}}{2.6}(0.6) + \frac{T_{AE}}{2.8}(1.2) - 3.6 \text{ kN} = 0$$

$$\frac{2.6}{2.8}T_{AE}\left(\frac{0.6}{2.6}\right) + \frac{1.2}{2.8}T_{AE} - 3.6 \text{ kN} = 0$$

$$T_{AE}\left(\frac{0.6 + 1.2}{2.8}\right) = 3.6 \text{ kN}$$

$$T_{AE} = 5.600 \text{ kN} \qquad\qquad T_{AE} = 5.60 \text{ kN} \blacktriangleleft$$

PROBLEM 4.107 (Continued)

Eq. (1): $T_{AD} = \dfrac{2.6}{2.8}(5.6) = 5.200 \text{ kN}$ $\qquad T_{AD} = 5.20 \text{ kN}$ ◀

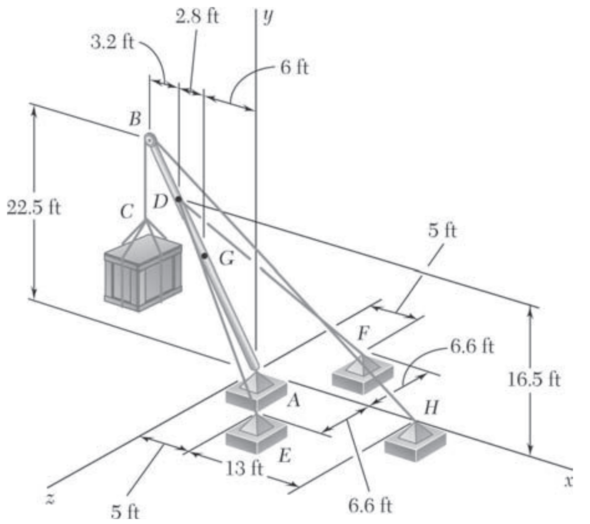
$$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(5.2 \text{ kN}) + \frac{0.8}{2.8}(5.6 \text{ kN}) = 0; \qquad C_x = 0$$

$$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(5.2 \text{ kN}) + \frac{1.2}{2.8}(5.6 \text{ kN}) - 3.6 \text{ kN} = 0 \qquad C_y = 0$$

$$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(5.2 \text{ kN}) - \frac{2.4}{2.8}(5.6 \text{ kN}) = 0 \qquad C_z = 9.60 \text{ kN}$$

$\mathbf{C} = (9.60 \text{ kN})\mathbf{k}$ ◀

Note: Since forces and reaction are concurrent at A, we could have used the methods of Chapter 2.

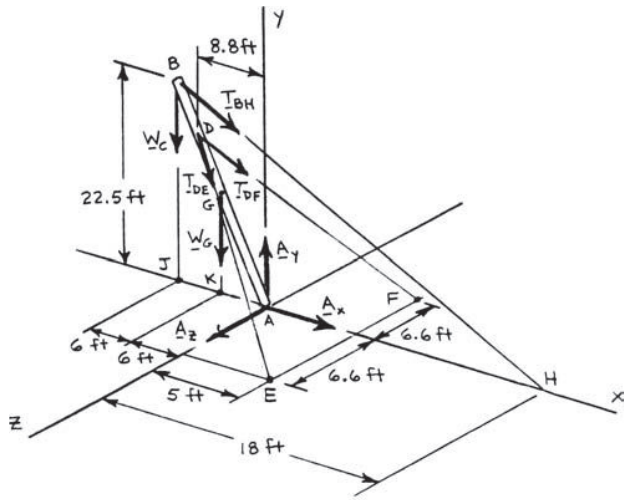


PROBLEM 4.108

A 600-lb crate hangs from a cable that passes over a pulley B and is attached to a support at H. The 200-lb boom AB is supported by a ball-and-socket joint at A and by two cables DE and DF. The center of gravity of the boom is located at G. Determine (a) the tension in cables DE and DF, (b) the reaction at A.

SOLUTION

Free-Body Diagram:



$$W_C = 600 \text{ lb}$$
$$W_G = 200 \text{ lb}$$

We have five unknowns $(T_{DE}, T_{DF}, A_x, A_y, A_z)$ and five equilibrium equations. The boom is free to spin about the AB axis, but equilibrium is maintained, since $\Sigma M_{AB} = 0$.

We have $\quad \overrightarrow{BH} = (30 \text{ ft})\mathbf{i} - (22.5 \text{ ft})\mathbf{j} \qquad BH = 37.5 \text{ ft}$

$$\overrightarrow{DE} = (13.8 \text{ ft})\mathbf{i} - \frac{8.8}{12}(22.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k}$$

$$= (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k} \qquad DE = 22.5 \text{ ft}$$

$$\overrightarrow{DF} = (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} - (6.6 \text{ ft})\mathbf{k} \qquad DF = 22.5 \text{ ft}$$

Thus:
$$\mathbf{T}_{BH} = \mathbf{T}_{BH}\frac{\overrightarrow{BH}}{BH} = (600\text{ lb})\frac{30\mathbf{i} - 22.5\mathbf{j}}{37.5} = (480\text{ lb})\mathbf{i} - (360\text{ lb})\mathbf{j}$$

$$\mathbf{T}_{DE} = \mathbf{T}_{DE}\frac{\overrightarrow{DE}}{DE} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} + 6.6\mathbf{k})$$

$$\mathbf{T}_{DF} = \mathbf{T}_{DF}\frac{\overrightarrow{DF}}{DF} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} - 6.6\mathbf{k})$$

(*a*) $\Sigma\mathbf{M}_A = 0$: $(\mathbf{r}_J \times \mathbf{W}_C) + (\mathbf{r}_K \times \mathbf{W}_G) + (\mathbf{r}_H \times \mathbf{T}_{BH}) + (\mathbf{r}_E \times \mathbf{T}_{DE}) + (\mathbf{r}_F \times \mathbf{T}_{DF}) = 0$

$$-(12\mathbf{i}) \times (-600\mathbf{j}) - (6\mathbf{i}) \times (-200\mathbf{j}) + (18\mathbf{i}) \times (480\mathbf{i} - 360\mathbf{j})$$

$$+\frac{T_{DE}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 6.6 \\ 13.8 & -16.5 & 6.6 \end{vmatrix} + \frac{T_{DF}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & -6.6 \\ 13.8 & -16.5 & -6.6 \end{vmatrix} = 0$$

or,
$$7200\mathbf{k} + 1200\mathbf{k} - 6480\mathbf{k} + 4.84(T_{DE} - T_{DF})\mathbf{i}$$

$$+\frac{58.08}{22.5}(T_{DE} - T_{DF})\mathbf{j} - \frac{82.5}{22.5}(T_{DE} + T_{DF})\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors:

i or **j**: $\quad T_{DE} - T_{DF} = 0 \qquad T_{DE} = T_{DF}$*

k: $\quad 7200 + 1200 - 6480 - \dfrac{82.5}{22.5}(2T_{DE}) = 0 \qquad T_{DE} = 261.82\text{ lb}$

$$T_{DE} = T_{DF} = 262\text{ lb} \blacktriangleleft$$

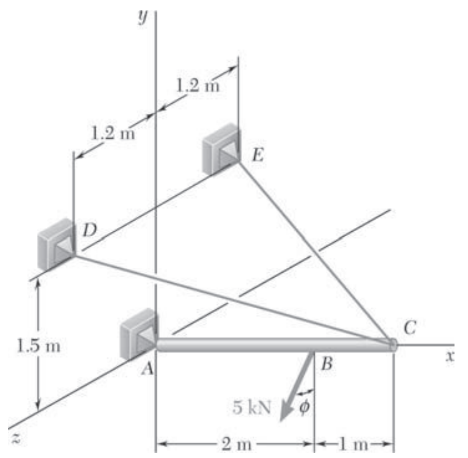
(*b*) $\Sigma F_x = 0$: $\quad A_x + 480 + 2\left(\dfrac{13.8}{22.5}\right)(261.82) = 0 \qquad A_x = -801.17\text{ lb}$

$\Sigma F_y = 0$: $\quad A_y - 600 - 200 - 360 - 2\left(\dfrac{16.5}{22.5}\right)(261.82) = 0 \qquad A_y = 1544.00\text{ lb}$

$\Sigma F_z = 0$: $\quad A_z = 0$

$$\mathbf{A} = -(801\text{ lb})\mathbf{i} + (1544\text{ lb})\mathbf{j} \blacktriangleleft$$

*Remark: The fact that $T_{DE} = T_{DF}$ could have been noted at the outset from the symmetry of structure with respect to *xy* plane.



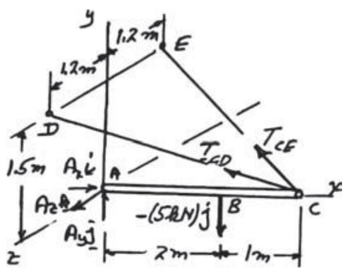
PROBLEM 4.109

A 3-m pole is supported by a ball-and-socket joint at A and by the cables CD and CE. Knowing that the 5-kN force acts vertically downward ($\phi = 0$), determine (*a*) the tension in cables CD and CE, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:

By symmetry with *xy* plane



$$T_{CD} = T_{CE} = T$$

$$\overrightarrow{CD} = -3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k}$$

$$CD = 3.562 \text{ m}$$

$$T_{CD} = T\frac{\overrightarrow{CD}}{CD} = T\frac{-3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k}}{3.562}$$

$$T_{CE} = T\frac{-3\mathbf{i} + 1.5\mathbf{j} - 1.2\mathbf{k}}{3.562}$$

$$\mathbf{r}_{B/A} = 2\mathbf{i} \qquad \mathbf{r}_{C/A} = 3\mathbf{i}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{B/A} \times (-5 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & 1.2 \end{vmatrix} \frac{T}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & -1.2 \end{vmatrix} \frac{T}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & -5 & 0 \end{vmatrix} = 0$$

Coefficient of **k**: $\quad 2\left[3 \times 1.5 \times \frac{T}{3.562}\right] - 10 = 0 \quad T = 3.958 \text{ kN}$

$$\Sigma F = 0: \quad A + T_{CD} + T_{CE} - 5\mathbf{j} = 0$$

PROBLEM 4.109 (Continued)

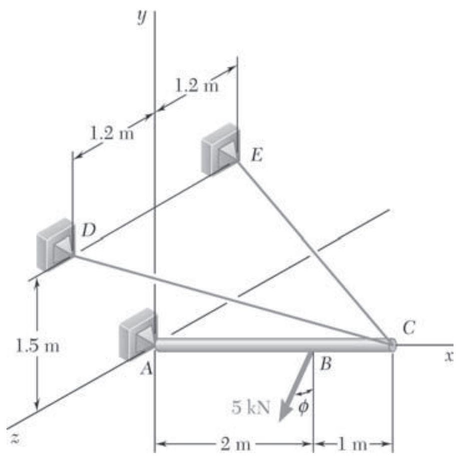
Coefficient of **k**: $A_z = 0$

Coefficient of **i**: $A_x - 2[3.958 \times 3/3.562] = 0$ $A_x = 6.67 \text{ kN}$

Coefficient of **j**: $A_y + 2[3.958 \times 1.5/3.562] - 5 = 0$ $A_y = 1.667 \text{ kN}$

(*a*) $T_{CD} = T_{CE} = 3.96 \text{ kN}$

(*b*) $\mathbf{A} = (6.67 \text{ kN})\mathbf{i} + (1.667 \text{ kN})\mathbf{j}$ ◀

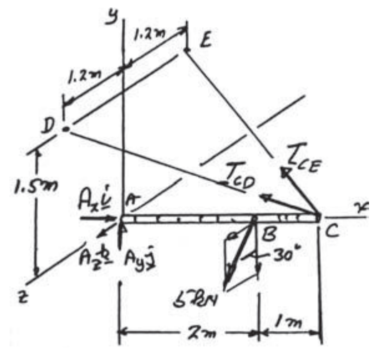


PROBLEM 4.110

A 3-m pole is supported by a ball-and-socket joint at A and by the cables CD and CE. Knowing that the line of action of the 5-kN force forms an angle $\phi = 30°$ with the vertical xy plane, determine (*a*) the tension in cables CD and CE, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium but equilibrium is maintained $(\Sigma M_{AC} = 0)$

$$\mathbf{r}_{B/A} = 2\mathbf{i}$$

$$\mathbf{r}_{C/A} = 3\mathbf{i}$$

Load at B.

$$= -(5\cos 30)\mathbf{j} + (5\sin 30)\mathbf{k}$$

$$= -4.33\mathbf{j} + 2.5\mathbf{k}$$

$$\overrightarrow{CD} = -3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k} \qquad CD = 3.562 \text{ m}$$

$$\mathbf{T}_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = \frac{T}{3.562}(-3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k})$$

Similarly,

$$\mathbf{T}_{CE} = \frac{T}{3.562}(-3\mathbf{i} + 1.5\mathbf{j} - 1.2\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{B/A} \times (-4.33\mathbf{j} + 2.5\mathbf{k}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & 1.2 \end{vmatrix}\frac{T_{CD}}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & -1.2 \end{vmatrix}\frac{T_{CE}}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & -4.33 & 2.5 \end{vmatrix} = 0$$

Equate coefficients of unit vectors to zero.

$$\mathbf{j}: \quad -3.6\frac{T_{CD}}{3.562} + 3.6\frac{T_{CE}}{3.562} - 5 = 0$$

$$-3.6T_{CD} + 3.6T_{CE} - 17.810 = 0 \qquad (1)$$

PROBLEM 4.110 (Continued)

$$\mathbf{k}:\quad 4.5\frac{T_{CD}}{3.562}+4.5\frac{T_{CE}}{3.562}-8.66=0$$

$$4.5T_{CD}+4.5T_{CE}=30.846 \qquad (2)$$

$(2)+1.25(1)$: $\quad 9T_{CE}-53.11=0 \quad T_{CE}=5.901\text{ kN}$

Eq. (1): $\quad -3.6T_{CD}+3.6(5.901)-17.810=0$

$$T_{CD}=0.954\text{ kN}$$

$$\Sigma F=0:\quad \mathbf{A}+\mathbf{T}_{CD}+\mathbf{T}_{CE}-4.33\mathbf{j}+2.5\mathbf{k}=0$$

$$\mathbf{i}:\quad A_x+\frac{0.954}{3.562}(-3)+\frac{5.901}{3.562}(-3)=0$$

$$A_x=5.77\text{ kN}$$

$$\mathbf{j}:\quad A_y+\frac{0.954}{3.562}(1.5)+\frac{5.901}{3.562}(1.5)-4.33=0$$

$$A_y=1.443\text{ kN}$$

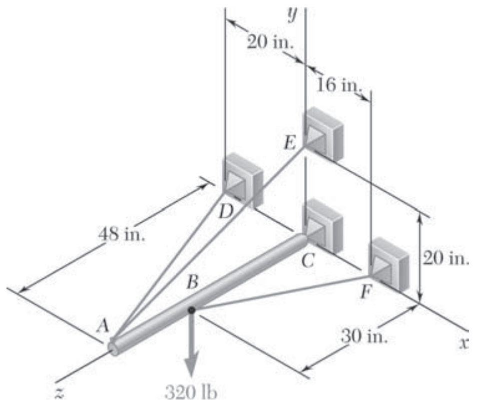
$$\mathbf{k}:\quad A_z+\frac{0.954}{3.562}(1.2)+\frac{5.901}{3.562}(-1.2)+2.5=0$$

$$A_z=-0.833\text{ kN}$$

Answers:

(*a*) $\quad T_{CD}=0.954\text{ kN} \qquad T_{CE}=5.90\text{ kN}$ ◀

(*b*) $\quad \mathbf{A}=(5.77\text{ kN})\mathbf{i}+(1.443\text{ kN})\mathbf{j}-(0.833\text{ kN})\mathbf{k}$ ◀

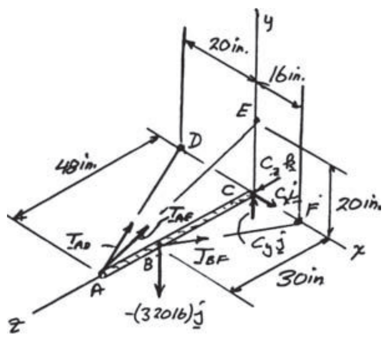


PROBLEM 4.111

A 48-in. boom is held by a ball-and-socket joint at C and by two cables BF and DAE; cable DAE passes around a frictionless pulley at A. For the loading shown, determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium but equilibrium is maintained $(\Sigma M_{AC} = 0)$.

T = Tension in both parts of cable DAE.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma\mathbf{M}_C = 0\text{:} \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_B \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + (30\mathbf{k}) \times (-320\mathbf{j}) = 0$$

Coefficient of **i**: $\quad -\frac{240}{13}T + 9600 = 0 \qquad T = 520 \text{ lb}$

Coefficient of **j**: $-\dfrac{240}{13}T + \dfrac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(520) \quad T_{BD} = 680 \text{ lb}$$

$\Sigma \mathbf{F} = 0$: $\mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + C = 0$

Coefficient of **i**: $-\dfrac{20}{52}(520) + \dfrac{8}{17}(680) + C_x = 0$

$$-200 + 320 + C_x = 0 \qquad C_x = -120 \text{ lb}$$

Coefficient of **j**: $\dfrac{20}{52}(520) - 320 + C_y = 0$

$$200 - 320 + C_y = 0 \qquad C_y = 120 \text{ lb}$$

Coefficient of **k**: $-\dfrac{48}{52}(520) - \dfrac{48}{52}(520) - \dfrac{30}{34}(680) + C_z = 0$

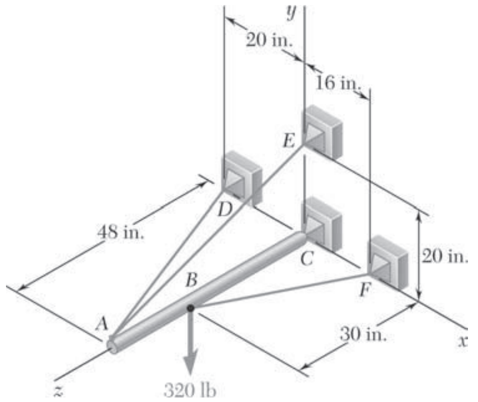
$$-480 - 480 - 600 + C_z = 0$$

$$C_z = 1560 \text{ lb}$$

Answers: $T_{DAE} = T$ $\qquad T_{DAE} = 520 \text{ lb}$ ◀

$T_{BD} = 680 \text{ lb}$ ◀

$\mathbf{C} = -(120.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j} + (1560 \text{ lb})\mathbf{k}$ ◀



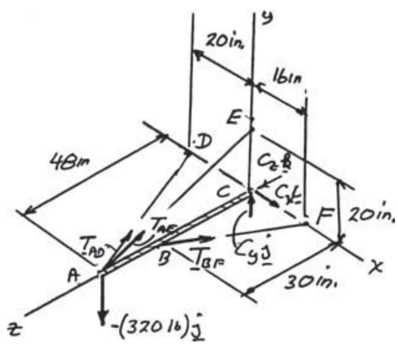
PROBLEM 4.112

Solve Problem 4.111, assuming that the 320-lb load is applied at A.

PROBLEM 4.111 A 48-in. boom is held by a ball-and-socket joint at C and by two cables BF and DAE; cable DAE passes around a frictionless pulley at A. For the loading shown, determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium but equilibrium is maintained ($\Sigma M_{AC} = 0$).

T = tension in both parts of cable DAE.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_A \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix} \frac{T_{BF}}{17} + 48\mathbf{k} \times (-320\mathbf{j}) = 0$$

Coefficient of $\mathbf{i}$: $$-\frac{240}{13}T + 15360 = 0 \qquad T = 832 \text{ lb}$$

PROBLEM 4.112 (Continued)

Coefficient of **j**: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(832) \qquad T_{BD} = 1088 \text{ lb}$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + \mathbf{C} = 0$$

Coefficient of **i**: $-\frac{20}{52}(832) + \frac{8}{17}(1088) + C_x = 0$

$$-320 + 512 + C_x = 0 \qquad C_x = -192 \text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(832) - 320 + C_y = 0$

$$320 - 320 + C_y = 0 \qquad C_y = 0$$

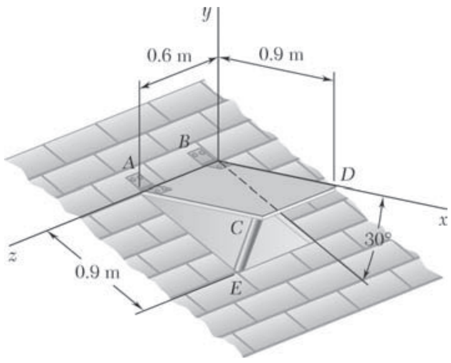
Coefficient of **k**: $-\frac{48}{52}(832) - \frac{48}{52}(852) - \frac{30}{34}(1088) + C_z = 0$

$$-768 - 768 - 960 + C_z = 0 \qquad C_z = 2496 \text{ lb}$$

Answers: $T_{DAE} = T$ $\qquad T_{DAE} = 832 \text{ lb}$ ◀

$T_{BD} = 1088 \text{ lb}$ ◀

$\mathbf{C} = -(192.0 \text{ lb})\mathbf{i} + (2496 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.113

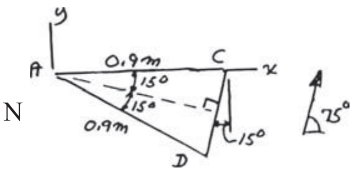
A 20-kg cover for a roof opening is hinged at corners *A* and *B*. The roof forms an angle of 30° with the horizontal, and the cover is maintained in a horizontal position by the brace *CE*. Determine (*a*) the magnitude of the force exerted by the brace, (*b*) the reactions at the hinges. Assume that the hinge at *A* does not exert any axial thrust.

SOLUTION

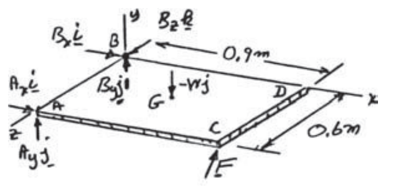
Force exerted by *CD*

$$\mathbf{F} = F(\sin 75°)\mathbf{i} + F(\cos 75°)\mathbf{j}$$

$$\mathbf{F} = F(0.2588\mathbf{i} + 0.9659\mathbf{j})$$



$$W = mg = 20\text{ kg}(9.81\text{ m/s}^2) = 196.2\text{ N}$$



$$\mathbf{r}_{A/B} = 0.6\mathbf{k}$$

$$\mathbf{r}_{C/B} = 0.9\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{r}_{G/B} = 0.45\mathbf{i} + 0.3\mathbf{k}$$

$$\mathbf{F} = F(0.2588\mathbf{i} + 0.9659\mathbf{j})$$

$$\Sigma\mathbf{M}_B = 0: \quad \mathbf{r}_{G/B} \times (-196.2\mathbf{j}) + \mathbf{r}_{C/B} \times \mathbf{F} + \mathbf{r}_{A/B} \times \mathbf{A} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.3 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0.6 \\ 0.2588 & +0.9659 & 0 \end{vmatrix} F + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.6 \\ A_x & A_y & 0 \end{vmatrix} = 0$$

Coefficient of **i** : $+58.86 - 0.5796F - 0.6A_y = 0$ (1)

Coefficient of **j**: $+0.1553F + 0.6A_x = 0$ (2)

Coefficient of **k**: $-88.29 + 0.8693F = 0: \quad F = 101.56\text{ N}$

Eq. (2): $+58.86 - 0.5796(101.56) - 0.6A_y = 0 \quad A_y = 0$

Eq. (3): $+0.1553(101.56) + 0.6A_x = 0 \quad A_x = -26.29\text{ N}$

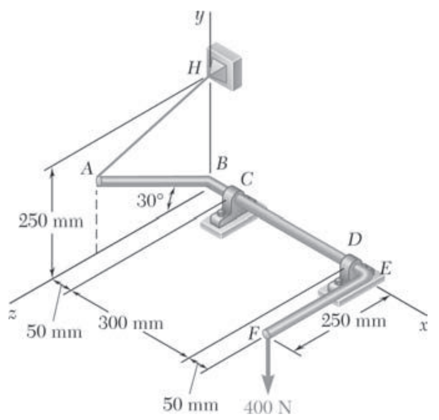
$F = 101.6\text{ N}$ $\quad\quad$ $\mathbf{A} = -(26.3\text{ N})\mathbf{i}$ ◀

$$\Sigma\mathbf{F}: \quad \mathbf{A} + \mathbf{B} + \mathbf{F} - W_{\mathbf{j}} = 0$$

Coefficient of **i**: $26.29 + B_x + 0.2588(101.56) = 0 \quad B_x = 0$

Coefficient of **j**: $B_y + 0.9659(101.56) - 196.2 = 0 \quad B_y = 98.1\text{ N}$

Coefficient of **k**: $B_z = 0$ $\quad\quad$ $\mathbf{B} = (98.1\text{ N})\mathbf{j}$ ◀



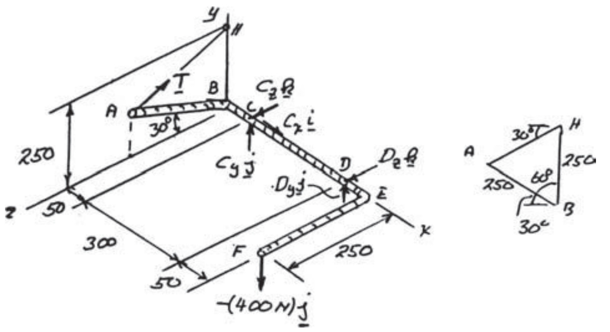
PROBLEM 4.114

The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Free-Body Diagram: ΔABH is equilateral

Dimensions in mm



$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$
$$\mathbf{r}_{D/C} = 300\mathbf{i}$$
$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$
$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma \mathbf{M}_C = 0: \quad \mathbf{r}_{H/C} \times \mathbf{T} + \mathbf{r}_D \times \mathbf{D} + \mathbf{r}_{F/C} \times (-400\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

Coefficient **i**: $\quad -216.5T + 100 \times 10^3 = 0$

$$T = 461.9 \text{ N} \qquad\qquad T = 462 \text{ N} \blacktriangleleft$$

Coefficient of **j**: $\quad -43.3T - 300D_z = 0$

$$-43.3(461.9) - 300D_z = 0 \qquad D_z = -66.67 \text{ N}$$

PROBLEM 4.114 (Continued)

Coefficient of **k**: $-25T + 300D_y - 140\times 10^3 = 0$

$$-25(461.9) + 300D_y - 140\times 10^3 = 0 \qquad D_y = 505.1 \text{ N}$$

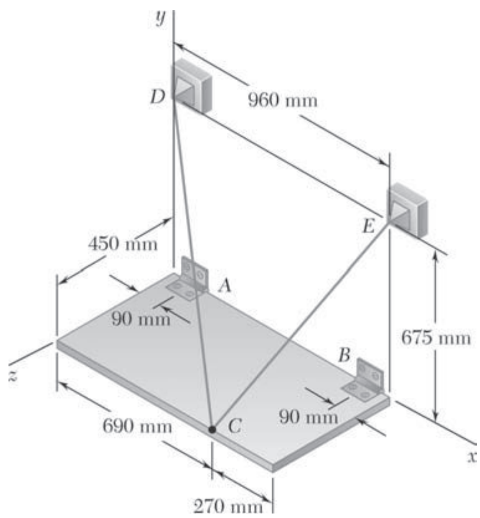
$$\mathbf{D} = (505 \text{ N})\mathbf{j} - (66.7 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{C} + \mathbf{D} + \mathbf{T} - 400\mathbf{j} = 0$$

Coefficient **i**: $C_x = 0$ $\qquad C_x = 0$

Coefficient **j**: $C_y + (461.9)0.5 + 505.1 - 400 = 0$ $\qquad C_y = -336 \text{ N}$

Coefficient **k**: $C_z - (461.9)0.866 - 66.67 = 0$ $\qquad C_z = 467 \text{ N}$ $\qquad \mathbf{C} = -(336 \text{ N})\mathbf{j} + (467 \text{ N})\mathbf{k} \blacktriangleleft$

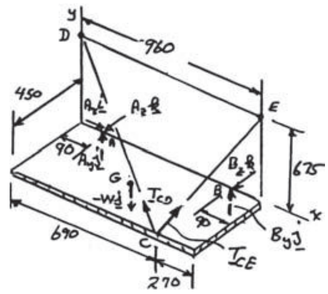


PROBLEM 4.115

A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

Dimensions in mm



$$\mathbf{r}_{B/A}(960-180)\mathbf{i} = 780\mathbf{i}$$

$$\mathbf{r}_{G/A} = \left(\frac{960}{2} - 90\right)\mathbf{i} + \frac{450}{2}\mathbf{k}$$

$$= 390\mathbf{i} + 225\mathbf{k}$$

$$\mathbf{r}_{C/A} = 600\mathbf{i} + 450\mathbf{k}$$

T = Tension in cable DCE

$$\overrightarrow{CD} = -690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CD = 1065 \text{ mm}$$

$$\overrightarrow{CE} = 270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CE = 855 \text{ mm}$$

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ -690 & 675 & -450 \end{vmatrix} \frac{T}{1065} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} +$$

$$+\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

PROBLEM 4.115 (Continued)

Coefficient of **i**: $-(450)(675)\dfrac{T}{1065}-(450)(675)\dfrac{T}{855}+220.725\times 10^3=0$

$T = 344.6$ N $\qquad T = 345$ N ◀

Coefficient of **j**: $(-690\times 450+600\times 450)\dfrac{344.6}{1065}+(270\times 450+600\times 450)\dfrac{344.6}{855}-780B_z=0$

$B_z = 185.49$ N

Coefficient of **k**: $(600)(675)\dfrac{344.6}{1065}+(600)(675)\dfrac{344.6}{855}-382.59\times 10^3+780B_y \quad B_y = 113.2\text{N}$

$\mathbf{B} = (113.2\text{ N})\mathbf{j}+(185.5\text{ N})\mathbf{k}$ ◀

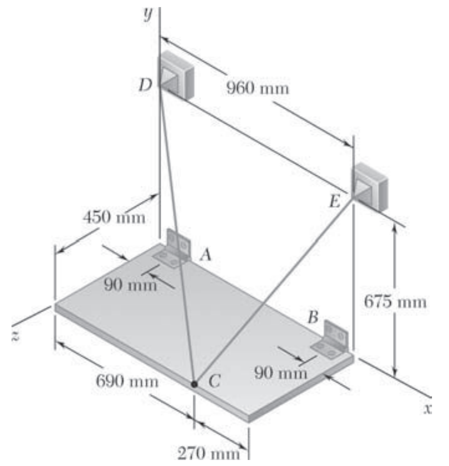
$\Sigma\mathbf{F}=0\text{:}\quad \mathbf{A}+\mathbf{B}+\mathbf{T}_{CD}+\mathbf{T}_{CE}+\mathbf{W}=0$

Coefficient of **i**: $A_x-\dfrac{690}{1065}(344.6)+\dfrac{270}{855}(344.6)=0 \qquad A_x = 114.4\text{ N}$

Coefficient of **j**: $A_y+113.2+\dfrac{675}{1065}(344.6)+\dfrac{675}{855}(344.6)-981=0 \qquad A_y = 377\text{ N}$

Coefficient of **k**: $A_z+185.5-\dfrac{450}{1065}(344.6)-\dfrac{450}{855}(344.6)=0 \qquad A_z = 141.5\text{ N}$

$\mathbf{A} = (114.4\text{ N})\mathbf{i}+(377\text{ N})\mathbf{j}+(144.5\text{ N})\mathbf{k}$ ◀



PROBLEM 4.116

Solve Problem 4.115, assuming that cable *DCE* is replaced by a cable attached to Point *E* and hook *C*.

PROBLEM 4.115 A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

See solution to Problem 4.115 for free-body diagram and analysis leading to the following:

$$CD = 1065 \text{ mm}$$

$$CE = 855 \text{ mm}$$

Now:
$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times T_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**:
$$-(450)(675)\frac{T}{855} + 220.725 \times 10^3 = 0$$

$$T = 621.3 \text{ N} \qquad\qquad T = 621 \text{ N} \blacktriangleleft$$

Coefficient of **j**:
$$(270 \times 450 + 600 \times 450)\frac{621.3}{855} - 980B_z = 0 \quad B_z = 364.7 \text{ N}$$

Coefficient of **k**:
$$(600)(675)\frac{621.3}{855} - 382.59 \times 10^3 + 780B_y = 0 \quad B_y = 113.2 \text{ N}$$

$$\mathbf{B} = (113.2 \text{ N})\mathbf{j} + (365 \text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.116 (Continued)

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T}_{CE} + \mathbf{W} = 0$$

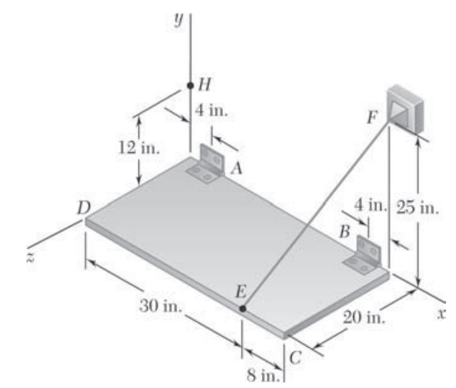
Coefficient of **i**: $A_x + \dfrac{270}{855}(621.3) = 0 \quad A_x = -196.2 \text{ N}$

Coefficient of **j**: $A_y + 113.2 + \dfrac{675}{855}(621.3) - 981 = 0 \quad A_y = 377.3 \text{ N}$

Coefficient of **k**: $A_z + 364.7 - \dfrac{450}{855}(621.3) = 0 \qquad A_z = -37.7 \text{ N}$

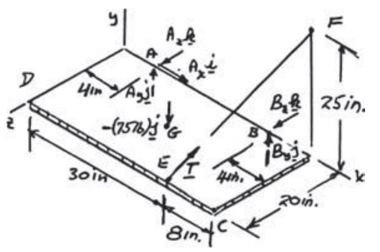
$$\mathbf{A} = -(196.2 \text{ N})\mathbf{i} + (377 \text{ N})\mathbf{j} - (37.7 \text{ N})\mathbf{k} \quad ◀$$

PROBLEM 4.117



The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at A and B.

SOLUTION



$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k}$$

$$= 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k}$$

$$= 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EF} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$EF = 33 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times T + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix}\frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**: $\quad -(25)(20)\dfrac{T}{33} + 750 = 0: \qquad T = 49.5 \text{ lb}$ ◀

Coefficient of **j**: $\quad (160+520)\dfrac{49.5}{33} - 30B_z = 0: \quad B_z = 34 \text{ lb}$

Coefficient of **k**: $\quad (26)(25)\dfrac{49.5}{33} - 1425 + 30B_y = 0: \quad B_y = 15 \text{ lb} \qquad \mathbf{B} = (15 \text{ lb})\mathbf{j} + (34 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.117 (Continued)

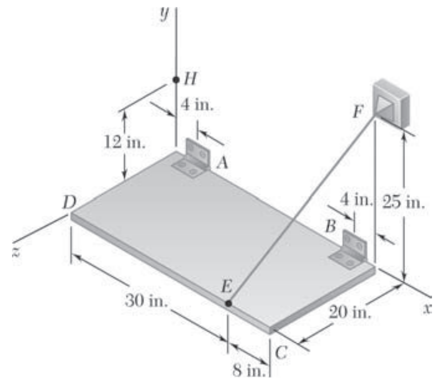
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x + \frac{8}{33}(49.5) = 0 \quad A_x = -12.00 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \frac{25}{33}(49.5) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z + 34 - \frac{20}{33}(49.5) = 0 \quad A_z = -4.00 \text{ lb}$

$$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} - (4.00 \text{ lb})\mathbf{k} \blacktriangleleft$$

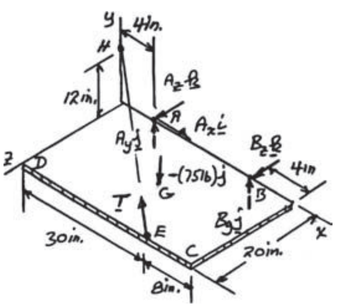


PROBLEM 4.118

Solve Problem 4.117, assuming that cable *EF* is replaced by a cable attached at points *E* and *H*.

PROBLEM 4.117 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at *A* and *B* and by cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION



$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k}$$

$$= 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k}$$

$$= 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EH} = -30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k}$$

$$EH = 38 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{EH}}{EH} = \frac{T}{38}(-30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ -30 & 12 & -20 \end{vmatrix}\frac{T}{38} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**: $-(12)(20)\frac{T}{38} + 750 = 0 \quad T = 118.75$ $\qquad T = 118.8\,\text{lb}$ ◀

Coefficient of **j**: $(-600 + 520)\frac{118.75}{38} - 30B_z = 0 \quad B_z = -8.33\,\text{lb}$

Coefficient of **k**: $(26)(12)\frac{118.75}{38} - 1425 + 30B_y = 0 \quad B_y = 15.00 \text{ lb}$ $\qquad \mathbf{B} = (15.00\,\text{lb})\mathbf{j} - (8.33 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.118 (Continued)

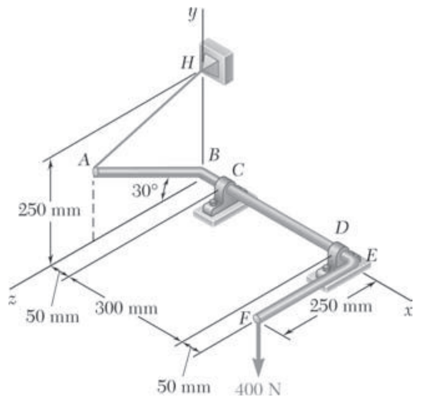
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x - \dfrac{30}{38}(118.75) = 0 \quad A_x = 93.75 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \dfrac{12}{38}(118.75) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z - 8.33 - \dfrac{20}{38}(118.75) = 0 \quad A_z = 70.83 \text{ lb}$

$\mathbf{A} = (93.8 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} + (70.8 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.119

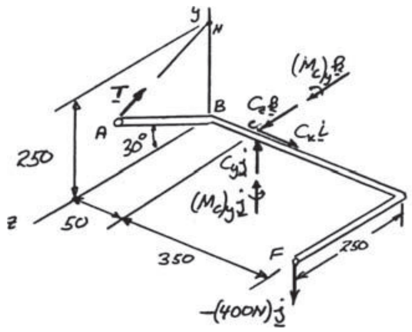
Solve Problem 4.114, assuming that the bearing at *D* is removed and that the bearing at *C* can exert couples about axes parallel to the *y* and *z* axes.

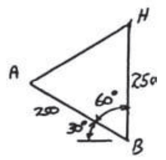
PROBLEM 4.114 The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Free-Body Diagram: ΔABH is Equilateral

Dimensions in mm





$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$
$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$
$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma\mathbf{M}_C = 0: \quad \mathbf{r}_{F/C} \times (-400\mathbf{j}) + \mathbf{r}_{H/C} \times T + (M_C)_y\,\mathbf{j} + (M_C)_z\,\mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + (M_C)_y\,\mathbf{j} + (M_C)_z\,\mathbf{k} = 0$$

Coefficient of **i**: $+100\times 10^3 - 216.5T = 0 \quad T = 461.9 \text{ N}$ $\quad T = 462 \text{ N}$ ◀

Coefficient of **j**: $-43.3(461.9) + (M_C)_y = 0$

$$(M_C)_y = 20\times 10^3 \text{ N}\cdot\text{mm}$$
$$(M_C)_y = 20.0 \text{ N}\cdot\text{m}$$

PROBLEM 4.119 (Continued)

Coefficient of **k**: $-140\times10^3 - 25(461.9) + (M_C)_z = 0$

$$(M_C)_z = 151.54\times10^3 \text{ N}\cdot\text{mm}$$
$$(M_C)_z = 151.5 \text{ N}\cdot\text{m}$$

$$\Sigma F = 0: \quad C + T - 400\mathbf{j} = 0$$

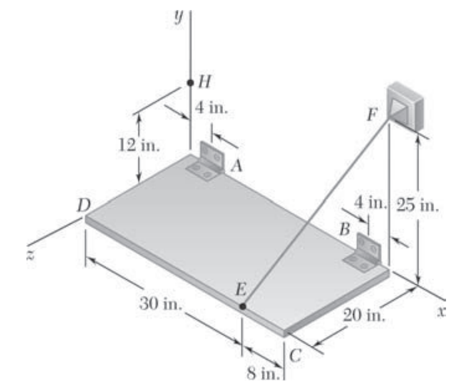
$$\mathbf{M}_C = (20.0 \text{ N}\cdot\text{m})\mathbf{j} + (151.5 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

Coefficient of **i**: $C_x = 0$

Coefficient of **j**: $C_y + 0.5(461.9) - 400 = 0 \quad C_y = 169.1 \text{ N}$

Coefficient of **k**: $C_z - 0.866(461.9) = 0 \quad C_z = 400 \text{ N}$

$$\mathbf{C} = (169.1 \text{ N})\mathbf{j} + (400 \text{ N})\mathbf{k} \blacktriangleleft$$

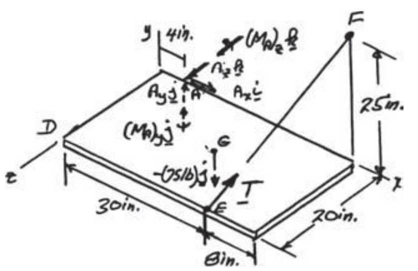


PROBLEM 4.120

Solve Problem 4.117, assuming that the hinge at *B* is removed and that the hinge at *A* can exert couples about axes parallel to the *y* and *z* axes.

PROBLEM 4.117 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at *A* and *B* and by cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION



$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k} = 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = (0.5\times 38)\mathbf{i} + 10\mathbf{k} = 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{AE} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$AE = 33 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A}\times\mathbf{T} + \mathbf{r}_{G/A}\times(-75\mathbf{j}) + (M_A)_y\,\mathbf{j} + (M_A)_z\,\mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix}\frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + (M_A)_y\,\mathbf{j} + (M_A)_z\,\mathbf{k} = 0$$

Coefficient of **i**: $\quad -(20)(25)\dfrac{T}{33} + 750 = 0 \qquad T = 49.5 \text{ lb}$ ◀

Coefficient of **j**: $\quad (160+520)\dfrac{49.5}{33} + (M_A)_y = 0 \quad (M_A)_y = -1020 \text{ lb}\cdot\text{in.}$

Coefficient of **k**: $\quad (26)(25)\dfrac{49.5}{33} - 1425 + (M_A)_z = 0 \quad (M_A)_z = 450 \text{ lb}\cdot\text{in.}$

$$\Sigma F = 0: \quad \mathbf{A} + \mathbf{T} - 75\mathbf{j} = 0$$

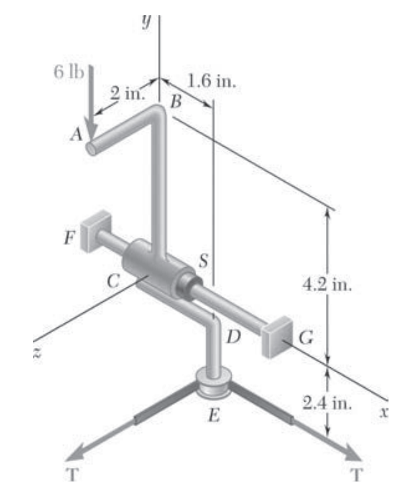
$\mathbf{M}_A = -(1020 \text{ lb}\cdot\text{in})\mathbf{j} + (450 \text{ lb}\cdot\text{in.})\mathbf{k}$ ◀

Coefficient of **i**: $\quad A_x + \dfrac{8}{33}(49.5) = 0 \quad A_x = 12.00 \text{ lb}$

Coefficient of **j**: $\quad A_y + \dfrac{25}{33}(49.5) - 75 = 0 \quad A_y = 37.5 \text{ lb}$

Coefficient of **k**: $\quad A_z - \dfrac{20}{33}(49.5) \qquad A_z = 30.0 \text{ lb}$

$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (37.5 \text{ lb})\mathbf{j} + (30.0 \text{ lb})\mathbf{k}$ ◀

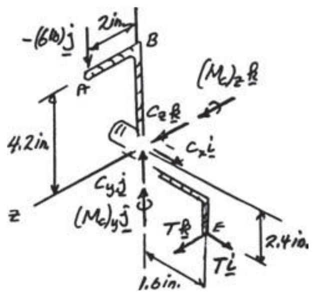


PROBLEM 4.121

The assembly shown is used to control the tension T in a tape that passes around a frictionless spool at E. Collar C is welded to rods ABC and CDE. It can rotate about shaft FG but its motion along the shaft is prevented by a washer S. For the loading shown, determine (*a*) the tension T in the tape, (*b*) the reaction at C.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{A/C} = 4.2\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}_{E/C} = 1.6\mathbf{i} - 2.4\mathbf{j}$$

$$\Sigma M_C = 0: \quad \mathbf{r}_{A/C} \times (-6\mathbf{j}) + \mathbf{r}_{E/C} \times T(\mathbf{i}+\mathbf{k}) + (M_C)_y\,\mathbf{j} + (M_C)_z\,\mathbf{k} = 0$$

$$(4.2\,\mathbf{j} + 2\mathbf{k}) \times (-6\mathbf{j}) + (1.6\mathbf{i} - 2.4\,\mathbf{j}) \times T(\mathbf{i}+\mathbf{k}) + (M_C)_y\,\mathbf{j} + (M_C)_z\,\mathbf{k} = 0$$

Coefficient of **i**: $\quad 12 - 2.4T = 0 \qquad\qquad T = 5 \text{ lb} \blacktriangleleft$

Coefficient of **j**: $\quad -1.6(5\text{ lb}) + (M_C)_y = 0 \quad (M_C)_y = 8\text{ lb}\cdot\text{in.}$

Coefficient of **k**: $\quad 2.4(5\text{ lb}) + (M_C)_z = 0 \quad (M_C)_z = -12\text{ lb}\cdot\text{in.}$

$$\mathbf{M}_C = (8\text{ lb}\cdot\text{in.})\mathbf{j} - (12\text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

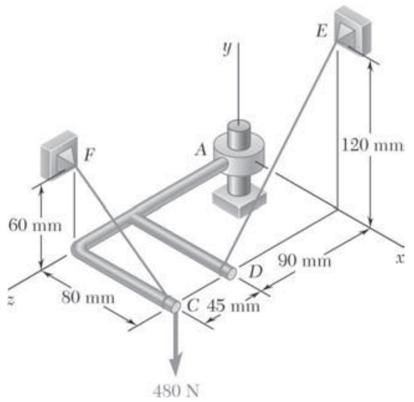
$$\Sigma F = 0: \quad C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} - (6\text{ lb})\mathbf{j} + (5\text{ lb})\mathbf{i} + (5\text{ lb})\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero.

$$C_x = -5\text{ lb} \quad C_y = 6\text{ lb} \quad C_z = -5\text{ lb} \blacktriangleleft$$

$$\mathbf{C} = -(5.00\text{ lb})\mathbf{i} + (6.00\text{ lb})\mathbf{j} - (5.00\text{ lb})\mathbf{k} \blacktriangleleft$$

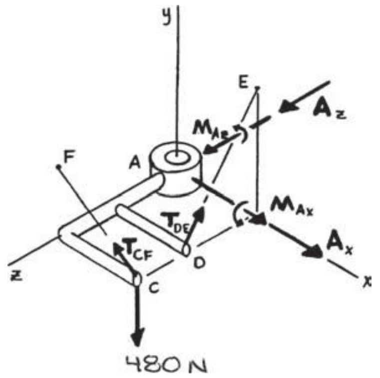
PROBLEM 4.122



The assembly shown is welded to collar A that fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y axis. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION

Free-Body Diagram:



First note

$$\mathbf{T}_{CF} = \boldsymbol{\lambda}_{CF} T_{CF} = \frac{-(0.08\text{ m})\mathbf{i} + (0.06\text{ m})\mathbf{j}}{\sqrt{(0.08)^2 + (0.06)^2}\text{ m}} T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE} = \frac{(0.12\text{ m})\mathbf{j} - (0.09\text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.09)^2}\text{ m}} T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

(*a*) From f.b.d. of assembly

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 480\text{ N} = 0$$

or

$$0.6T_{CF} + 0.8T_{DE} = 480\text{ N} \qquad (1)$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.135\text{ m}) + (0.6T_{DE})(0.08\text{ m}) = 0$$

or

$$T_{DE} = 2.25T_{CF} \qquad (2)$$

Substituting Equation (2) into Equation (1)

$$0.6T_{CF} + 0.8[(2.25)T_{CF}] = 480\text{ N}$$

$$T_{CF} = 200.00\text{ N}$$

or $T_{CF} = 200\text{ N}$ ◀

and from Equation (2) $T_{DE} = 2.25(200.00\text{ N}) = 450.00$

or $T_{DE} = 450\text{ N}$ ◀

PROBLEM 4.122 (Continued)

(*b*) From f.b.d. of assembly

$$\Sigma F_z = 0: \quad A_z - (0.6)(450.00 \text{ N}) = 0 \quad A_z = 270.00 \text{ N}$$

$$\Sigma F_x = 0: \quad A_x - (0.8)(200.00 \text{ N}) = 0 \quad A_x = 160.000 \text{ N}$$

or $\mathbf{A} = (160.0 \text{ N})\mathbf{i} + (270 \text{ N})\mathbf{k}$ ◀

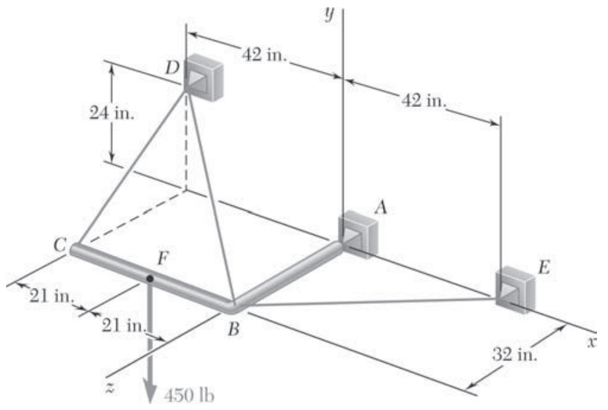
$$\Sigma M_x = 0: \quad M_{A_x} + (480 \text{ N})(0.135 \text{ m}) - [(200.00 \text{ N})(0.6)](0.135 \text{ m})$$
$$- [(450 \text{ N})(0.8)](0.09 \text{ m}) = 0$$

$$M_{A_x} = -16.2000 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad M_{A_z} - (480 \text{ N})(0.08 \text{ m}) + [(200.00 \text{ N})(0.6)](0.08 \text{ m})$$
$$+ [(450 \text{ N})(0.8)](0.08 \text{ m}) = 0$$

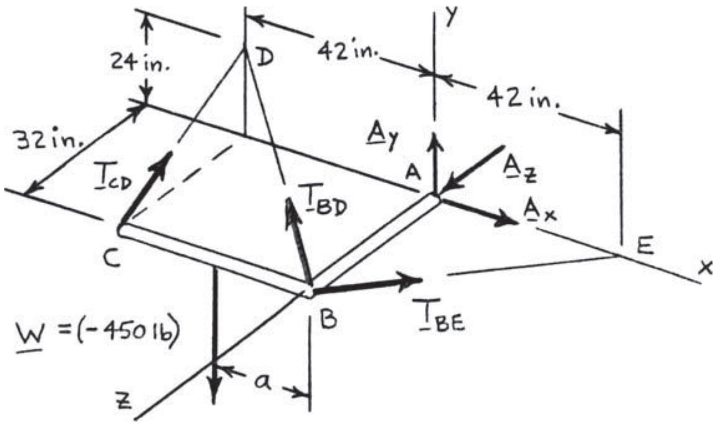
$$M_{A_z} = 0$$

or $\mathbf{M}_A = -(16.20 \text{ N}\cdot\text{m})\mathbf{i}$ ◀

	**PROBLEM 4.123** The rigid L-shaped member *ABC* is supported by a ball-and-socket joint at *A* and by three cables. If a 450-lb load is applied at *F*, determine the tension in each cable.

SOLUTION

Free-Body Diagram:



In this problem: $a = 21$ in.

We have $\overrightarrow{CD} = (24 \text{ in.})\mathbf{j} - (32 \text{ in.})\mathbf{k} \quad CD = 40 \text{ in.}$

$$\overrightarrow{BD} = -(42 \text{ in.})\mathbf{i} + (24 \text{ in.})\mathbf{j} - (32 \text{ in.})\mathbf{k} \quad BD = 58 \text{ in.}$$

$$\overrightarrow{BE} = (42 \text{ in.})\mathbf{i} - (32 \text{ in.})\mathbf{k} \quad BE = 52.802 \text{ in.}$$

Thus

$$T_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = T_{CD}(0.6\mathbf{j} - 0.8\mathbf{k})$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = T_{BD}(-0.72414\mathbf{i} + 0.41379\mathbf{j} - 0.55172\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.79542\mathbf{i} - 0.60604\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad (\mathbf{r}_C \times \mathbf{T}_{CD}) + (\mathbf{r}_B \times \mathbf{T}_{BD}) + (\mathbf{r}_B \times \mathbf{T}_{BE}) + (\mathbf{r}_W \times \mathbf{W}) = 0$$

PROBLEM 4.123 (Continued)

Noting that

$$\mathbf{r}_C = -(42 \text{ in.})\mathbf{i} + (32 \text{ in.})\mathbf{k}$$
$$\mathbf{r}_B = (32 \text{ in.})\mathbf{k}$$
$$\mathbf{r}_W = -a\mathbf{i} + (32 \text{ in.})\mathbf{k}$$

and using determinants, we write

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -42 & 0 & 32 \\ 0 & 0.6 & -0.8 \end{vmatrix} T_{CD} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 32 \\ -0.72414 & 0.41379 & -0.55172 \end{vmatrix} T_{BD}$$

$$+\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 32 \\ 0.79542 & 0 & -0.60604 \end{vmatrix} T_{BE} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a & 0 & 32 \\ 0 & -450 & 0 \end{vmatrix} = 0$$

Equating to zero the coefficients of the unit vectors:

$$\mathbf{i}: \quad -19.2T_{CD} - 13.241T_{BD} + 14400 = 0 \qquad (1)$$

$$\mathbf{j}: \quad -33.6T_{CD} - 23.172T_{BD} + 25.453T_{BE} = 0 \qquad (2)$$

$$\mathbf{k}: \quad -25.2T_{CD} + 450a = 0 \qquad (3)$$

Recalling that $a = 21$ in., Eq. (3) yields

$$T_{CD} = \frac{450(21)}{25.2} = 375 \text{ lb} \qquad T_{CD} = 375 \text{ lb} \blacktriangleleft$$

From (1):

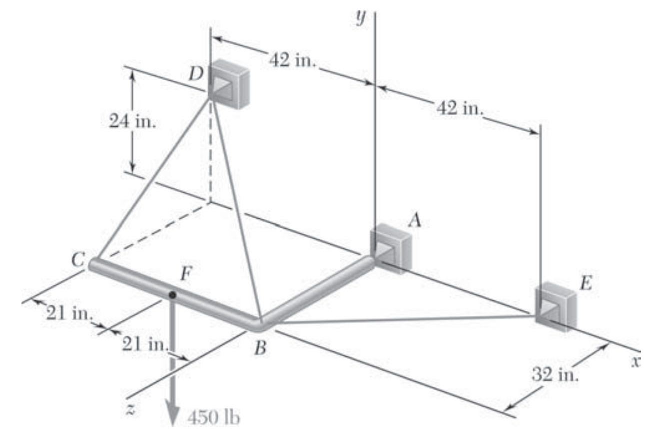
$$-19.2(375) - 13.241T_{BD} + 14400 = 0$$

$$T_{BD} = 543.77 \text{ lb} \qquad T_{BD} = 544 \text{ lb} \blacktriangleleft$$

From (2):

$$-33.6(375) - 23.172(543.77) + 25.453T_{BE} = 0$$

$$T_{BE} = 990.07 \text{ lb} \qquad T_{BE} = 990 \text{ lb} \blacktriangleleft$$



PROBLEM 4.124

Solve Problem 4.123, assuming that the 450-lb load is applied at *C*.

PROBLEM 4.123 The rigid L-shaped member *ABC* is supported by a ball-and-socket joint at *A* and by three cables. If a 450-lb load is applied at *F*, determine the tension in each cable.

SOLUTION

See solution of Problem 4.123 for free-body diagram and derivation of Eqs. (1), (2), and (3):

$$-19.2T_{CD} - 13.241T_{BD} + 14400 = 0 \quad (1)$$

$$-33.6T_{CD} - 23.172T_{BD} + 25.453T_{BE} = 0 \quad (2)$$

$$-25.2T_{CD} + 450a = 0 \quad (3)$$

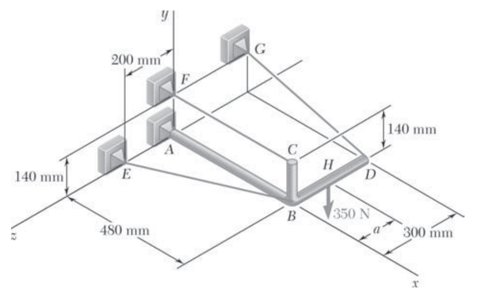
In this problem, the 450-lb load is applied at *C* and we have $a = 42$ in. Carrying into (3) and solving for T_{CD},

$$T_{CD} = 750 \text{ lb}$$ $T_{CD} = 750$ lb ◀

From (1): $-19.2(750) - 13.241T_{BD} + 14400 = 0$ $T_{BD} = 0$ ◀

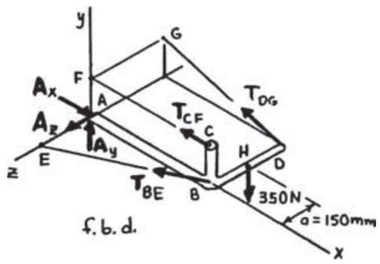
From (2): $-33.6(750) - 0 + 25.453T_{BE} = 0$ $T_{BE} = 990$ lb ◀

PROBLEM 4.125



Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. For $a = 150$ mm, determine the tension in each cable and the reaction at *A*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG} T_{DG} = \frac{-(0.48 \text{ m})\mathbf{i} + (0.14 \text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2} \text{ m}} T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \lambda_{BE} T_{BE} = \frac{-(0.48 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2} \text{ m}} T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{j} + 5\mathbf{k})$$

From f.b.d. of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25} T_{DG}\right)(0.3 \text{ m}) - (350 \text{ N})(0.15 \text{ m}) = 0$$

or $$T_{DG} = 625 \text{ N} \blacktriangleleft$$

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 625 \text{ N}\right)(0.3 \text{ m}) - \left(\frac{5}{13} T_{BE}\right)(0.48 \text{ m}) = 0$$

or $$T_{BE} = 975 \text{ N} \blacktriangleleft$$

$$\Sigma M_z = 0: \quad T_{CF}(0.14 \text{ m}) + \left(\frac{7}{25} \times 625 \text{ N}\right)(0.48 \text{ m}) - (350 \text{ N})(0.48 \text{ m}) = 0$$

or $$T_{CF} = 600 \text{ N} \blacktriangleleft$$

PROBLEM 4.125 (Continued)

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x - 600 \text{ N} - \left(\frac{12}{13} \times 975 \text{ N}\right) - \left(\frac{24}{25} \times 625 \text{ N}\right) = 0$$

$$A_x = 2100 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 625 \text{ N}\right) - 350 \text{ N} = 0$$

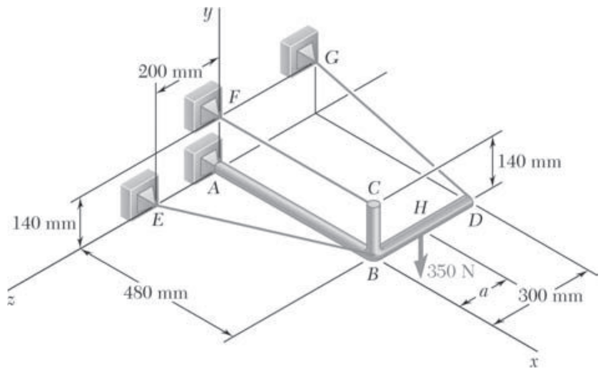
$$A_y = 175.0 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 975 \text{ N}\right) = 0$$

$$A_z = -375 \text{ N}$$

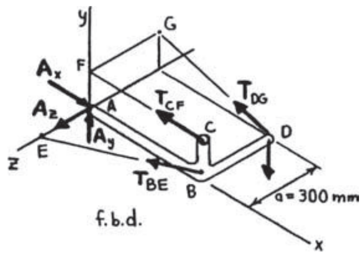
Therefore $\mathbf{A} = (2100 \text{ N})\mathbf{i} + (175.0 \text{ N})\mathbf{j} - (375 \text{ N})\mathbf{k}$ ◀



PROBLEM 4.126

Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. Knowing that the 350-N load is applied at *D* ($a = 300$ mm), determine the tension in each cable and the reaction at *A*.

SOLUTION



First note $\quad \mathbf{T}_{DG} = \boldsymbol{\lambda}_{DG} T_{DG} = \dfrac{-(0.48 \text{ m})\mathbf{i} + (0.14 \text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2} \text{ m}} T_{DG}$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE} T_{BE} = \frac{-(0.48 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2} \text{ m}} T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{i} + 5\mathbf{k})$$

From f.b.d. of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25} T_{DG}\right)(0.3 \text{ m}) - (350 \text{ N})(0.3 \text{ m}) = 0$$

or $\quad T_{DG} = 1250 \text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 1250 \text{ N}\right)(0.3 \text{ m}) - \left(\frac{5}{13} T_{BE}\right)(0.48 \text{ m}) = 0$$

or $\quad T_{BE} = 1950 \text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14 \text{ m}) + \left(\frac{7}{25} \times 1250 \text{ N}\right)(0.48 \text{ m}) - (350 \text{ N})(0.48 \text{ m}) = 0$$

or $\quad T_{CF} = 0$ ◀

PROBLEM 4.126 (Continued)

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x + 0 - \left(\frac{12}{13} \times 1950 \text{ N}\right) - \left(\frac{24}{25} \times 1250 \text{ N}\right) = 0$$

$$A_x = 3000 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 1250 \text{ N}\right) - 350 \text{ N} = 0$$

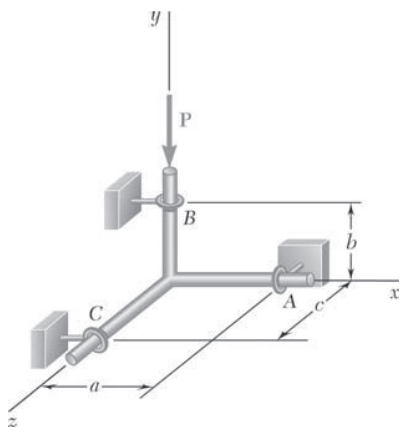
$$A_y = 0$$

$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 1950 \text{ N}\right) = 0$$

$$A_z = -750 \text{ N}$$

Therefore $\mathbf{A} = (3000 \text{ N})\mathbf{i} - (750 \text{ N})\mathbf{k}$ ◀

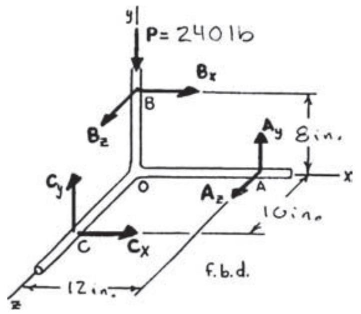


PROBLEM 4.127

Three rods are welded together to form a "corner" that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ lb, $a = 12$ in., $b = 8$ in., and $c = 10$ in.

SOLUTION

From f.b.d. of weldment



$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$(-12 A_z \mathbf{j} + 12 A_y \mathbf{k}) + (8 B_z \mathbf{i} - 8 B_x \mathbf{k}) + (-10 C_y \mathbf{i} + 10 C_x \mathbf{j}) = 0$$

From **i**-coefficient $\quad 8B_z - 10C_y = 0$

or $\quad B_z = 1.25 C_y \quad (1)$

j-coefficient $\quad -12A_z + 10C_x = 0$

or $\quad C_x = 1.2 A_z \quad (2)$

k-coefficient $\quad 12A_y - 8B_x = 0$

or $\quad B_x = 1.5 A_y \quad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

or $\quad (B_x + C_x)\mathbf{i} + (A_y + C_y - 240 \text{ lb})\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$

From **i**-coefficient $\quad B_x + C_x = 0$

or $\quad C_x = -B_x \quad (4)$

j-coefficient $\quad A_y + C_y - 240 \text{ lb} = 0$

or $\quad A_y + C_y = 240 \text{ lb} \quad (5)$

PROBLEM 4.127 (Continued)

k-coefficient $\quad A_z + B_z = 0$

or $$A_z = -B_z \tag{6}$$

Substituting C_x from Equation (4) into Equation (2)

$$-B_z = 1.2A_z \tag{7}$$

Using Equations (1), (6), and (7)

$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25}\left(\frac{B_x}{1.2}\right) = \frac{B_x}{1.5} \tag{8}$$

From Equations (3) and (8)

$$C_y = \frac{1.5A_y}{1.5} \quad \text{or} \quad C_y = A_y$$

and substituting into Equation (5)

$$2A_y = 240 \text{ lb}$$

$$A_y = C_y = 120 \text{ lb} \tag{9}$$

Using Equation (1) and Equation (9)

$$B_z = 1.25(120 \text{ lb}) = 150.0 \text{ lb}$$

Using Equation (3) and Equation (9)

$$B_x = 1.5(120 \text{ lb}) = 180.0 \text{ lb}$$

From Equation (4) $\quad C_x = -180.0 \text{ lb}$

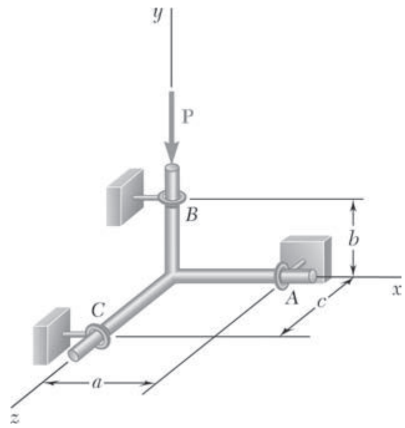
From Equation (6) $\quad A_z = -150.0 \text{ lb}$

Therefore

$\mathbf{A} = (120.0 \text{ lb})\mathbf{j} - (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (180.0 \text{ lb})\mathbf{i} + (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(180.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j}$ ◀

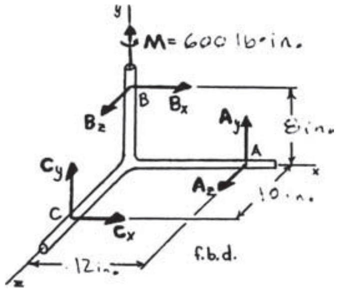


PROBLEM 4.128

Solve Problem 4.127, assuming that the force **P** is removed and is replaced by a couple $\mathbf{M} = +(600\ \text{lb}\cdot\text{in.})\mathbf{j}$ acting at B.

PROBLEM 4.127 Three rods are welded together to form a "corner" that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ lb, $a = 12$ in., $b = 8$ in., and $c = 10$ in.

SOLUTION



From f.b.d. of weldment

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} + \mathbf{M} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} + (600\ \text{lb}\cdot\text{in.})\mathbf{j} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{j} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) + (600\ \text{lb}\cdot\text{in.})\mathbf{j} = 0$$

From **i**-coefficient $\qquad 8B_z - 10C_y = 0$

or $\qquad C_y = 0.8B_z \qquad (1)$

j-coefficient $\qquad -12A_z + 10C_x + 600 = 0$

or $\qquad C_x = 1.2A_z - 60 \qquad (2)$

k-coefficient $\qquad 12A_y - 8B_x = 0$

or $\qquad B_x = 1.5A_y \qquad (3)$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

$$(B_x + C_x)\mathbf{i} + (A_y + C_y)\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$$

From **i**-coefficient $\qquad C_x = -B_x \qquad (4)$

j-coefficient $\qquad C_y = -A_y \qquad (5)$

k-coefficient $\qquad A_z = -B_z \qquad (6)$

PROBLEM 4.128 (Continued)

Substituting C_x from Equation (4) into Equation (2)

$$A_z = 50 - \left(\frac{B_x}{1.2}\right) \tag{7}$$

Using Equations (1), (6), and (7)

$$C_y = 0.8B_z = -0.8A_z = \left(\frac{2}{3}\right)B_x - 40 \tag{8}$$

From Equations (3) and (8)

$$C_y = A_y - 40$$

Substituting into Equation (5) $\quad 2A_y = 40$

$$A_y = 20.0 \text{ lb}$$

From Equation (5) $\quad C_y = -20.0$ lb

Equation (1) $\quad B_z = -25.0$ lb

Equation (3) $\quad B_x = 30.0$ lb

Equation (4) $\quad C_x = -30.0$ lb

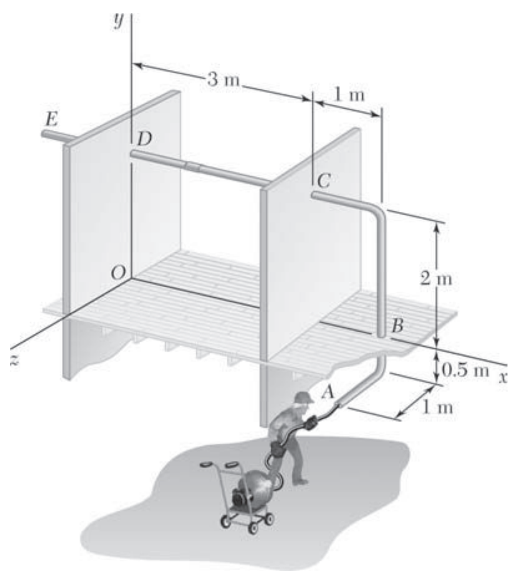
Equation (6) $\quad A_z = 25.0$ lb

Therefore

$\mathbf{A} = (20.0 \text{ lb})\mathbf{j} + (25.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (30.0 \text{ lb})\mathbf{i} - (25.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(30.0 \text{ lb})\mathbf{i} - (20.0 \text{ lb})\mathbf{j}$ ◀

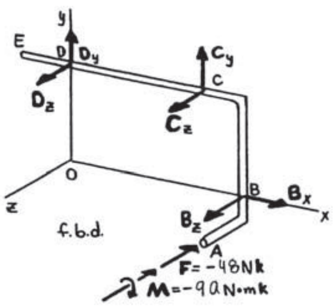


PROBLEM 4.129

In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48\text{ N})\mathbf{k}, \mathbf{M} = -(90\text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From f.b.d. of pipe assembly *ABCD*



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (48\text{ N})(2.5\text{ m}) - B_z(2\text{ m}) = 0$$

$$B_z = 60.0\text{ N}$$

and $\mathbf{B} = (60.0\text{ N})\mathbf{k}$ ◀

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\text{ m}) - 90\text{ N}\cdot\text{m} = 0$$

$$C_y = 30.0\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad -C_z(3\text{ m}) - (60.0\text{ N})(4\text{ m}) + (48\text{ N})(4\text{ m}) = 0$$

$$C_z = -16.00\text{ N}$$

and $\mathbf{C} = (30.0\text{ N})\mathbf{j} - (16.00\text{ N})\mathbf{k}$ ◀

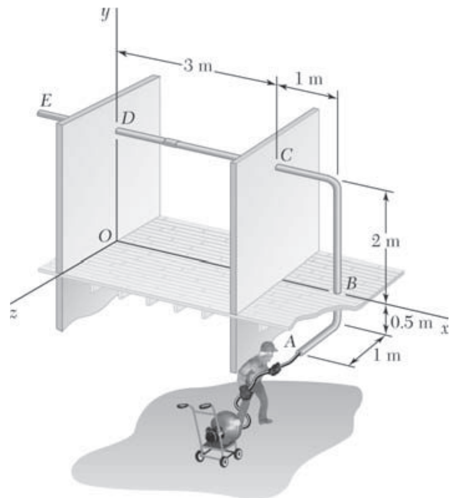
$$\Sigma F_y = 0: \quad D_y + 30.0 = 0$$

$$D_y = -30.0\text{ N}$$

$$\Sigma F_z = 0: \quad D_z - 16.00\text{ N} + 60.0\text{ N} - 48\text{ N} = 0$$

$$D_z = 4.00\text{ N}$$

and $\mathbf{D} = -(30.0\text{ N})\mathbf{j} + (4.00\text{ N})\mathbf{k}$ ◀



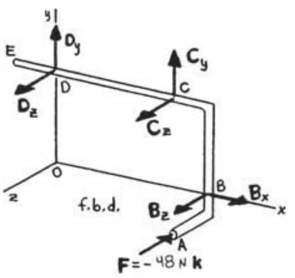
PROBLEM 4.130

Solve Problem 4.129, assuming that the plumber exerts a force $\mathbf{F} = -(48\text{ N})\mathbf{k}$ and that the motor is turned off $(\mathbf{M} = 0)$.

PROBLEM 4.129 In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48\text{ N})\mathbf{k}, \mathbf{M} = -(90\text{ N}\cdot\text{m})\mathbf{k}.$ Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From f.b.d. of pipe assembly *ABCD*



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (48\text{ N})(2.5\text{ m}) - B_z(2\text{ m}) = 0$$

$$B_z = 60.0\text{ N} \qquad \text{and } \mathbf{B} = (60.0\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\text{ m}) - B_x(2\text{ m}) = 0$$

$$C_y = 0$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(3\text{ m}) - (60.0\text{ N})(4\text{ m}) + (48\text{ N})(4\text{ m}) = 0$$

$$C_z = -16.00\text{ N} \qquad \text{and } \mathbf{C} = -(16.00\text{ N})\mathbf{k} \blacktriangleleft$$

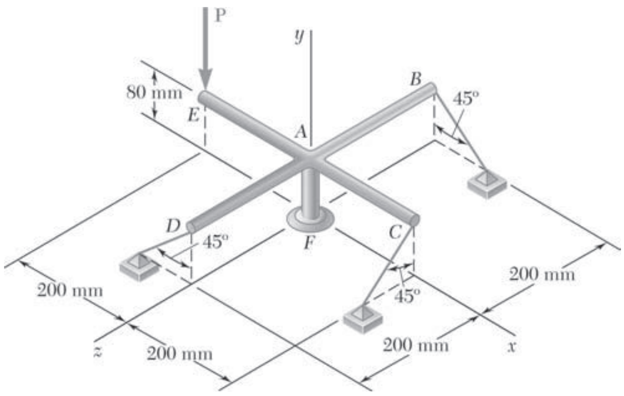
$$\Sigma F_y = 0: \quad D_y + C_y = 0$$

$$D_y = 0$$

$$\Sigma F_z = 0: \quad D_z + B_z + C_z - F = 0$$

$$D_z + 60.0\text{ N} - 16.00\text{ N} - 48\text{ N} = 0$$

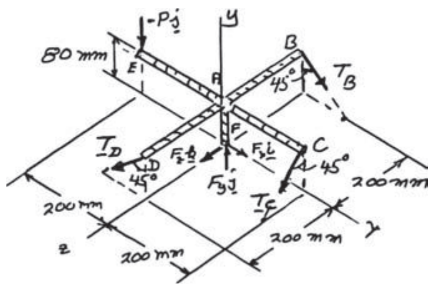
$$D_z = 4.00\text{ N} \qquad \text{and } \mathbf{D} = (4.00\text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.131

The assembly shown consists of an 80-mm rod AF that is welded to a cross consisting of four 200-mm arms. The assembly is supported by a ball-and-socket joint at F and by three short links, each of which forms an angle of 45° with the vertical. For the loading shown, determine (*a*) the tension in each link, (*b*) the reaction at F.

SOLUTION



$$\mathbf{r}_{E/F} = -200\,\mathbf{i} + 80\,\mathbf{j}$$

$$\mathbf{T}_B = T_B(\mathbf{i} - \mathbf{j})/\sqrt{2} \qquad \mathbf{r}_{B/F} = 80\mathbf{j} - 200\mathbf{k}$$

$$\mathbf{T}_C = T_C(-\mathbf{j} + \mathbf{k})/\sqrt{2} \qquad \mathbf{r}_{C/F} = 200\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_D = T_D(-\mathbf{i} + \mathbf{j})/\sqrt{2} \qquad \mathbf{r}_{D/E} = 80\mathbf{j} + 200\mathbf{k}$$

$$\Sigma M_F = 0\text{:} \quad \mathbf{r}_{B/F} \times \mathbf{T}_B + \mathbf{r}_{C/F} \times \mathbf{T}_C + \mathbf{r}_{D/F} \times T_D + \mathbf{r}_{E/F} \times (-P\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & -200 \\ 1 & -1 & 0 \end{vmatrix} \frac{T_B}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200 & 80 & 0 \\ 0 & -1 & 1 \end{vmatrix} \frac{T_c}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 200 \\ -1 & -1 & 0 \end{vmatrix} \frac{T_D}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -200 & 80 & 0 \\ 0 & -P & 0 \end{vmatrix} = 0$$

Equate coefficients of unit vectors to zero and multiply each equation by $\sqrt{2}$.

$$\mathbf{i}\text{:} \quad -200T_B + 80T_C + 200\,T_D = 0 \tag{1}$$

$$\mathbf{j}\text{:} \quad -200T_B - 200T_C - 200T_D = 0 \tag{2}$$

$$\mathbf{k}\text{:} \quad -80T_B - 200T_C + 80T_D + 200\sqrt{2}P = 0 \tag{3}$$

$$\frac{80}{200}(2)\text{:} \quad -80\,T_B - 80\,T_C - 80\,T_D = 0 \tag{4}$$

Eqs. (3) + (4): $\qquad -160T_B - 280T_C + 200\sqrt{2}P = 0 \qquad (5)$

Eqs. (1) + (2): $\qquad -400T_B - 120T_C = 0$

$$T_B = -\frac{120}{400}T_C - 0.3T_C \tag{6}$$

PROBLEM 4.131 (Continued)

Eqs. (6) $\longrightarrow$ (5): $$-160(-0.3T_C) - 280T_C + 200\sqrt{2}P = 0$$

$$-232T_C + 200\sqrt{2}P = 0$$

$$T_C = 1.2191P \qquad T_C = 1.219P \blacktriangleleft$$

From Eq. (6): $$T_B = -0.3(1.2191P) = -0.36574 = P \qquad T_B = -0.366P \blacktriangleleft$$

From Eq. (2): $$-200(-0.3657P) - 200(1.2191P) - 200T_{\theta D} = 0$$

$$T_D = -0.8534P \qquad T_D = -0.853P \blacktriangleleft$$

$\Sigma\mathbf{F} = 0$: $\quad \mathbf{F} + \mathbf{T}_B + \mathbf{T}_C + \mathbf{T}_D - P\mathbf{j} = 0$

$\mathbf{i}$: $$F_x + \frac{(-0.36574P)}{\sqrt{2}} - \frac{(-0.8534P)}{\sqrt{2}} = 0$$

$$F_x = -0.3448P \qquad F_x = -0.345P$$

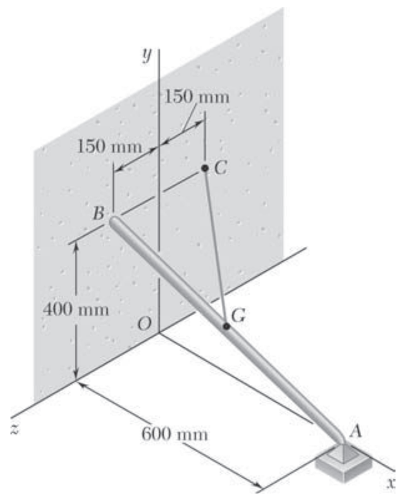
$\mathbf{j}$: $$F_y - \frac{(-0.36574P)}{\sqrt{2}} - \frac{(1.2191P)}{\sqrt{2}} - \frac{(-0.8534P)}{\sqrt{2}} - 200 = 0$$

$$F_y = P \qquad F_y = P$$

$\mathbf{k}$: $$F_z + \frac{(1.2191P)}{\sqrt{2}} = 0$$

$$F_z = -0.8620P \quad F_z = -0.862P \qquad \mathbf{F} = -0.345P\mathbf{i} + P\mathbf{j} - 0.862P\mathbf{k} \blacktriangleleft$$

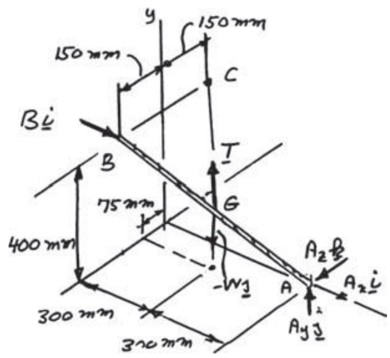
PROBLEM 4.132



The uniform 10-kg rod AB is supported by a ball-and-socket joint at A and by the cord CG that is attached to the midpoint G of the rod. Knowing that the rod leans against a frictionless vertical wall at B, determine (*a*) the tension in the cord, (*b*) the reactions at A and B.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium. But equilibrium is maintained $(\Sigma M_{AB} = 0)$

$$W = mg$$
$$= (10\text{ kg})\,9.81\text{ m/s}^2$$
$$W = 98.1\text{ N}$$

$$\overrightarrow{GC} = -300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k} \quad GC = 425\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{GC}}{GC} = \frac{T}{425}(-300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k})$$

$$\mathbf{r}_{B/A} = -600\mathbf{i} + 400\mathbf{j} + 150\text{ mm}$$

$$\mathbf{r}_{G/A} = -300\mathbf{i} + 200\mathbf{j} + 75\text{ mm}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{G/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -600 & 400 & 150 \\ B & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ -300 & 200 & -225 \end{vmatrix} \frac{T}{425} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ 0 & -98.1 & 0 \end{vmatrix}$$

Coefficient of $\mathbf{i}$: $\quad (-105.88 - 35.29)T + 7357.5 = 0$

$$T = 52.12\text{ N}$$

$T = 52.1\text{ N}$ ◀

PROBLEM 4.132 (Continued)

Coefficient of $\mathbf{j}$: $150B - (300\times 75 + 300\times 225)\dfrac{52.12}{425} = 0$

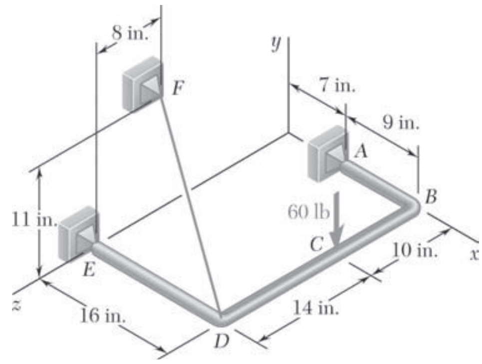
$B = 73.58\text{ N}$ $\qquad \mathbf{B} = (73.6\text{ N})\mathbf{i}$ ◀

$\Sigma\mathbf{F} = 0$: $\mathbf{A} + \mathbf{B} + \mathbf{T} - W\mathbf{j} = 0$

Coefficient of $\mathbf{i}$: $A_x + 73.58 - 52.15\dfrac{300}{425} = 0$ $\qquad A_x = 36.8\text{ N}$ ◀

Coefficient of $\mathbf{j}$: $A_y + 52.15\dfrac{200}{425} - 98.1 = 0$ $\qquad A_y = 73.6\text{ N}$ ◀

Coefficient of $\mathbf{k}$: $A_z - 52.15\dfrac{225}{425} = 0$ $\qquad A_z = 27.6\text{ N}$ ◀

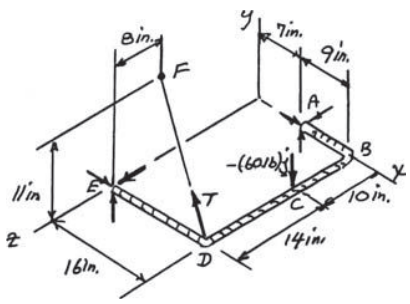


PROBLEM 4.133

The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 60-lb load is applied at *C* as shown, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{DF} = -16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k} \qquad DF = 21\text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{DE}}{DF} = \frac{T}{21}(-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$$

$$\mathbf{r}_{D/E} = 16\,\mathbf{i}$$

$$\mathbf{r}_{C/E} = 16\mathbf{i} - 14\mathbf{k}$$

$$\boldsymbol{\lambda}_{EA} = \frac{\overrightarrow{EA}}{EA} = \frac{7\mathbf{i} - 24\mathbf{k}}{25}$$

$$\Sigma M_{EA} = 0: \quad \boldsymbol{\lambda}_{EA} \cdot (\mathbf{r}_{B/E} \times \mathbf{T}) + \boldsymbol{\lambda}_{EA} \cdot (\mathbf{r}_{C/E} \cdot (-60\mathbf{j})) = 0$$

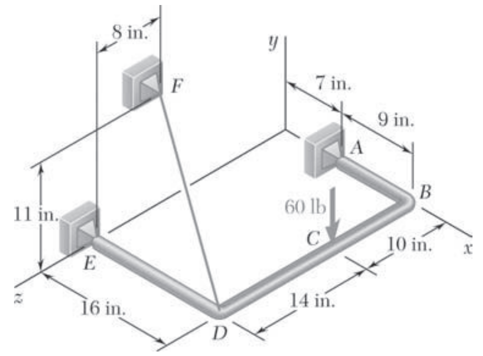
$$\begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T}{21 \times 25} + \begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & -14 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 16 \times 11}{21 \times 25}T + \frac{-7 \times 14 \times 60 + 24 \times 16 \times 60}{25} = 0$$

$$201.14T + 17{,}160 = 0$$

$$T = 85.314 \text{ lb}$$

$$T = 85.3 \text{ lb} \blacktriangleleft$$

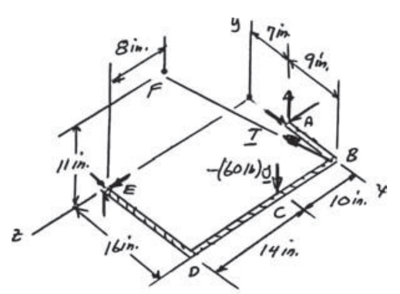


PROBLEM 4.134

Solve Problem 4.133, assuming that cable *DF* is replaced by a cable connecting *B* and *F*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 9\mathbf{i}$$

$$\mathbf{r}_{C/A} = 9\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{BF} = -16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k} \qquad BF = 25.16 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{BF}}{BF} = \frac{T}{25.16}(-16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k})$$

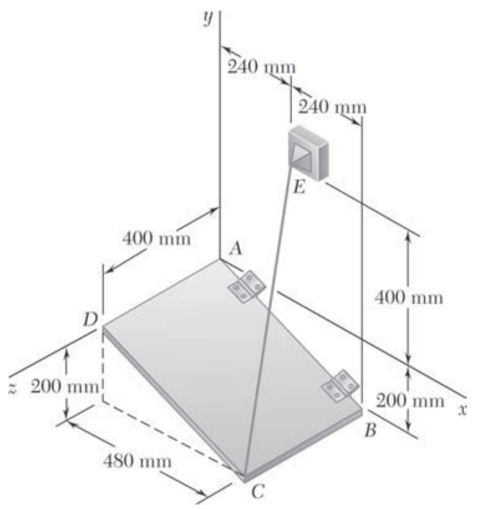
$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{7\mathbf{i} - 24\mathbf{k}}{\mathbf{25}}$$

$$\Sigma M_{AE} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{C/A} \cdot (-60\mathbf{j})) = 0$$

$$\begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 0 \\ -16 & 11 & 16 \end{vmatrix} \frac{T}{25 \times 25.16} + \begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 10 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 9 \times 11}{25 \times 25.16}T + \frac{24 \times 9 \times 60 + 7 \times 10 \times 60}{25}$$

$$94.436T - 17{,}160 = 0 \qquad T = 181.7 \text{ lb} \blacktriangleleft$$

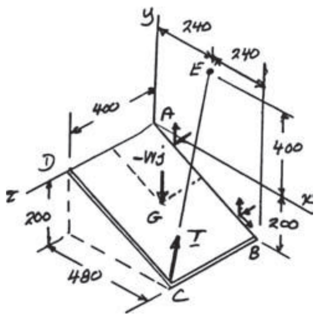


PROBLEM 4.135

The 50-kg plate *ABCD* is supported by hinges along edge *AB* and by wire *CE*. Knowing that the plate is uniform, determine the tension in the wire.

SOLUTION

Free-Body Diagram:



$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 490.50\text{ N}$$

$$\overrightarrow{CE} = -240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k}$$

$$CE = 760\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{CE}}{CE} = \frac{T}{760}(-240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k})$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

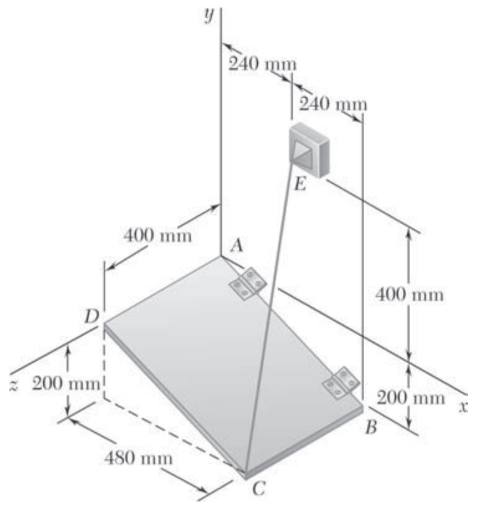
$$\Sigma\mathbf{M}_{AB} = 0: \quad \boldsymbol{\lambda}_{AB}\cdot(\mathbf{r}_{E/A}\times T) + \boldsymbol{\lambda}_{AB}\cdot(\mathbf{r}_{G/A}\times -W\mathbf{j}) = 0$$

$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ -240 & 600 & -400 \end{vmatrix}\frac{T}{13\times 20} + \begin{vmatrix} 12 & -5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix}\frac{1}{13} = 0$$

$$(-12\times 400\times 400 - 5\times 240\times 400)\frac{T}{760} + 12\times 200W = 0$$

$$T = 0.76W = 0.76(490.50\text{ N}) \qquad T = 373\text{ N} \blacktriangleleft$$



PROBLEM 4.136

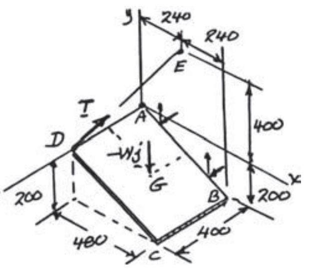
Solve Problem 4.135, assuming that wire *CE* is replaced by a wire connecting *E* and *D*.

PROBLEM 4.135 The 50-kg plate *ABCD* is supported by hinges along edge *AB* and by wire *CE*. Knowing that the plate is uniform, determine the tension in the wire.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$W = mg = (50 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 490.50 \text{ N}$$

$$\overrightarrow{DE} = -240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k}$$

$$DE = 614.5 \text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{DE}}{DE} = \frac{T}{614.5}(240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k})$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

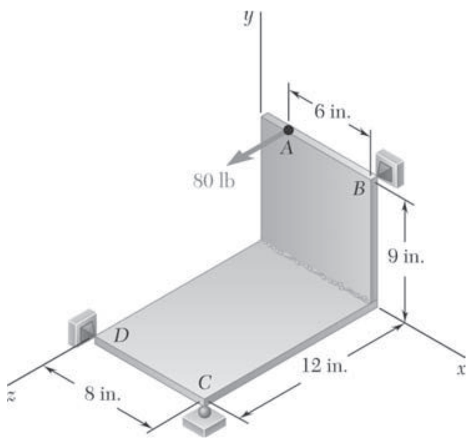
$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ 240 & 400 & -400 \end{vmatrix} \frac{T}{13 \times 614.5} + \begin{vmatrix} 12 & 5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400)\frac{T}{614.5} + 12 \times 200 \times W = 0$$

$$T = 0.6145W = 0.6145(490.50 \text{ N})$$

$$T = 301 \text{ N} \blacktriangleleft$$



PROBLEM 4.137

Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at *B* and *D* and by a ball on a horizontal surface at *C*. For the loading shown, determine the reaction at *C*.

SOLUTION

First note

$$\boldsymbol{\lambda}_{BD} = \frac{-(6\text{ in.})\mathbf{i} - (9\text{ in.})\mathbf{j} + (12\text{ in.})\mathbf{k}}{\sqrt{(6)^2 + (9)^2 + (12)^2}\text{ in.}}$$

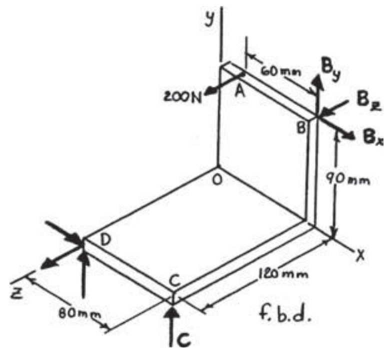
$$= \frac{1}{16.1555}(-6\mathbf{i} - 9\mathbf{j} + 12\mathbf{k})$$

$$\mathbf{r}_{A/B} = -(6\text{ in.})\mathbf{i}$$

$$\mathbf{P} = (80\text{ lb})\mathbf{k}$$

$$\mathbf{r}_{C/D} = (8\text{ in.})\mathbf{i}$$

$$\mathbf{C} = (C)\mathbf{j}$$



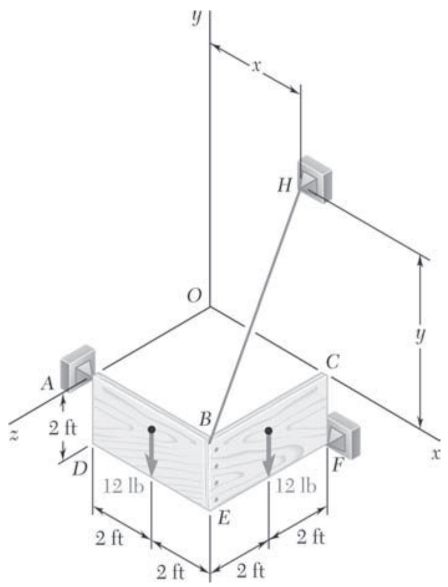
From the f.b.d. of the plates

$$\Sigma M_{BD} = 0: \quad \boldsymbol{\lambda}_{BD} \cdot (\mathbf{r}_{A/B} \times \mathrm{P}) + \boldsymbol{\lambda}_{BD} \cdot (\mathbf{r}_{C/D} \times \mathrm{C}) = 0$$

$$\begin{vmatrix} -6 & -9 & 12 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \left[\frac{6(80)}{16.1555}\right] + \begin{vmatrix} -6 & -9 & 12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \left[\frac{C(8)}{16.1555}\right] = 0$$

$$(-9)(6)(80) + (12)(8)C = 0$$

$$C = 45.0\text{ lb} \qquad \text{or } \mathbf{C} = (45.0\text{ lb})\mathbf{j} \blacktriangleleft$$

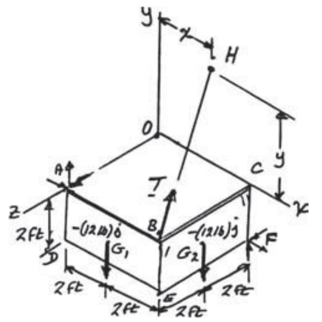


PROBLEM 4.138

Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k} \quad AF = 6\text{ ft}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G/A} \times (-12\mathbf{j}) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G2/A} \times (-12\mathbf{j})) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times T) = 0$$

$$\begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$(2\times 2\times 12)\frac{1}{3} + (-2\times 2\times 12 + 2\times 4\times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = -32 \quad \text{or} \quad \mathbf{T} \cdot (\boldsymbol{\lambda}_{A/F} \times \mathbf{r}_{B/A}) = -32 \qquad (1)$$

PROBLEM 4.138 (Continued)

Projection of **T** on $(\boldsymbol{\lambda}_{AF} \times \mathbf{r}_{B/A})$ is constant. Thus, $T_{\min}$ is parallel to

$$\boldsymbol{\lambda}_{AF} \times \mathbf{r}_{B/A} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i} = \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k})$$

Corresponding unit vector is $\frac{1}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k})$

$$T_{\min} = T(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}} \qquad (2)$$

Eq. (1):
$$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \left[\frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i}\right] = -32$$

$$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k}) = -32$$

$$\frac{T}{3\sqrt{5}}(16 + 4) = -32 \qquad T = -\frac{3\sqrt{5}(32)}{20} = 4.8\sqrt{5}$$

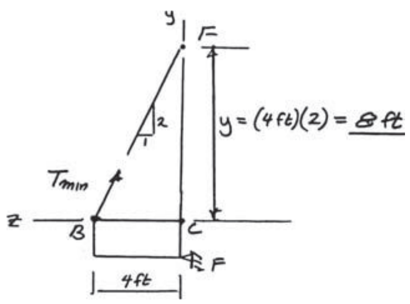
$$T = 10.7331 \text{ lb}$$

Eq. (2)
$$T_{\min} = T(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}}$$

$$= 4.8\sqrt{5}(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}}$$

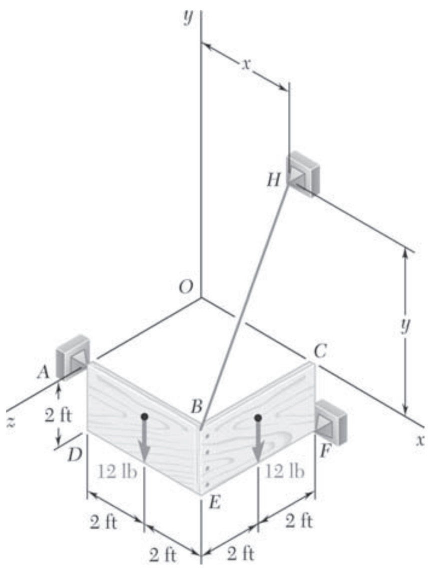
$$\mathbf{T}_{\min} = -(9.6 \text{ lb})\mathbf{j} + (4.8 \text{ lb } \mathbf{k})$$

Since $T_{\min}$ has no **i** component, wire BH is parallel to the yz plane, and $x = 4$ ft.



(*a*) $x = 4.00$ ft; $y = 8.00$ ft ◀

(*b*) $T_{\min} = 10.73$ lb ◀



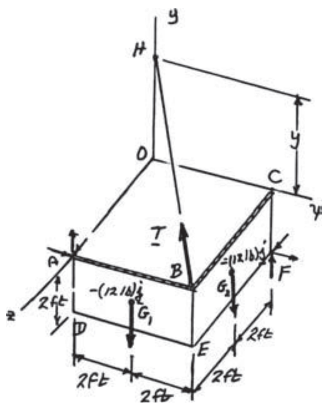
PROBLEM 4.139

Solve Problem 4.138, subject to the restriction that H must lie on the y axis.

PROBLEM 4.138 Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G/A} \times (-12\mathbf{j}) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G_2/A} \times (-12\mathbf{j})) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times T) = 0$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$(2 \times 2 \times 12)\frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = -32 \qquad (1)$$

$$\overrightarrow{BH} = -4\mathbf{i} + y\mathbf{j} - 4\mathbf{k} \qquad BH = (32 + y^2)^{1/2}$$

$$\mathbf{T} = T\frac{\overrightarrow{BH}}{BH} = T\frac{-4\mathbf{i} + y\mathbf{j} - 4\mathbf{k}}{(32 + y^2)^{1/2}}$$

PROBLEM 4.139 (Continued)

Eq. (1):

$$\boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times T)=\begin{vmatrix}2 & -1 & -2\\ 4 & 0 & 0\\ -4 & y & -4\end{vmatrix}\frac{T}{3(32+y^2)^{1/2}}=-32$$

$$(-16-8y)T=-3\times 32(32+y^2)^{1/2}\qquad T=96\frac{(32+y^2)^{1/2}}{8y+16}\qquad (2)$$

$$\frac{dT}{dy}=0:\quad 96\frac{(8y+16)\frac{1}{2}(32+y^2)^{-1/2}(2y)+(32+y^2)^{1/2}(8)}{(8y+16)^2}$$

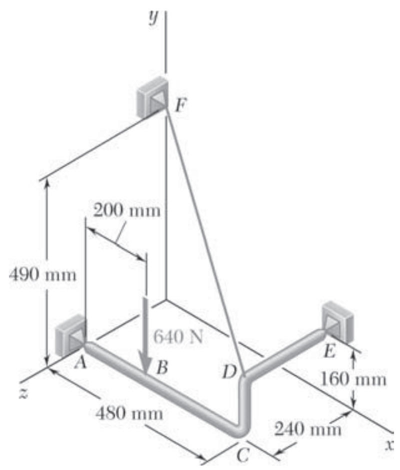
Numerator = 0: $(8y+16)y=(32+y^2)8$

$$8y^2+16y=32\times 8+8y^2$$

$y=16.00$ ft ◀

Eq. (2): $T=96\dfrac{(32+16^2)^{1/2}}{8\times 16+16}=11.3137$ lb

$T_{\min}=11.31$ lb ◀



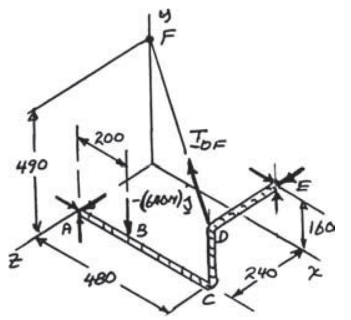
PROBLEM 4.140

The pipe *ACDE* is supported by ball-and-socket joints at *A* and *E* and by the wire *DF*. Determine the tension in the wire when a 640-N load is applied at *B* as shown.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$\overrightarrow{AE} = 480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}$$

$$AE = 560 \text{ mm}$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}}{560}$$

$$\boldsymbol{\lambda}_{AE} = \frac{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{7}$$

$$\mathbf{r}_{B/A} = 200\mathbf{i}$$

$$\mathbf{r}_{D/A} = 480\mathbf{i} + 160\mathbf{j}$$

$$\overrightarrow{DF} = -480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}; \qquad DF = 630 \text{ mm}$$

$$\mathbf{T}_{DF} = T_{DF}\frac{\overrightarrow{DF}}{DF} = T_{DF}\frac{-480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}}{630} = T_{DF}\frac{-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}}{21}$$

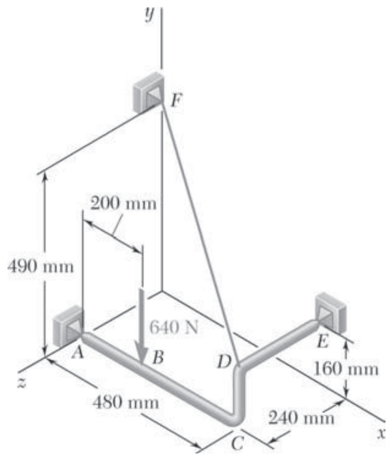
$$\Sigma M_{AE} = \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}_{DF}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times (-600\mathbf{j})) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T_{DE}}{21 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

$$\frac{-6 \times 160 \times 8 + 2 \times 480 \times 8 - 3 \times 480 \times 11 - 3 \times 160 \times 16}{21 \times 7} T_{DF} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-1120 T_{DF} + 384 \times 10^3 = 0$$

$$T_{DF} = 342.86 \text{ N} \qquad\qquad T_{DF} = 343 \text{ N} ◀$$



PROBLEM 4.141

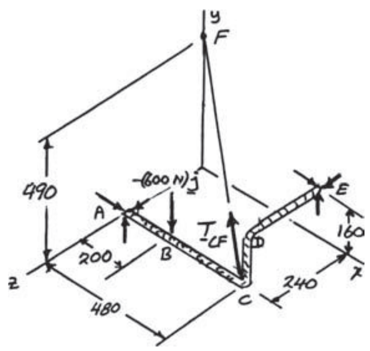
Solve Problem 4.140, assuming that wire *DF* is replaced by a wire connecting *C* and *F*.

PROBLEM 4.140 The pipe *ACDE* is supported by ball-and-socket joints at *A* and *E* and by the wire *DF*. Determine the tension in the wire when a 640-N load is applied at *B* as shown.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$\overrightarrow{AE} = 480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}$$

$$AE = 560 \text{ mm}$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}}{560}$$

$$\boldsymbol{\lambda}_{AE} = \frac{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{7}$$

$$\mathbf{r}_{B/A} = 200\mathbf{i}$$

$$\mathbf{r}_{C/A} = 480\mathbf{i}$$

$$\overrightarrow{CF} = -480\mathbf{i} + 490\mathbf{j} - 240\mathbf{k}; \quad CF = 726.70 \text{ mm}$$

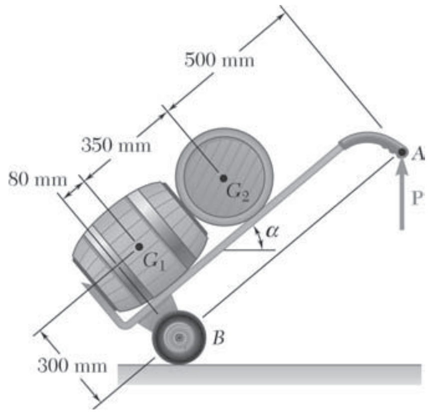
$$\mathbf{T}_{CF} = T_{CF}\frac{\overrightarrow{CE}}{CF} = \frac{-480\mathbf{i} + 490\mathbf{j} - 240\mathbf{k}}{726.70}$$

$$\Sigma M_{AE} = 0: \quad \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{C/A} \times \mathbf{T}_{CF}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times (-600\mathbf{j})) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 0 & 0 \\ -480 & +490 & -240 \end{vmatrix} \frac{T_{CF}}{726.7 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

$$\frac{2 \times 480 \times 240 - 3 \times 480 \times 490}{726.7 \times 7} T_{CF} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-653.91 T_{CF} + 384 \times 10^3 = 0 \qquad T_{CF} = 587 \text{ N} \blacktriangleleft$$

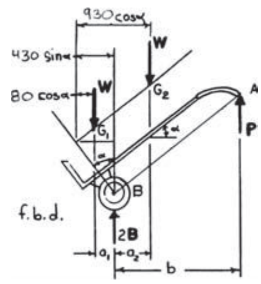


PROBLEM 4.142

A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (*a*) the vertical force **P** that should be applied to the handle to maintain equilibrium when $\alpha = 35°$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION

Free-Body Diagram:



Dimensions in mm

$$W = mg = (40\text{ kg})(9.81\text{ m/s}^2) = 392.40\text{ N}$$
$$a_1 = (300\text{ mm})\sin\alpha - (80\text{ mm})\cos\alpha$$
$$a_2 = (430\text{ mm})\cos\alpha - (300\text{ mm})\sin\alpha$$
$$b = (930\text{ mm})\cos\alpha$$

From free-body diagram of hand truck

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad P - 2W + 2B = 0 \qquad (2)$$

For $\alpha = 35°$

$$a_1 = 300\sin 35° - 80\cos 35° = 106.541\text{ mm}$$
$$a_2 = 430\cos 35° - 300\sin 35° = 180.162\text{ mm}$$
$$b = 930\cos 35° = 761.81\text{ mm}$$

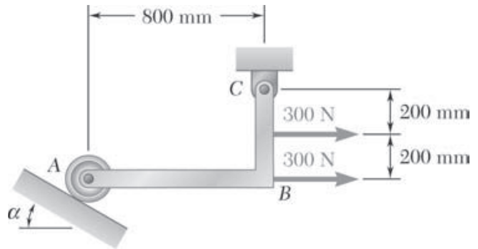
(*a*) From Equation (1)

$$P(761.81\text{ mm}) - 392.40\text{ N}(180.162\text{ mm}) + 392.40\text{ N}(106.54\text{ mm}) = 0$$

$$P = 37.921\text{ N}$$ or $\mathbf{P} = 37.9\text{ N}\uparrow$ ◀

(*b*) From Equation (2)

$$37.921\text{ N} - 2(392.40\text{ N}) + 2B = 0$$ or $\mathbf{B} = 373\text{ N}\uparrow$ ◀



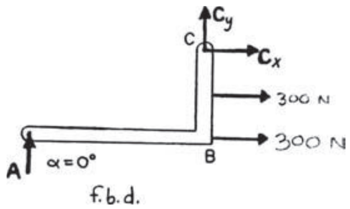
PROBLEM 4.143

Determine the reactions at A and C when (*a*) $\alpha = 0$, (*b*) $\alpha = 30°$.

SOLUTION

(*a*) $\alpha = 0°$

From f.b.d. of member ABC



$$+\circlearrowleft \Sigma M_C = 0: \quad (300\text{ N})(0.2\text{ m}) + (300\text{ N})(0.4\text{ m}) - A(0.8\text{ m}) = 0$$

$$A = 225\text{ N} \qquad \text{or} \qquad \mathbf{A} = 225\text{ N}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 225\text{ N} = 0$$

$$C_y = -225\text{ N} \quad \text{or} \quad \mathbf{C}_y = 225\text{ N}\downarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 300\text{ N} + 300\text{ N} + C_x = 0$$

$$C_x = -600\text{ N} \quad \text{or} \quad \mathbf{C}_x = 600\text{ N} \longleftarrow$$

Then

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(600)^2 + (225)^2} = 640.80\text{ N}$$

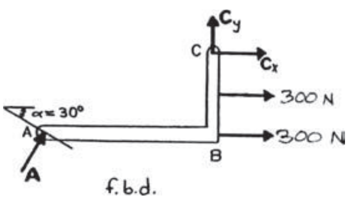
and

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-225}{-600}\right) = 20.556°$$

$$\text{or} \quad \mathbf{C} = 641\text{ N} \measuredangle 20.6° \blacktriangleleft$$

(*b*) $\alpha = 30°$

From f.b.d. of member ABC



$$+\circlearrowleft \Sigma M_C = 0: \quad (300\text{ N})(0.2\text{ m}) + (300\text{ N})(0.4\text{ m}) - (A\cos 30°)(0.8\text{ m}) + (A\sin 30°)(20\text{ in.}) = 0$$

$$A = 365.24\text{ N} \qquad \text{or} \qquad \mathbf{A} = 365\text{ N} \measuredangle 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 300\text{ N} + 300\text{ N} + (365.24\text{ N})\sin 30° + C_x = 0$$

$$C_x = -782.62$$

PROBLEM 4.143 (Continued)

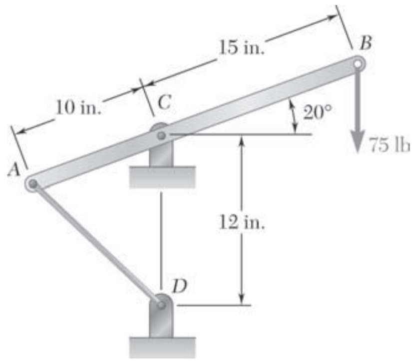
$$+\uparrow\Sigma F_y = 0: \quad C_y + (365.24 \text{ N})\cos 30° = 0$$

$$C_y = -316.31 \text{ N} \quad \text{or} \quad \mathbf{C}_y = 316 \text{ N} \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(782.62)^2 + (316.31)^2} = 884.12 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-316.31}{-782.62}\right) = 22.007°$$

or $$\mathbf{C} = 884 \text{ N} \swarrow 22.0° \blacktriangleleft$$

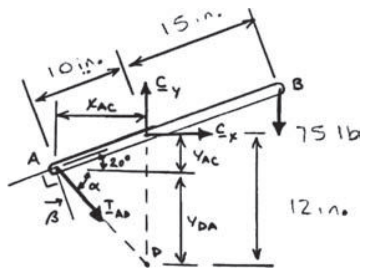


PROBLEM 4.144

A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 75-lb vertical force at B, determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION

Free-Body Diagram:



Geometry:

$$x_{AC} = (10 \text{ in.})\cos 20° = 9.3969 \text{ in.}$$
$$y_{AC} = (10 \text{ in.})\sin 20° = 3.4202 \text{ in.}$$

$$\Rightarrow y_{DA} = 12 \text{ in.} - 3.4202 \text{ in.} = 8.5798 \text{ in.}$$

$$\alpha = \tan^{-1}\left(\frac{y_{DA}}{x_{AC}}\right) = \tan^{-1}\left(\frac{8.5798}{9.3969}\right) = 42.397°$$

$$\beta = 90° - 20° - 42.397° = 27.603°$$

Equilibrium for lever:

(a) $\quad +\circlearrowleft \Sigma M_C = 0: \quad T_{AD}\cos 27.603°(10 \text{ in.}) - (75 \text{ lb})[(15 \text{ in.})\cos 20°] = 0$

$$T_{AD} = 119.293 \text{ lb}$$

$T_{AD} = 119.3 \text{ lb}$ ◀

(b) $\quad \xrightarrow{+} \Sigma F_x = 0: \quad C_x + (119.293 \text{ lb})\cos 42.397° = 0$

$$C_x = -88.097 \text{ lb}$$

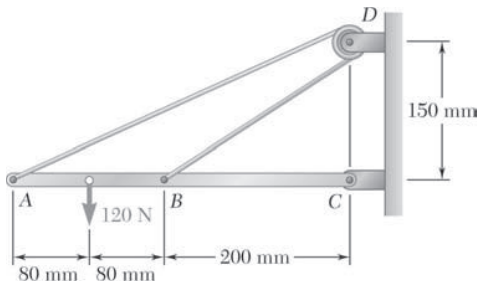
$+\uparrow \Sigma F_y = 0: \quad C_y - 75 \text{ lb} - (119.293 \text{ lb})\sin 42.397° = 0$

$$C_y = 155.435$$

Thus: $\quad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-88.097)^2 + (155.435)^2} = 178.665 \text{ lb}$

and $\quad \theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{155.435}{88.097} = 60.456°$

$\mathbf{C} = 178.7 \text{ lb} \measuredangle 60.5°$ ◀



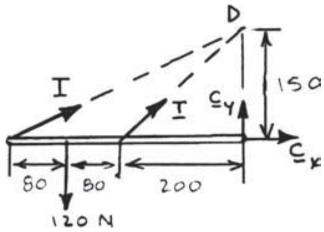
PROBLEM 4.145

Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ADB*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

Dimensions in mm



Geometry:

Distance $AD = \sqrt{(0.36)^2 + (0.150)^2} = 0.39 \text{ m}$

Distance $BD = \sqrt{(0.2)^2 + (0.15)^2} = 0.25 \text{ m}$

Equilibrium for beam:

(*a*) $+\circlearrowleft \Sigma M_C = 0$: $(120 \text{ N})(0.28 \text{ m}) - \left(\dfrac{0.15}{0.39}T\right)(0.36 \text{ m}) - \left(\dfrac{0.15}{0.25}T\right)(0.2 \text{ m}) = 0$

$T = 130.000 \text{ N}$

or $T = 130.0 \text{ N}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $C_x + \left(\dfrac{0.36}{0.39}\right)(130.000 \text{ N}) + \left(\dfrac{0.2}{0.25}\right)(130.000 \text{ N}) = 0$

$C_x = -224.00 \text{ N}$

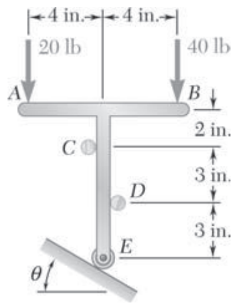
$+\uparrow \Sigma F_y = 0$: $C_y + \left(\dfrac{0.15}{0.39}\right)(130.00 \text{ N}) + \left(\dfrac{0.15}{0.25}\right)(130.00 \text{ N}) - 120 \text{ N} = 0$

$C_y = -8.0000 \text{ N}$

Thus: $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-224)^2 + (-8)^2} = 224.14 \text{ N}$

and $\theta = \tan^{-1}\dfrac{C_y}{C_x} = \tan^{-1}\dfrac{8}{224} = 2.0454°$

$\mathbf{C} = 224 \text{ N} \measuredangle 2.05°$ ◀

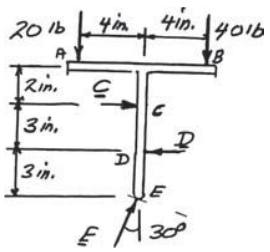


PROBLEM 4.146

The T-shaped bracket shown is supported by a small wheel at *E* and pegs at *C* and *D*. Neglecting the effect of friction, determine the reactions at *C*, *D*, and *E* when $\theta = 30°$.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E\cos 30° - 20 - 40 = 0$$

$$E = \frac{60 \text{ lb}}{\cos 30°} = 69.282 \text{ lb} \qquad \mathbf{E} = 69.3 \text{ lb} \measuredangle 60.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.})$$
$$- C(3 \text{ in.}) + E\sin 30°(3 \text{ in.}) = 0$$
$$-80 - 3C + 69.282(0.5)(3) = 0$$

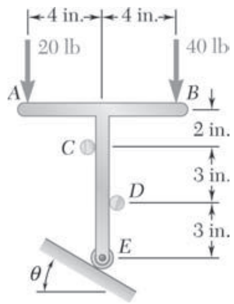
$$C = 7.9743 \text{ lb} \qquad \mathbf{C} = 7.97 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad E\sin 30° + C - D = 0$$

$$(69.282 \text{ lb})(0.5) + 7.9743 \text{ lb} - D = 0$$

$$D = 42.615 \text{ lb} \qquad \mathbf{D} = 42.6 \text{ lb} \longleftarrow \blacktriangleleft$$

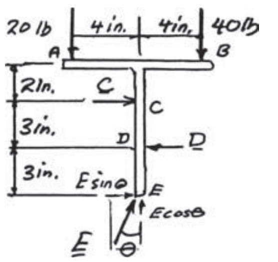
PROBLEM 4.147



The T-shaped bracket shown is supported by a small wheel at E and pegs at C and D. Neglecting the effect of friction, determine (a) the smallest value of θ for which the equilibrium of the bracket is maintained, (b) the corresponding reactions at C, D, and E.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E\cos\theta - 20 - 40 = 0$$

$$E = \frac{60}{\cos\theta} \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.}) - C(3 \text{ in.}) + \left(\frac{60}{\cos\theta}\sin\theta\right)3 \text{ in.} = 0$$

$$C = \frac{1}{3}(180\tan\theta - 80)$$

(a) For $C = 0$,

$$180\tan\theta = 80$$

$$\tan\theta = \frac{4}{9} \quad \theta = 23.962^\circ \qquad \theta = 24.0^\circ \blacktriangleleft$$

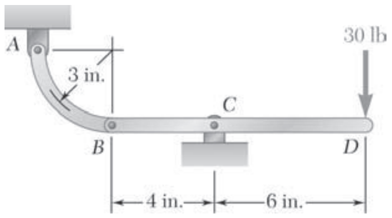
Eq. (1)

$$E = \frac{60}{\cos 23.962^\circ} = 65.659$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -D + C + E\sin\theta = 0$$

$$D = (65.659)\sin 23.962 = 26.666 \text{ lb}$$

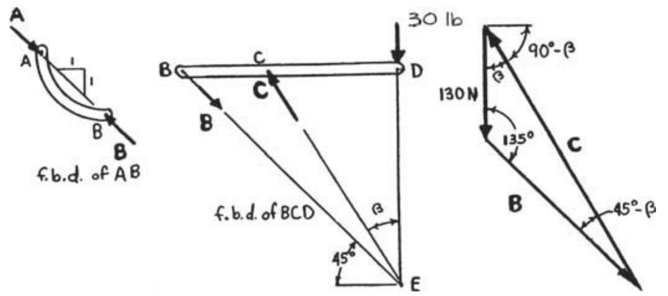
(b) $\mathbf{C} = 0 \quad \mathbf{D} = 26.7 \text{ lb} \longleftarrow \qquad \mathbf{E} = 65.71 \text{ lb} \measuredangle 66.0^\circ \blacktriangleleft$



PROBLEM 4.148

For the frame and loading shown, determine the reactions at A and C.

SOLUTION



Since member AB is acted upon by two forces, **A** and **B**, they must be colinear, have the same magnitude, and be opposite in direction for AB to be in equilibrium. The force **B** acting at B of member BCD will be equal in magnitude but opposite in direction to force **B** acting on member AB. Member BCD is a three-force body with member forces intersecting at E. The f.b.d.'s of members AB and BCD illustrate the above conditions. The force triangle for member BCD is also shown. The angle β is found from the member dimensions:

$$\beta = \tan^{-1}\left(\frac{6 \text{ in.}}{10 \text{ in.}}\right) = 30.964°$$

Applying of the law of sines to the force triangle for member BCD,

$$\frac{30 \text{ lb}}{\sin(45° - \beta)} = \frac{B}{\sin\beta} = \frac{C}{\sin 135°}$$

or

$$\frac{30 \text{ lb}}{\sin 14.036°} = \frac{B}{\sin 30.964°} = \frac{C}{\sin 135°}$$

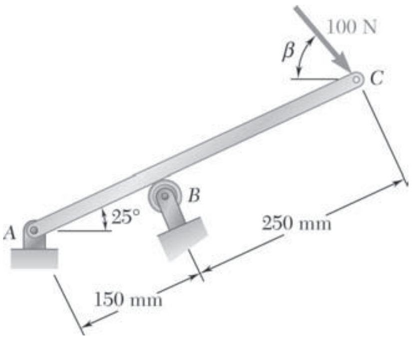
$$A = B = \frac{(30 \text{ lb})\sin 30.964°}{\sin 14.036°} = 63.641 \text{ lb}$$

or **A** $= 63.6$ lb ⦣ $45.0°$ ◀

and

$$C = \frac{(30 \text{ lb})\sin 135°}{\sin 14.036°} = 87.466 \text{ lb}$$

or **C** $= 87.5$ lb ⦣ $59.0°$ ◀



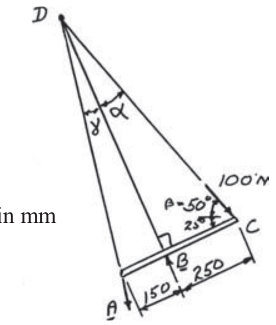
PROBLEM 4.149

Determine the reactions at A and B when $\beta = 50°$.

SOLUTION

Free-Body Diagram: (Three-force body)

Reaction **A** must pass through Point D where 100-N force and **B** intersect



Dimensions in mm

In right Δ BCD

$$\alpha = 90° - 75° = 15°$$

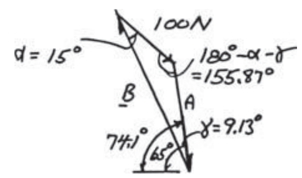
$$BD = 250 \tan 75° = 933.01 \text{ mm}$$

In right Δ ABD

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}}$$

$$\gamma = 9.13$$

Force Triangle

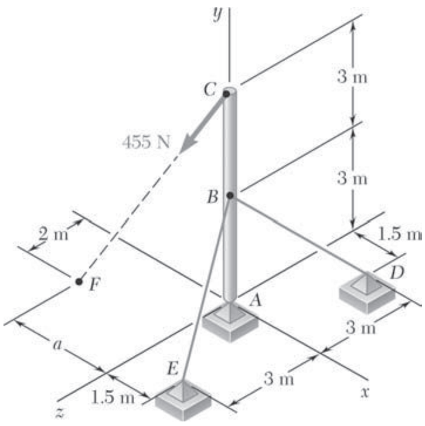


Law of sines

$$\frac{100 \text{ N}}{\sin 9.13°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.87°}$$

$$A = 163.1 \text{ N}; \quad B = 257.6 \text{ N}$$

A = 163.1 N ∠ 74.1° **B** = 258 N ∠ 65.0° ◀

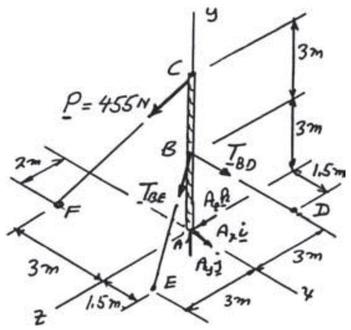


PROBLEM 4.150

The 6-m pole ABC is acted upon by a 455-N force as shown. The pole is held by a ball-and-socket joint at A and by two cables BD and BE. For $a = 3$ m, determine the tension in each cable and the reaction at A.

SOLUTION

Free-Body Diagram:



Five unknowns and six Eqs. of equilibrium, but equilibrium is maintained

$$(\Sigma M_{AC} = 0)$$

$$\mathbf{r}_B = 3\mathbf{j}$$
$$\mathbf{r}_C = 6\mathbf{j}$$

$$\overrightarrow{CF} = -3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k} \quad CF = 7 \text{ m}$$
$$\overrightarrow{BD} = 1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k} \quad BD = 4.5 \text{ m}$$
$$\overrightarrow{BE} = 1.5\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} \quad BE = 4.5 \text{ m}$$

$$\mathbf{P} = P\frac{\overrightarrow{CF}}{CE} = \frac{P}{7}(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$$

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}) = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

$$\mathbf{T}_{BE} = T_{BE} = \frac{\overrightarrow{BE}}{BE} = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_B \times \mathbf{T}_{BE} + \mathbf{r}_C \times \mathbf{P} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix} \frac{T_{BD}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & 2 \end{vmatrix} \frac{T_{BE}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -3 & -6 & 2 \end{vmatrix} \frac{P}{7} = 0$$

Coefficient of **i**:
$$-2T_{BD} + 2T_{BE} + \frac{12}{7}P = 0 \tag{1}$$

Coefficient of **k**:
$$-T_{BD} - T_{BF} + \frac{18}{7}P = 0 \tag{2}$$

PROBLEM 4.150 (Continued)

Eq. (1) + 2 Eq. (2): $-4T_{BD}+\frac{48}{7}P=0 \quad T_{BD}=\frac{12}{7}P$

Eq. (2): $-\frac{12}{7}P-T_{BE}+\frac{18}{7}P=0 \quad T_{BE}=\frac{6}{7}P$

Since $P=445\text{ N} \quad T_{BD}=\frac{12}{7}(455)$ $\qquad T_{BD}=780\text{ N}$ ◀

$$T_{BE}=\frac{6}{7}(455) \qquad T_{BE}=390\text{ N} \;◀$$

$$\Sigma\mathbf{F}=0\text{:}\quad \mathbf{T}_{BD}+\mathbf{T}_{BE}+\mathbf{P}+\mathbf{A}=0$$

Coefficient of **i**: $\frac{780}{3}+\frac{390}{3}-\frac{455}{7}(3)+A_x=0$

$$260+130-195+A_x=0 \quad A_x=195.0\text{ N}$$

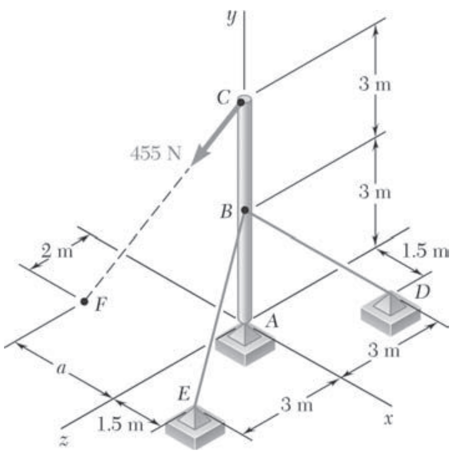
Coefficient of **j**: $-\frac{780}{3}(2)-\frac{390}{3}(2)-\frac{455}{7}(6)+A_y=0$

$$-520-260-390+A_y=0 \quad A_y=1170\text{ N}$$

Coefficient of **k**: $-\frac{780}{3}(2)+\frac{390}{3}(2)+\frac{455}{7}(2)+A_z=0$

$$-520+260+130+A_z=0 \quad A_z=+130.0\text{ N}$$

$$\mathbf{A}=-(195.0\text{ N})\mathbf{i}+(1170\text{ N})\mathbf{j}+(130.0\text{ N})\mathbf{k} \;◀$$



PROBLEM 4.151

Solve Problem 4.150 for $a = 1.5$ m.

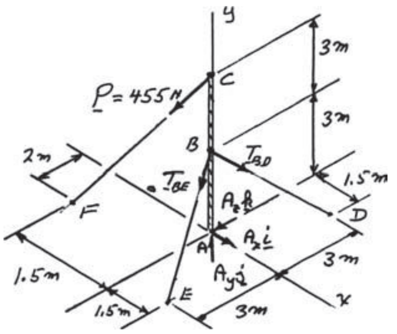
PROBLEM 4.150 The 6-m pole *ABC* is acted upon by a 455-N force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. For $a = 3$ m, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:

Five unknowns and six Eqs. of equilibrium but equilibrium is maintained

$(\Sigma M_{AC} = 0)$



$$\mathbf{r}_B = 3\mathbf{j}$$

$$\mathbf{r}_C = 6\mathbf{j}$$

$$\overrightarrow{CF} = -1.5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k} \quad CF = 6.5 \text{ m}$$

$$\overrightarrow{BD} = 1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k} \quad BD = 4.5 \text{ m}$$

$$\overrightarrow{BE} = 1.5\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} \quad BE = 4.5 \text{ m}$$

$$\mathbf{P} = P\frac{\overrightarrow{CF}}{CE} = \frac{P}{6.5}(-1.5\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) = \frac{P}{13}(-3\mathbf{i} - 12\mathbf{j} + 4\mathbf{k})$$

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}) = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

$$\mathbf{T}_{BE} = T_{BE} = \frac{\overrightarrow{BE}}{BE} = \frac{T_{BD}}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_B \times \mathbf{T}_{BE} + \mathbf{r}_C \times \mathbf{P} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix}\frac{T_{BD}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 1 & -2 & 2 \end{vmatrix}\frac{T_{BE}}{3} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -3 & -12 & +4 \end{vmatrix}\frac{P}{13} = 0$$

Coefficient of **i**: $$-2T_{BD} + 2T_{BE} + \frac{24}{13}P = 0 \qquad (1)$$

Coefficient of **k**: $$-T_{BD} - T_{BE} + \frac{18}{13}P = 0 \qquad (2)$$

PROBLEM 4.151 (Continued)

Eq. (1) + 2 Eq. (2): $-4T_{BD} + \frac{60}{13}P = 0 \quad T_{BD} = \frac{15}{13}P$

Eq (2): $-\frac{15}{13}P - T_{BE} + \frac{18}{13}P = 0 \quad T_{BE} = \frac{3}{13}P$

Since $P = 445\text{ N} \quad T_{BD} = \frac{15}{13}(455)$ $\quad T_{BD} = 525\text{ N}$ ◀

$T_{BE} = \frac{3}{13}(455)$ $\quad T_{BE} = 105.0\text{ N}$ ◀

$$\Sigma\mathbf{F} = 0\text{:} \quad \mathbf{T}_{BD} + \mathbf{T}_{BE} + \mathbf{P} + \mathbf{A} = 0$$

Coefficient of **i**: $\frac{525}{3} + \frac{105}{3} - \frac{455}{13}(3) + A_x = 0$

$$175 + 35 - 105 + A_x = 0 \quad A_x = 105.0\text{ N}$$

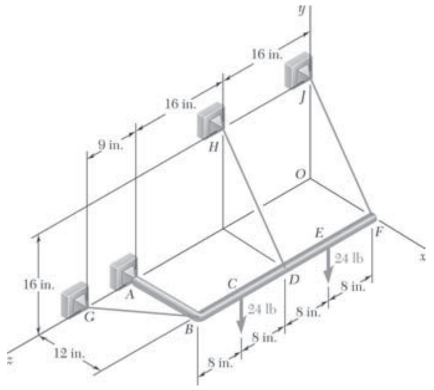
Coefficient of **j**: $-\frac{525}{3}(2) - \frac{105}{3}(2) - \frac{455}{13}(12) + A_y = 0$

$$-350 - 70 - 420 + A_y = 0 \quad A_y = 840\text{ N}$$

Coefficient of **k**: $-\frac{525}{3}(2) + \frac{105}{3}(2) + \frac{455}{13}(4) + A_z = 0$

$$-350 + 70 + 140 + A_z = 0 \quad A_z = 140.0\text{ N}$$

$\mathbf{A} = -(105.0\text{ N})\mathbf{i} + (840\text{ N})\mathbf{j} + (140.0\text{ N})\mathbf{k}$ ◀

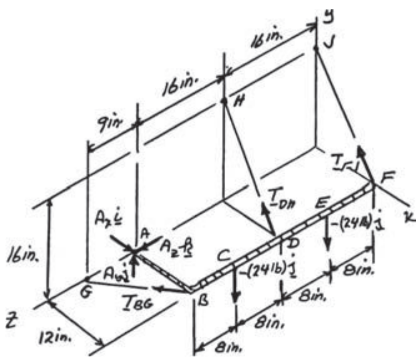


PROBLEM 4.152

The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 12\mathbf{i}$$
$$\mathbf{r}_{F/A} = 12\mathbf{j} - 8\mathbf{k}$$
$$\mathbf{r}_{D/A} = 12\mathbf{i} - 16\mathbf{k}$$
$$\mathbf{r}_{E/A} = 12\mathbf{i} - 24\mathbf{k}$$
$$\mathbf{r}_{F/A} = 12\mathbf{i} - 32\mathbf{k}$$
$$\overrightarrow{BG} = -12\mathbf{i} + 9\mathbf{k}$$
$$BG = 15 \text{ in.}$$
$$\boldsymbol{\lambda}_{BG} = -0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\overrightarrow{DH} = -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \lambda_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j}$$
$$\overrightarrow{FJ} = -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \lambda_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{T}_{BG}\lambda_{BG} + \mathbf{r}_{DH} \times \mathbf{T}_{DH}\lambda_{DH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FJ}\lambda_{FJ}$$
$$+\mathbf{r}_{F/A} \times (-24\mathbf{j}) + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ}$$

$$+\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -8 \\ 0 & -24 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

Coefficient of **i**: $$+12.8T_{DH} + 25.6T_{FJ} - 192 - 576 = 0 \quad (1)$$

Coefficient of **k**: $$+9.6T_{DH} + 9.6T_{FJ} - 288 - 288 = 0 \quad (2)$$

$\frac{3}{4}$ Eq. (1) – Eq. (2): $$9.6T_{FJ} = 0$$ $T_{FJ} = 0$ ◀

PROBLEM 4.152 (Continued)

Eq. (1): $12.8T_{DH} - 268 = 0$ $T_{DH} = 60 \text{ lb}$ ◀

Coefficient of **j:** $-7.2T_{BG} + (16 \times 0.6)(60.0 \text{ lb}) = 0$ $T_{BG} = 80.0 \text{ lb}$ ◀

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + T_{BG}\boldsymbol{\lambda}_{BG} + T_{DH}\boldsymbol{\lambda}_{DH} + T_{FJ} - 24\mathbf{j} - 24\mathbf{j} = 0$$

Coefficient of **i**: $A_x + (80)(-0.8) + (60.0)(-0.6) = 0$ $A_x = 100.0 \text{ lb}$

Coefficient of **j**: $A_y + (60.0)(0.8) - 24 - 24 = 0$ $A_y = 0$

Coefficient of **k:** $A_z + (80.0)(+0.6) = 0$ $A_z = -48.0 \text{ lb}$

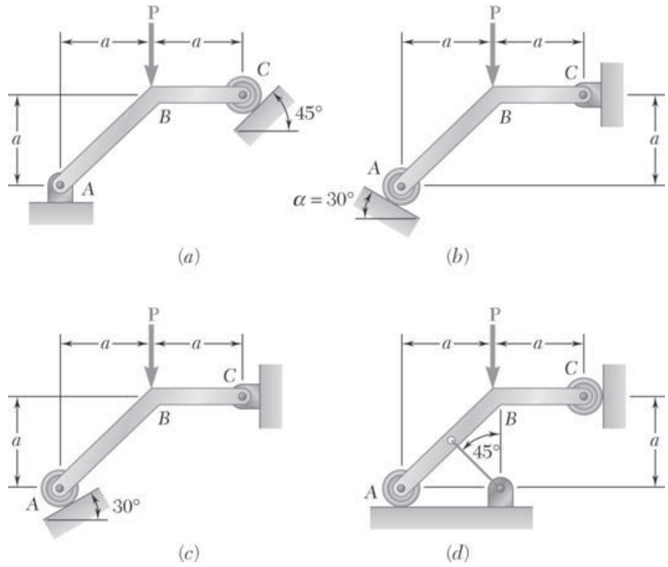
$\mathbf{A} = (100.0 \text{ lb})\mathbf{i} - (48.0 \text{ lb})\mathbf{j}$ ◀

Note: The value $A_y = 0$

Can be confirmed by considering $\Sigma M_{BF} = 0$

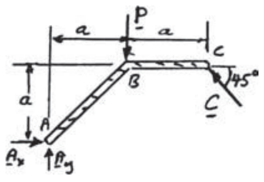
PROBLEM 4.153

A force **P** is applied to a bent rod *ABC*, which may be supported in four different ways as shown. In each case, if possible, determine the reactions at the supports.



SOLUTION

(*a*)



$$+\circlearrowleft \Sigma M_A = 0: \quad -P_a + (C\sin 45°)2a + (\cos 45°)a = 0$$

$$3\frac{C}{\sqrt{2}} = P \quad C = \frac{\sqrt{2}}{3}P$$

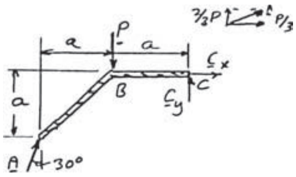
$\mathbf{C} = 0.471P$ ⦣ 45° ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_x = \frac{P}{3} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P + \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_y = \frac{2P}{3}\uparrow$$

$\mathbf{A} = 0.745P$ ⦣ 63.4° ◀

(*b*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 - 0.5) = P \quad A = 0.812P$$

$\mathbf{A} = 0.812P$ ⦣ 60.0° ◀

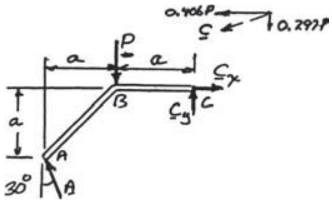
$$\xrightarrow{+} \Sigma F_x = 0: \quad (0.812P)\sin 30° + C_x = 0 \quad C_x = -0.406P$$

$$+\uparrow \Sigma F_y = 0: \quad (0.812P)\cos 30° - P + C_y = 0 \quad C_y = -0.297P$$

$\mathbf{C} = 0.503P$ ⦣ 36.2° ◀

PROBLEM 4.153 (Continued)

(*c*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 + 0.5) = P \quad A = 0.448P$$

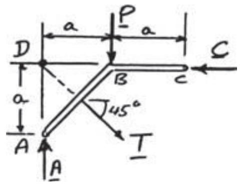
$$A = 0.448P \quad 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(0.448P)\sin 30° + C_x = 0 \qquad C_x = 0.224P \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad (0.448P)\cos 30° - P + C_y = 0 \quad C_y = 0.612P \uparrow$$

$$\mathbf{C} = 0.652P \quad 69.9° \blacktriangleleft$$

(*d*) Force **T** exerted by wire and reactions **A** and **C** all intersect at Point *D*.



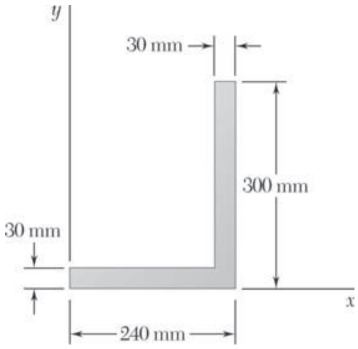
$$+\circlearrowleft \Sigma M_D = 0: \quad P_a = 0$$

Equilibrium not maintained

Rod is improperly constrained ◀

CHAPTER 5

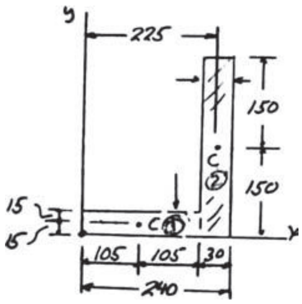
PROBLEM 5.1



Locate the centroid of the plane area shown.

SOLUTION

Dimensions in mm

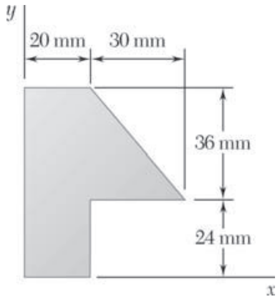


	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	6300	105	15	0.66150×10^6	0.094500×10^6
2	9000	225	150	2.0250×10^6	1.35000×10^6
Σ	15300			2.6865×10^6	1.44450×10^6

Then
$$\bar{X} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{2.6865\times10^6}{15300}$$
$\bar{X} = 175.6$ mm ◀

$$\bar{Y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{1.44450\times10^6}{15300}$$
$\bar{Y} = 94.4$ mm ◀

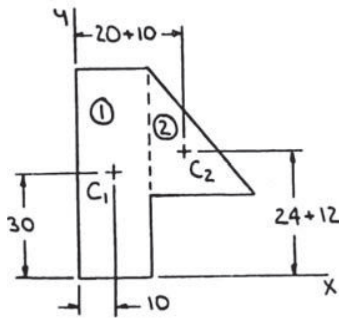
PROBLEM 5.2



Locate the centroid of the plane area shown.

SOLUTION

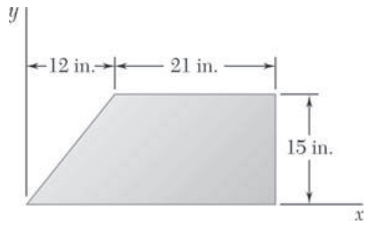
Dimensions in mm



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	1200	10	30	12000	36000
2	540	30	36	16200	19440
Σ	1740			28200	55440

Then $$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{28200}{1740}$$ $\bar{X} = 16.21\text{ mm}$ ◀

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{55440}{1740}$$ $\bar{Y} = 31.9\text{ mm}$ ◀

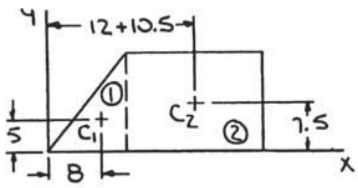


PROBLEM 5.3

Locate the centroid of the plane area shown.

SOLUTION

Dimensions in in.

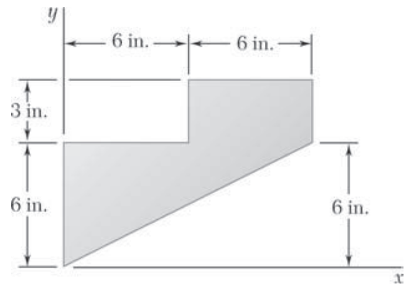


	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$\frac{1}{2}\times 12\times 15 = 90$	8	5	720	450
2	$21\times 15 = 315$	22.5	7.5	7087.5	2362.5
Σ	405.00			7807.5	2812.5

Then

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{7807.5}{405.00} \qquad \bar{X} = 19.28 \text{ in.} \blacktriangleleft$$

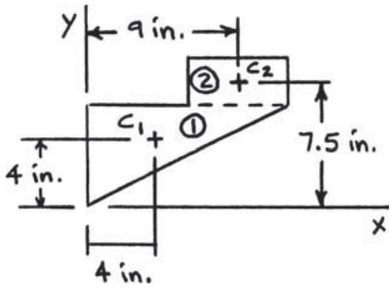
$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{2812.5}{405.00} \qquad \bar{Y} = 6.94 \text{ in.} \blacktriangleleft$$



PROBLEM 5.4

Locate the centroid of the plane area shown.

SOLUTION



	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$\frac{1}{2}(12)(6) = 36$	4	4	144	144
2	$(6)(3) = 18$	9	7.5	162	135
Σ	54			306	279

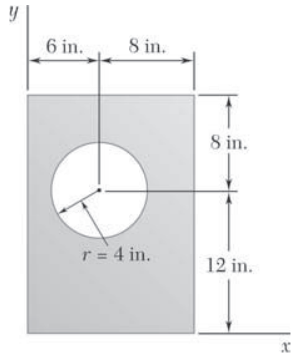
Then

$$\bar{X}A = \Sigma \bar{x}A$$

$$\bar{X}(54) = 306 \qquad \bar{X} = 5.67 \text{ in.} \blacktriangleleft$$

$$\bar{Y}A = \Sigma \bar{y}A$$

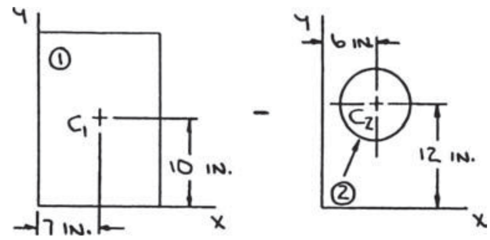
$$\bar{Y}(54) = 279 \qquad \bar{Y} = 5.17 \text{ in.} \blacktriangleleft$$



PROBLEM 5.5

Locate the centroid of the plane area shown.

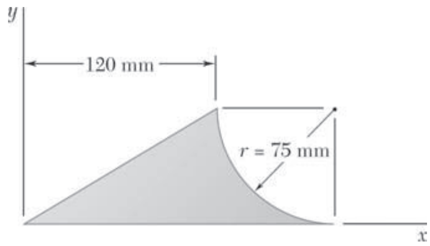
SOLUTION



	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$14 \times 20 = 280$	7	10	1960	2800
2	$-\pi(4)^2 = -16\pi$	6	12	–301.59	–603.19
Σ	229.73			1658.41	2196.8

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1658.41}{229.73}$$ $\bar{X} = 7.22$ in. ◀

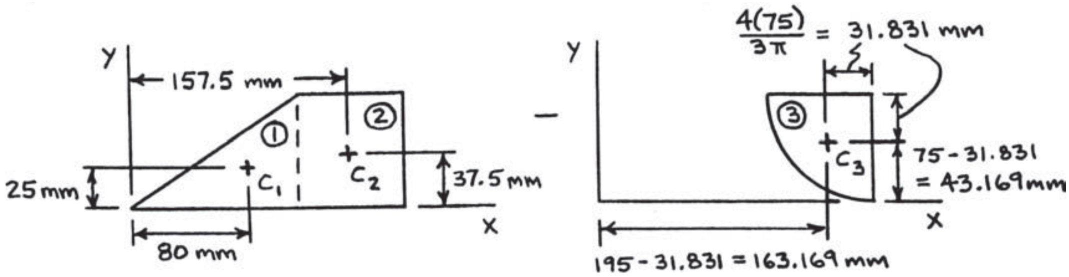
$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{2196.8}{229.73}$$ $\bar{Y} = 9.56$ in. ◀



PROBLEM 5.6

Locate the centroid of the plane area shown.

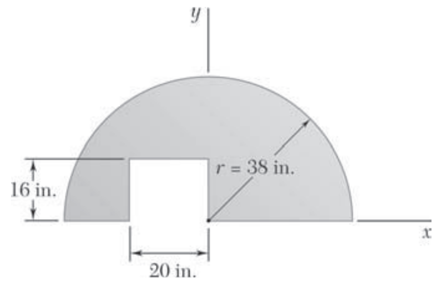
SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{2}(120)(75) = 4500$	80	25	360×10^3	112.5×10^3
2	$(75)(75) = 5625$	157.5	37.5	885.94×10^3	210.94×10^3
3	$-\frac{\pi}{4}(75)^2 = -4417.9$	163.169	43.169	-720.86×10^3	-190.716×10^3
Σ	5707.1			525.08×10^3	132.724×10^3

Then $\quad \bar{X}A = \Sigma\bar{x}A \quad \bar{X}(5707.1) = 525.08\times10^3 \quad \bar{X} = 92.0$ mm ◀

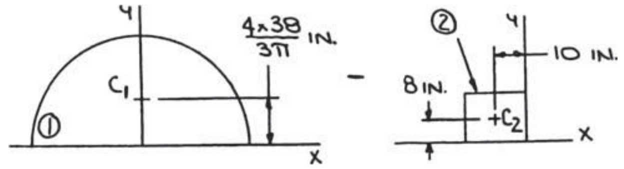
$\bar{Y}A = \Sigma\bar{y}A \quad \bar{Y}(5707.1) = 132.724\times10^3 \quad \bar{Y} = 23.3$ mm ◀



PROBLEM 5.7

Locate the centroid of the plane area shown.

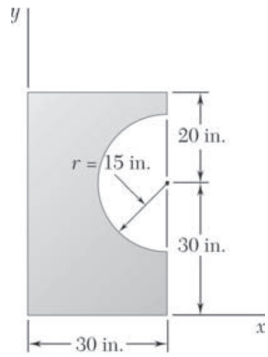
SOLUTION



	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$\frac{\pi}{2}(38)^2 = 2268.2$	0	16.1277	0	36581
2	$-20\times 16 = -320$	-10	8	3200	-2560
Σ	1948.23			3200	34021

Then $\quad \bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{3200}{1948.23}$ $\quad \bar{X} = 1.643$ in. ◀

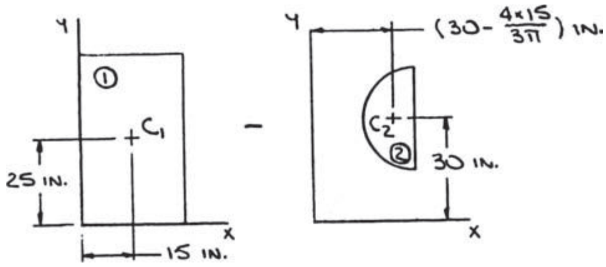
$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{34021}{1948.23}$ $\quad \bar{Y} = 17.46$ in. ◀



PROBLEM 5.8

Locate the centroid of the plane area shown.

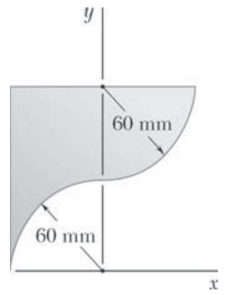
SOLUTION



	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$30\times 50 = 1500$	15	25	22500	37500
2	$-\frac{\pi}{2}(15)^2 = 353.43$	23.634	30	–8353.0	–10602.9
Σ	1146.57			14147.0	26.897

Then
$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{14147.0}{1146.57} \qquad \bar{X} = 12.34 \text{ in.} \blacktriangleleft$$

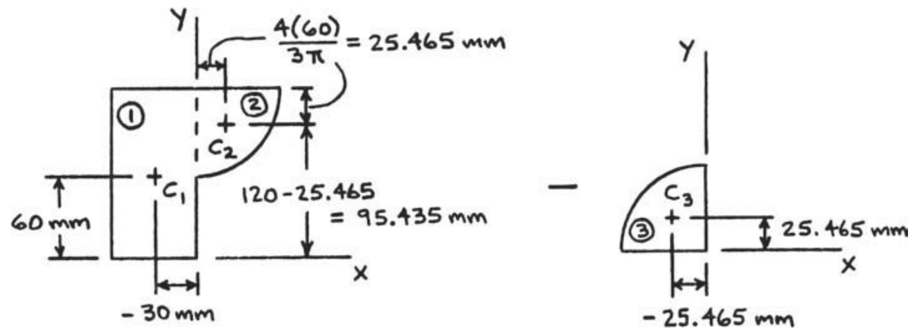
$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{26897}{1146.57} \qquad \bar{Y} = 23.5 \text{ in.} \blacktriangleleft$$



PROBLEM 5.9

Locate the centroid of the plane area shown.

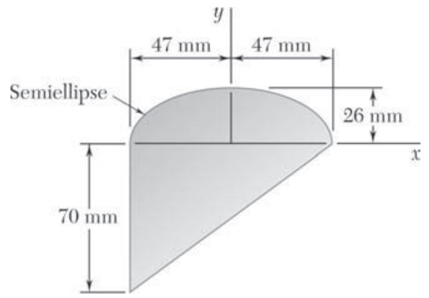
SOLUTION



	A, mm^2	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{x}A$, mm^3	$\overline{y}A$, mm^3
1	$(60)(120) = 7200$	–30	60	-216×10^3	432×10^3
2	$\frac{\pi}{4}(60)^2 = 2827.4$	25.465	95.435	72.000×10^3	269.83×10^3
3	$-\frac{\pi}{4}(60)^2 = -2827.4$	–25.465	25.465	72.000×10^3	-72.000×10^3
Σ	7200			-72.000×10^3	629.83×10^3

Then $\overline{X}A = \Sigma\overline{x}A \qquad \overline{X}(7200) = -72.000\times10^3 \qquad \overline{X} = -10.00 \text{ mm}$ ◀

$\overline{Y}A = \Sigma\overline{y}A \qquad \overline{Y}(7200) = 629.83\times10^3 \qquad \overline{Y} = 87.5 \text{ mm}$ ◀

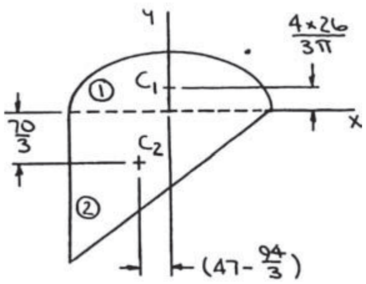


PROBLEM 5.10

Locate the centroid of the plane area shown.

SOLUTION

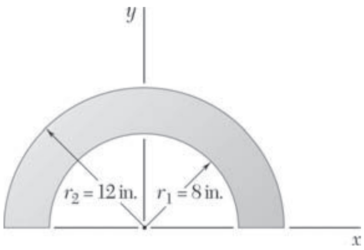
Dimensions in mm



	A, mm^2	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{x}A$, mm^3	$\overline{y}A$, mm^3
1	$\frac{\pi}{2}\times 47\times 26 = 1919.51$	0	11.0347	0	21181
2	$\frac{1}{2}\times 94\times 70 = 3290$	−15.6667	−23.333	−51543	−76766
Σ	5209.5			−51543	−55584

Then
$$\overline{X} = \frac{\Sigma \overline{x} A}{\Sigma A} = \frac{-51543}{5209.5}$$
$\overline{X} = -9.89$ mm ◀

$$\overline{Y} = \frac{\Sigma \overline{y} A}{\Sigma A} = \frac{-55584}{5209.5}$$
$\overline{Y} = -10.67$ mm ◀

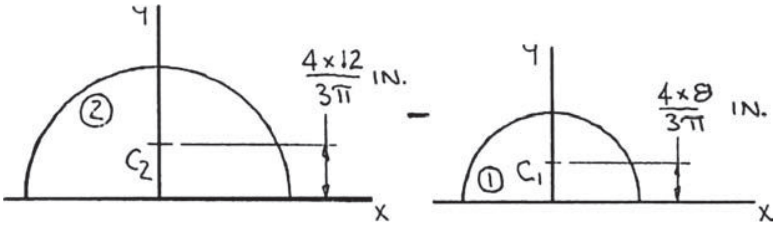


PROBLEM 5.11

Locate the centroid of the plane area shown.

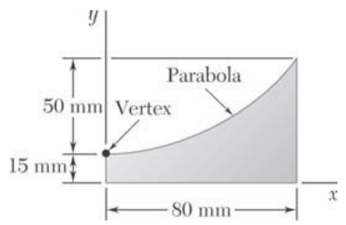
SOLUTION

First note that symmetry implies $\qquad \overline{X} = 0$ ◀



	A, in.^{2}	$\overline{y}$, in.	$\overline{y}A$, in.^{3}
1	$-\dfrac{\pi(8)^2}{2} = -100.531$	3.3953	–341.33
2	$\dfrac{\pi(12)^2}{2} = 226.19$	5.0930	1151.99
Σ	125.659		810.66

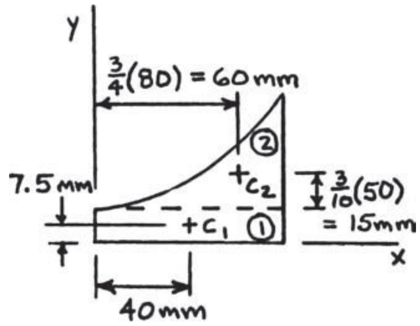
Then $$\overline{Y} = \frac{\Sigma \overline{y} A}{\Sigma A} = \frac{810.66 \text{ in.}^3}{125.66 \text{ in.}^2}$$ or $\overline{Y} = 6.45$ in. ◀



PROBLEM 5.12

Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$(15)(80) = 1200$	40	7.5	48×10^3	9×10^3
2	$\frac{1}{3}(50)(80) = 1333.33$	60	30	80×10^3	40×10^3
Σ	2533.3			128×10^3	49×10^3

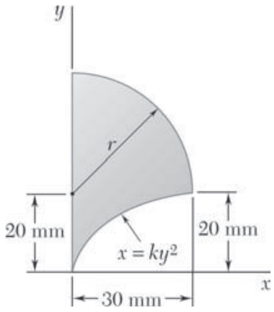
Then

$$\bar{X}A = \Sigma \bar{x}A$$

$$\bar{X}(2533.3) = 128 \times 10^3 \qquad \bar{X} = 50.5 \text{ mm} \blacktriangleleft$$

$$\bar{Y}A = \Sigma \bar{y}A$$

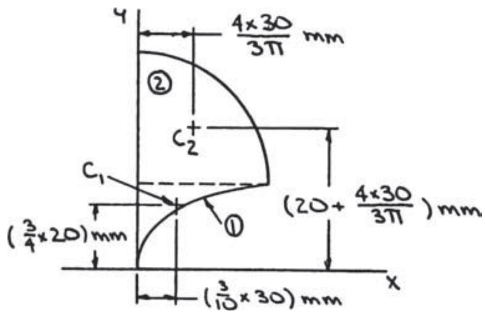
$$\bar{Y}(2533.3) = 49 \times 10^3 \qquad \bar{Y} = 19.34 \text{ mm} \blacktriangleleft$$



PROBLEM 5.13

Locate the centroid of the plane area shown.

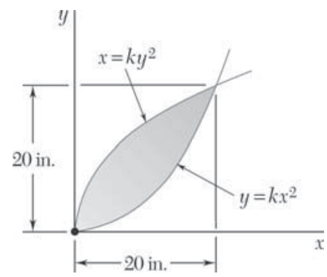
SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{3}\times30\times20=200$	9	15	1800	3000
2	$\frac{\pi}{4}(30)^2=706.86$	12.7324	32.7324	9000.0	23137
Σ	906.86			10800	26137

Then
$$\bar{X}=\frac{\Sigma\bar{x}A}{\Sigma A}=\frac{10800}{906.86}$$ $\bar{X}=11.91\text{ mm}$ ◀

$$\bar{Y}=\frac{\Sigma\bar{y}A}{\Sigma A}=\frac{26137}{906.86}$$ $\bar{Y}=28.8\text{ mm}$ ◀

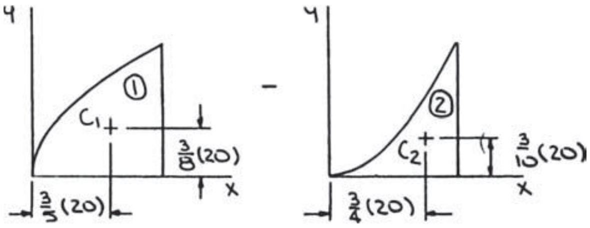


PROBLEM 5.14

Locate the centroid of the plane area shown.

SOLUTION

Dimensions in in.



	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in.^{3}
1	$\frac{2}{3}\times(20)(20) = \frac{800}{3}$	12	7.5	3200	2000
2	$\frac{-1}{3}(20)(20) = \frac{-400}{3}$	15	6.0	–2000	–800
Σ	$\frac{400}{3}$			1200	1200

Then

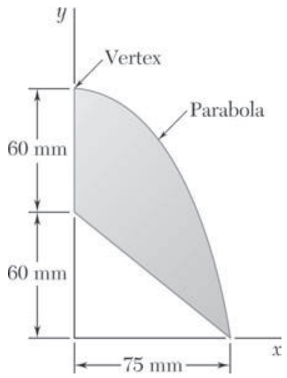
$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1200}{\left(\frac{400}{3}\right)}$$

$\bar{X} = 9.00$ in. ◀

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{1200}{\left(\frac{400}{3}\right)}$$

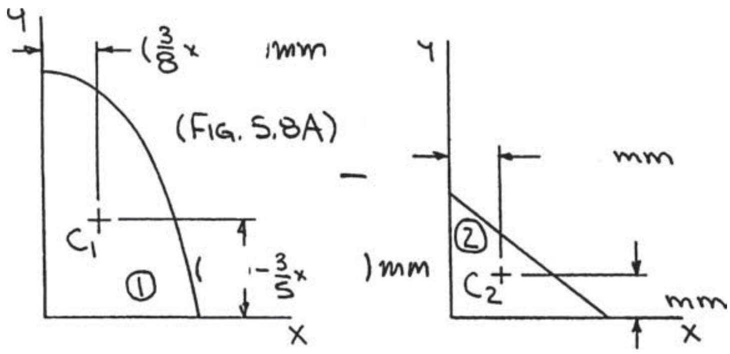
$\bar{Y} = 9.00$ in. ◀

PROBLEM 5.15



Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{2}{3}(75)(120) = 6000$	28.125	48	168750	288000
2	$-\frac{1}{2}(75)(60) = -2250$	25	20	–56250	–45000
Σ	3750			112500	243000

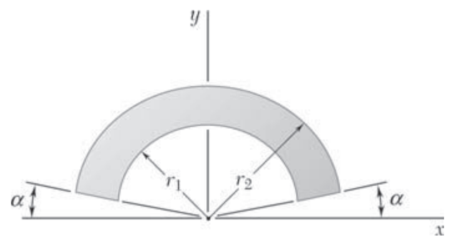
Then $$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(3750 \text{ mm}^2) = 112500 \text{ mm}^3$$ or $\bar{X} = 30.0$ mm ◀

and $$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

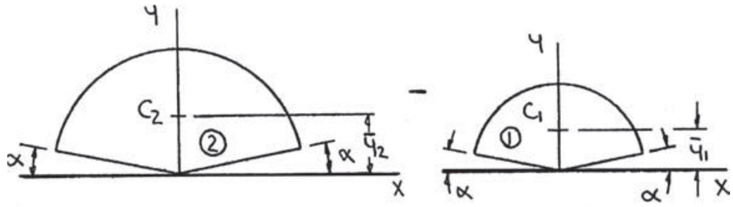
$$\bar{Y}(3750 \text{ mm}^2) = 243000$$ or $\bar{Y} = 64.8$ mm ◀

PROBLEM 5.16



Determine the y coordinate of the centroid of the shaded area in terms of r_1, r_2, and α.

SOLUTION



First, determine the location of the centroid.

From Figure 5.8A:

$$\bar{y}_2 = \frac{2}{3} r_2 \frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\left(\frac{\pi}{2} - \alpha\right)} \qquad A_2 = \left(\frac{\pi}{2} - \alpha\right) r_2^2$$

$$= \frac{2}{3} r_2 \frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)}$$

Similarly

$$\bar{y}_1 = \frac{2}{3} r_1 \frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)} \qquad A_1 = \left(\frac{\pi}{2} - \alpha\right) r_1^2$$

Then

$$\Sigma \bar{y} A = \frac{2}{3} r_2 \frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\frac{\pi}{2} - \alpha\right) r_2^2\right] - \frac{2}{3} r_1 \frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\frac{\pi}{2} - \alpha\right) r_1^2\right]$$

$$= \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

and

$$\Sigma A = \left(\frac{\pi}{2} - \alpha\right) r_2^2 - \left(\frac{\pi}{2} - \alpha\right) r_1^2$$

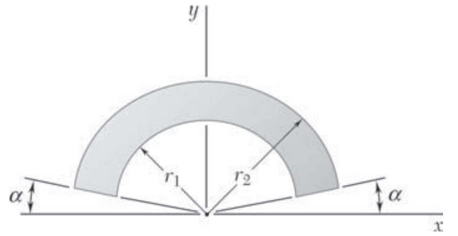
$$= \left(\frac{\pi}{2} - \alpha\right)\left(r_2^2 - r_1^2\right)$$

Now

$$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}\left[\left(\frac{\pi}{2} - \alpha\right)\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

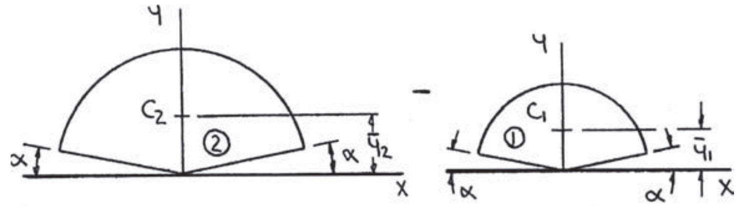
$$\bar{Y} = \frac{2}{3}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)\left(\frac{2\cos\alpha}{\pi - 2\alpha}\right) \blacktriangleleft$$



PROBLEM 5.17

Show that as r_1 approaches r_2, the location of the centroid approaches that for an arc of circle of radius $(r_1 + r_2)/2$.

SOLUTION



First, determine the location of the centroid.

From Figure 5.8A:
$$\bar{y}_2 = \frac{2}{3} r_2 \frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\left(\frac{\pi}{2} - \alpha\right)} \qquad A_2 = \left(\frac{\pi}{2} - \alpha\right) r_2^2$$
$$= \frac{2}{3} r_2 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)}$$

Similarly
$$\bar{y}_1 = \frac{2}{3} r_1 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \qquad A_1 = \left(\frac{\pi}{2} - \alpha\right) r_1^2$$

Then
$$\Sigma \bar{y} A = \frac{2}{3} r_2 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\frac{\pi}{2} - \alpha\right) r_2^2\right] - \frac{2}{3} r_1 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\frac{\pi}{2} - \alpha\right) r_1^2\right]$$
$$= \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos \alpha$$

and
$$\Sigma A = \left(\frac{\pi}{2} - \alpha\right) r_2^2 - \left(\frac{\pi}{2} - \alpha\right) r_1^2$$
$$= \left(\frac{\pi}{2} - \alpha\right)\left(r_2^2 - r_1^2\right)$$

Now
$$\bar{Y} \Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}\left[\left(\frac{\pi}{2} - \alpha\right)\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos \alpha$$

$$\bar{Y} = \frac{2}{3}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)\left(\frac{2\cos \alpha}{\pi - 2\alpha}\right)$$

PROBLEM 5.17 (Continued)

Using Figure 5.8B, $\overline{Y}$ of an arc of radius $\frac{1}{2}(r_1+r_2)$ is

$$\overline{Y} = \frac{1}{2}(r_1+r_2)\frac{\sin(\frac{\pi}{2}-\alpha)}{(\frac{\pi}{2}-\alpha)}$$

$$= \frac{1}{2}(r_1+r_2)\frac{\cos\alpha}{(\frac{\pi}{2}-\alpha)} \qquad (1)$$

Now
$$\frac{r_2^3-r_1^3}{r_2^2-r_1^2} = \frac{(r_2-r_1)\left(r_2^2+r_1r_2+r_1^2\right)}{(r_2-r_1)(r_2+r_1)}$$

$$= \frac{r_2^2+r_1r_2+r_1^2}{r_2+r_1}$$

Let
$$r_2 = r+\Delta$$
$$r_1 = r-\Delta$$

Then
$$r = \frac{1}{2}(r_1+r_2)$$

and
$$\frac{r_2^3-r_1^3}{r_2^2-r_1^2} = \frac{(r+\Delta)^2+(r+\Delta)(r-\Delta)(r-\Delta)^2}{(r+\Delta)+(r-\Delta)}$$

$$= \frac{3r^2+\Delta^2}{2r}$$

In the limit as $\Delta \longrightarrow 0$ (i.e., $r_1 = r_2$), then

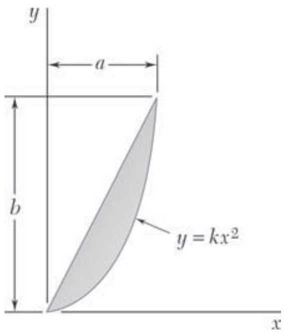
$$\frac{r_2^3-r_1^3}{r_2^2-r_1^2} = \frac{3}{2}r$$

$$= \frac{3}{2}\times\frac{1}{2}(r_1+r_2)$$

So that
$$\overline{Y} = \frac{2}{3}\times\frac{3}{4}(r_1+r_2)\frac{\cos\alpha}{\frac{\pi}{2}-\alpha}$$
or $\overline{Y} = (r_1+r_2)\dfrac{\cos\alpha}{\pi-2\alpha}$ ◀

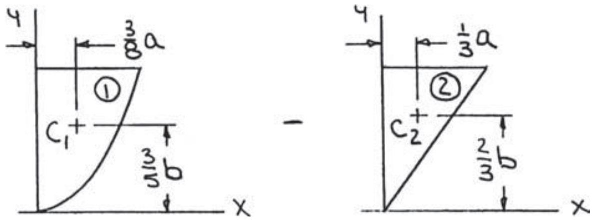
Which agrees with Equation (1).

PROBLEM 5.18



For the area shown, determine the ratio a/b for which $\bar{x} = \bar{y}$.

SOLUTION



	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$\frac{2}{3}ab$	$\frac{3}{8}a$	$\frac{3}{5}b$	$\frac{a^2b}{4}$	$\frac{2ab^2}{5}$
2	$-\frac{1}{2}ab$	$\frac{1}{3}a$	$\frac{2}{3}b$	$-\frac{a^2b}{6}$	$-\frac{ab^2}{3}$
Σ	$\frac{1}{6}ab$			$\frac{a^2b}{12}$	$\frac{ab^2}{15}$

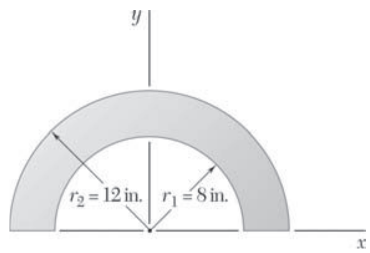
Then
$$\bar{X}\Sigma A = \Sigma \bar{x} A$$
$$\bar{X}\left(\frac{1}{6}ab\right) = \frac{a^2b}{12}$$

or
$$\bar{X} = \frac{1}{2}a$$
$$\bar{Y}\Sigma A = \Sigma \bar{y} A$$
$$\bar{Y}\left(\frac{1}{6}ab\right) = \frac{ab^2}{15}$$

or
$$\bar{Y} = \frac{2}{5}b$$

Now
$$\bar{X} = \bar{Y} \Rightarrow \qquad \frac{1}{2}a = \frac{2}{5}b$$

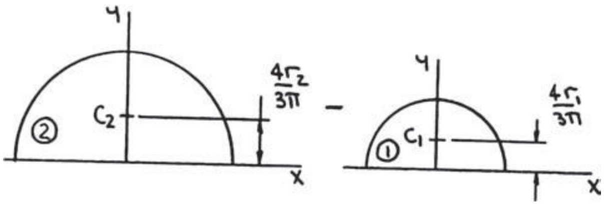
or $\frac{a}{b} = \frac{4}{5}$ ◀



PROBLEM 5.19

For the semiannular area of Problem 5.11, determine the ratio r_2/r_1 so that $\overline{y} = 3r_1/4$.

SOLUTION



	A	$\overline{Y}$	$\overline{Y}A$
1	$-\frac{\pi}{2}r_1^2$	$\frac{4r_1}{3\pi}$	$-\frac{2}{3}r_1^3$
2	$\frac{\pi}{2}r_2^2$	$\frac{4r_2}{3\pi}$	$\frac{2}{3}r_2^3$
Σ	$\frac{\pi}{2}\left(r_2^2 - r_1^2\right)$		$\frac{2}{3}\left(r_2^3 - r_1^3\right)$

Then $$\overline{Y}\,\Sigma A = \Sigma\,\overline{y}A$$

or $$\frac{3}{4}r_1 \times \frac{\pi}{2}\left(r_2^2 - r_1^2\right) = \frac{2}{3}\left(r_2^3 - r_1^3\right)$$

$$\frac{9\pi}{16}\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right] = \left(\frac{r_2}{r_1}\right)^3 - 1$$

Let $$p = \frac{r_2}{r_1}$$

$$\frac{9\pi}{16}[(p+1)(p-1)] = (p-1)(p^2+p+1)$$

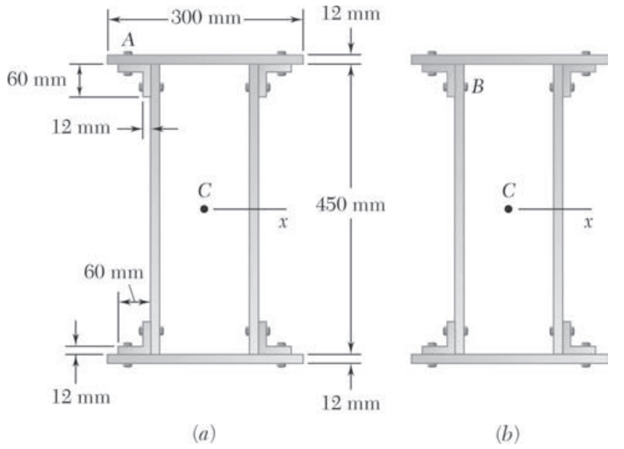
or $$16p^2 + (16 - 9\pi)p + (16 - 9\pi) = 0$$

PROBLEM 5.19 (Continued)

Then
$$p = \frac{-(16-9\pi) \pm \sqrt{(16-9\pi)^2 - 4(16)(16-9\pi)}}{2(16)}$$

or
$$p = -0.5726$$
$$p = 1.3397$$

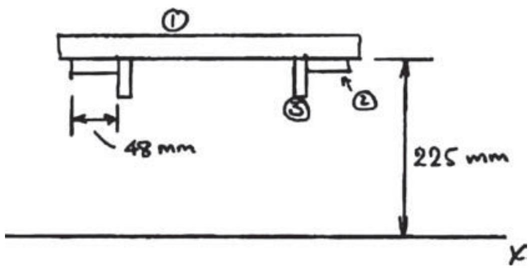
Taking the positive root
$$\frac{r_2}{r_1} = 1.340 \blacktriangleleft$$



PROBLEM 5.20

A composite beam is constructed by bolting four plates to four $60 \times 60 \times 12$-mm angles as shown. The bolts are equally spaced along the beam, and the beam supports a vertical load. As proved in mechanics of materials, the shearing forces exerted on the bolts at *A* and *B* are proportional to the first moments with respect to the centroidal *x* axis of the red shaded areas shown, respectively, in Parts *a* and *b* of the figure. Knowing that the force exerted on the bolt at *A* is 280 N, determine the force exerted on the bolt at *B*.

SOLUTION



From the problem statement: *F* is proportional to Q_x.

Therefore: $$\frac{F_A}{(Q_x)_A} = \frac{F_B}{(Q_x)_B}, \quad \text{or} \quad F_B = \frac{(Q_x)_B}{(Q_x)_A} F_A$$

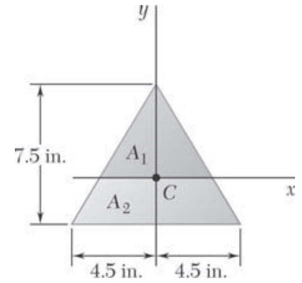
For the first moments: $$(Q_x)_A = \left(225 + \frac{12}{2}\right)(300 \times 12)$$

$$= 831600 \text{ mm}^3$$

$$(Q_x)_B = (Q_x)A + 2\left(225 - \frac{12}{2}\right)(48 \times 12) + 2(225 - 30)(12 \times 60)$$

$$= 1364688 \text{ mm}^3$$

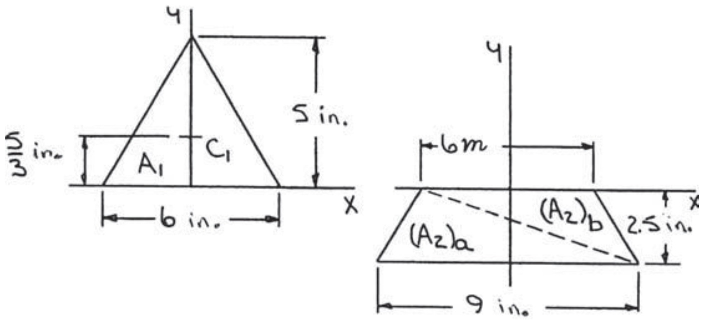
Then $$F_B = \frac{1364688}{831600}(280 \text{ N})$$ or $F_B = 459 \text{ N}$ ◀



PROBLEM 5.21

The horizontal x axis is drawn through the centroid C of the area shown, and it divides the area into two component areas A_1 and A_2. Determine the first moment of each component area with respect to the x axis, and explain the results obtained.

SOLUTION



Note that $$Q_x = \Sigma \overline{y} A$$

Then $$(Q_x)_1 = \left(\frac{5}{3}\text{in.}\right)\left(\frac{1}{2}\times 6\times 5\right)\text{in.}^2$$ or $(Q_x)_1 = 25.0 \text{ in.}^3$ ◀

and $$(Q_x)_2 = \left(-\frac{2}{3}\times 2.5\text{ in.}\right)\left(\frac{1}{2}\times 9\times 2.5\right)\text{in.}^2 + \left(-\frac{1}{3}\times 2.5\text{ in.}\right)\left(\frac{1}{2}\times 6\times 2.5\right)\text{in.}^2$$

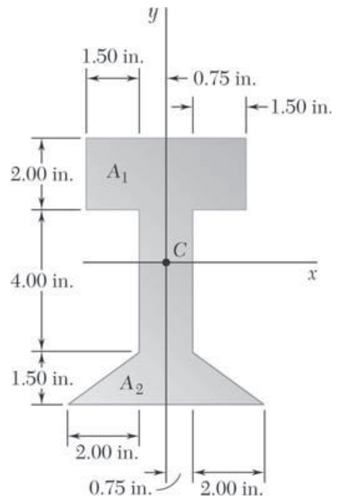
or $(Q_x)_2 = -25.0 \text{ in.}^3$ ◀

Now $$Q_x = (Q_x)_1 + (Q_x)_2 = 0$$

This result is expected since x is a centroidal axis (thus $\overline{y} = 0$)

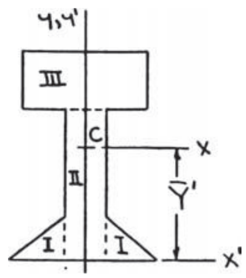
and $$Q_x = \Sigma\,\overline{y}A = \overline{Y}\Sigma A(\overline{y} = 0 \Rightarrow Q_x = 0)$$

PROBLEM 5.22



The horizontal x axis is drawn through the centroid C of the area shown, and it divides the area into two component areas A_1 and A_2. Determine the first moment of each component area with respect to the x axis, and explain the results obtained.

SOLUTION



First determine the location of the centroid C. We have

	A, in.^{2}	$\bar{y}'$, in.	$\bar{y}'A$, in.^{3}
I	$2\left(\frac{1}{2}\times2\times1.5\right)=3$	0.5	1.5
II	$1.5\times5.5=8.25$	2.75	22.6875
III	$4.5\times2=9$	6.5	58.5
Σ	20.25		82.6875

Then $\qquad \bar{Y}'\Sigma A=\Sigma \bar{y}'A$

$$\bar{Y}'(20.25)=82.6875$$

or $\qquad \bar{Y}'=4.0833 \text{ in.}$

Now $\qquad Q_x=\Sigma \bar{y}_1 A$

Then
$$(Q_x)_1=\left[\frac{1}{2}(5.5-4.0833)\text{in.}\right][(1.5)(5.5-4.0833)]\text{in.}^2$$
$$+[(6.5-4.0833)\text{in.}][(4.5)(2)]\text{in.}^2 \qquad \text{or} \quad (Q_x)_1=23.3 \text{ in.}^3 \blacktriangleleft$$

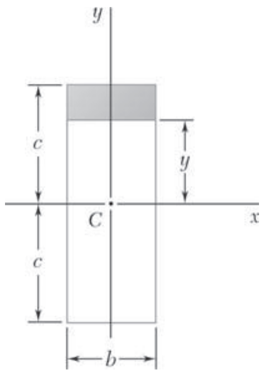
and
$$(Q_x)_2=-\left[\frac{1}{2}(4.0833\text{ in.})\right][(1.5)(4.0833)]\text{in.}^2$$
$$-[(4.0833-0.5)\text{in.}]\times2\left[\left(\frac{1}{2}\times2\times1.5\right)\text{in.}^2\right] \qquad \text{or} \quad (Q_x)_2=-23.3 \text{ in.}^3 \blacktriangleleft$$

PROBLEM 5.22 (Continued)

Now $$Q_x = (Q_x)_1 + (Q_x)_2 = 0$$

This result is expected since x is *a* centroidal axis (thus $\overline{Y} = 0$)

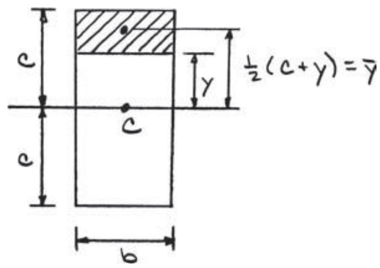
and $$Q_x = \Sigma \overline{y} A = \overline{Y} \Sigma A \quad (\overline{Y} = 0 \Rightarrow Q_x = 0)$$



PROBLEM 5.23

The first moment of the shaded area with respect to the x axis is denoted by Q_x. (a) Express Q_x in terms of b, c, and the distance y from the base of the shaded area to the x axis. (b) For what value of y is O_x maximum, and what is that maximum value?

SOLUTION



Shaded area:

$$A = b(c - y)$$

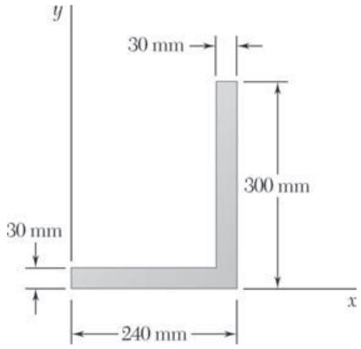
$$Q_x = \bar{y}A$$

$$= \frac{1}{2}(c + y)[b(c - y)]$$

(a) $$Q_x = \frac{1}{2}b(c^2 - y^2)$$ ◀

(b) For $Q_{\max}$: $$\frac{dQ}{dy} = 0 \quad \text{or} \quad \frac{1}{2}b(-2y) = 0$$ $y = 0$ ◀

For $y = 0$: $$(Q_x) = \frac{1}{2}bc^2$$ $(Q_x) = \frac{1}{2}bc^2$ ◀



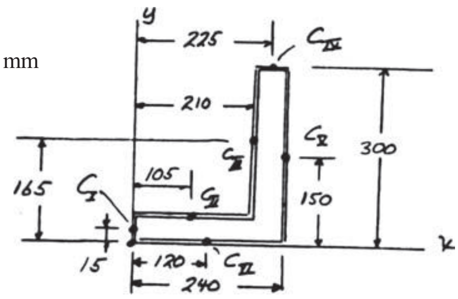
PROBLEM 5.24

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

Perimeter of Figure 5.1

Dimensions in mm



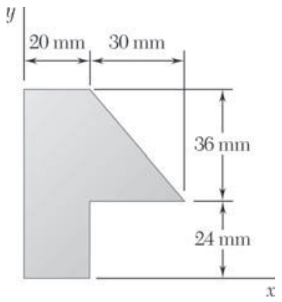
	L	$\bar{x}$	$\bar{y}$	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
I	30	0	15	0	0.45×10^3
II	210	105	30	22.05×10^3	6.3×10^3
III	270	210	165	56.7×10^3	44.55×10^3
IV	30	225	300	6.75×10^3	9×10^3
V	300	240	150	72×10^3	45×10^3
VII	240	120	0	28.8×10^3	0
Σ	1080			186.3×10^3	105.3×10^3

$$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(1080 \text{ mm}) = 186.3\times10^3 \text{ mm}^2 \qquad \bar{X} = 172.5 \text{ mm} \blacktriangleleft$$

$$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

$$\bar{Y}(1080 \text{ mm}) = 105.3\times10^3 \text{ mm}^2 \qquad \bar{Y} = 97.5 \text{ mm} \blacktriangleleft$$

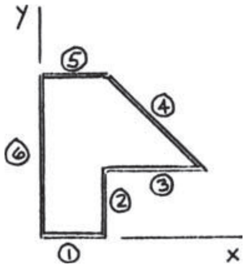


PROBLEM 5.25

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

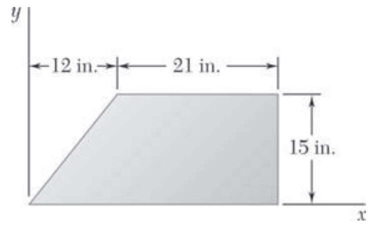
First note that because wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.



	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	20	10	0	200	0
2	24	20	12	480	288
3	30	35	24	1050	720
4	46.861	35	42	1640.14	1968.16
5	20	10	60	200	1200
6	60	0	30	0	1800
Σ	200.86			3570.1	5976.2

Then $\bar{X}\Sigma L = \Sigma \bar{x} L \quad \bar{X}(200.86) = 3570.1$ $\qquad \bar{X} = 17.77 \text{ mm}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y} L \quad \bar{Y}(200.86) = 5976.2$ $\qquad \bar{Y} = 29.8 \text{ mm}$ ◀

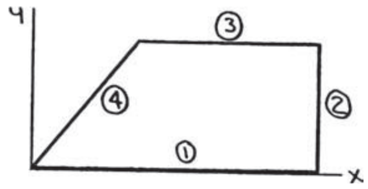


PROBLEM 5.26

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.



	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in.^{2}	$\bar{y}L$, in.^{2}
1	33	16.5	0	544.5	0
2	15	33	7.5	495	112.5
3	21	22.5	15	472.5	315
4	$\sqrt{12^2+15^2}=19.2093$	6	7.5	115.256	144.070
Σ	88.209			1627.26	571.57

Then

$$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(88.209) = 1627.26$$

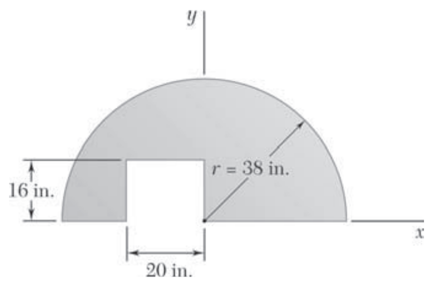
or $\bar{X} = 18.45$ in. ◀

and

$$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

$$\bar{Y}(88.209) = 571.57$$

or $\bar{Y} = 6.48$ in. ◀

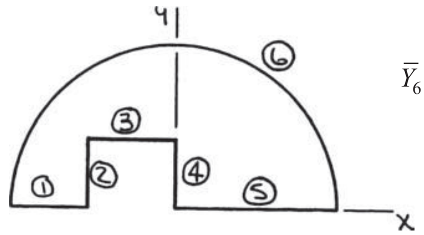


PROBLEM 5.27

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.



$$\overline{Y}_6 = \frac{2}{\pi}(38 \text{ in.})$$

	L, in.	$\overline{x}$, in.	$\overline{y}$, in.	$\overline{x}L$, in.^{2}	$\overline{y}L$, in.^{2}
1	18	–29	0	–522	0
2	16	–20	8	–320	128
3	20	–10	16	–200	320
4	16	0	8	0	128
5	38	19	0	722	0
6	$\pi(38) = 119.381$	0	24.192	0	2888.1
Σ	227.38			–320	3464.1

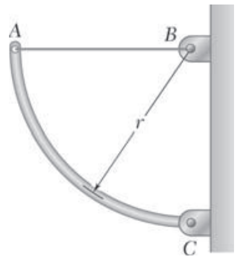
Then

$$\overline{X} = \frac{\Sigma \overline{x} L}{\Sigma L} = \frac{-320}{227.38}$$

$\overline{X} = -1.407$ in. ◀

$$\overline{Y} = \frac{\Sigma \overline{y} L}{\Sigma L} = \frac{3464.1}{227.38}$$

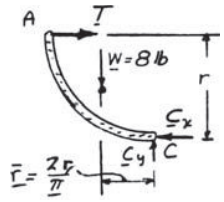
$\overline{Y} = 15.23$ in. ◀



PROBLEM 5.28

A uniform circular rod of weight 8 lb and radius 10 in. is attached to a pin at C and to the cable AB. Determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION



For quarter circle $\qquad \bar{r} = \dfrac{2r}{\pi}$

(a) $\qquad +\circlearrowleft \Sigma M_C = 0: \quad W\left(\dfrac{2r}{\pi}\right) - Tr = 0$

$$T = W\left(\frac{2}{\pi}\right) = (8\text{ lb})\left(\frac{2}{\pi}\right)$$

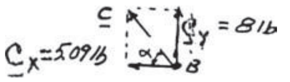
$T = 5.09$ lb ◀

(b) $\qquad \xrightarrow{+} \Sigma F_x = 0: \quad T - C_x = 0 \quad 5.09\text{ lb} - C_x = 0$

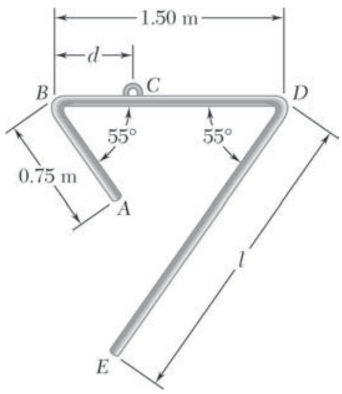
$\mathbf{C}_x = 5.09$ lb ⟵

$+\uparrow \Sigma F_y = 0: \quad C_y - W = 0 \qquad C_y - 8\text{ lb} = 0$

$\mathbf{C}_y = 8$ lb ↑



$\mathbf{C} = 9.48$ lb ⦪ 57.5° ◀

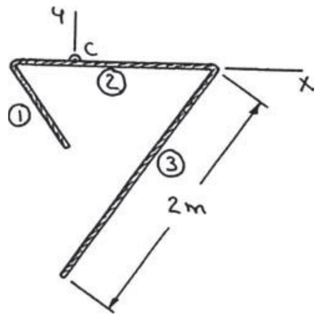


PROBLEM 5.29

Member *ABCDE* is a component of a mobile and is formed from a single piece of aluminum tubing. Knowing that the member is supported at *C* and that $l = 2$ m, determine the distance *d* so that portion *BCD* of the member is horizontal.

SOLUTION

First note that for equilibrium, the center of gravity of the component must lie on a vertical line through *C*. Further, because the tubing is uniform, the center of gravity of the component will coincide with the centroid of the corresponding line. Thus, $\bar{X} = 0$



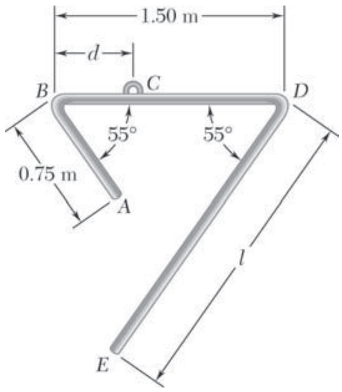
So that $\Sigma \bar{x} L = 0$

Then

$$-\left(d - \frac{0.75}{2}\cos 55°\right)\text{m} \times (0.75\text{ m})$$
$$+(0.75 - d)\text{m} \times (1.5\text{ m})$$
$$+\left[(1.5 - d)\text{m} - \left(\frac{1}{2} \times 2\text{ m} \times \cos 55°\right)\right] \times (2\text{ m}) = 0$$

or $$(0.75 + 1.5 + 2)d = \left[\frac{1}{2}(0.75)^2 - 2\right]\cos 55° + (0.75)(1.5) + 3$$

or $d = 0.739$ m ◀

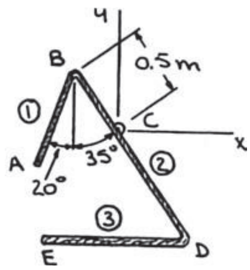


PROBLEM 5.30

Member *ABCDE* is a component of a mobile and is formed from a single piece of aluminum tubing. Knowing that the member is supported at *C* and that *d* is 0.50 m, determine the length *l* of arm *DE* so that this portion of the member is horizontal.

SOLUTION

First note that for equilibrium, the center of gravity of the component must lie on a vertical line through *C*. Further, because the tubing is uniform, the center of gravity of the component will coincide with the centroid of the corresponding line. Thus,



$$\bar{X} = 0$$

So that $\Sigma \bar{x} L = 0$

or
$$-\left(\frac{0.75}{2}\sin 20° + 0.5\sin 35°\right)\text{m} \times (0.75\text{ m})$$
$$+ (0.25\text{ m} \times \sin 35°) \times (1.5\text{ m})$$
$$+ \left(1.0 \times \sin 35° - \frac{l}{2}\right)\text{m} \times (l\text{ m}) = 0$$

or
$$\underbrace{-0.096193}_{(\bar{x}L)_{AB} + (\bar{x}L)_{BD}} + \underbrace{\left(\sin 35° - \tfrac{l}{2}\right) l}_{(\bar{x}L)_{DE}} = 0$$

The equation implies that the center of gravity of *DE* must be to the right of *C*.

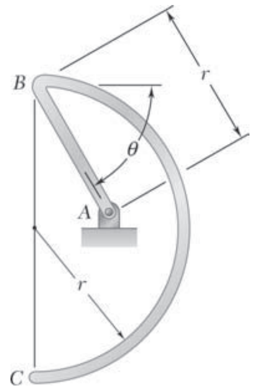
Then $l^2 - 1.14715l + 0.192386 = 0$

or
$$l = \frac{1.14715 \pm \sqrt{(-1.14715)^2 - 4(0.192386)}}{2}$$

or $l = 0.204$ m or $l = 0.943$ m ◀

Note that $\sin 35° - \frac{1}{2}l > 0$ for both values of *l* so both values are acceptable.

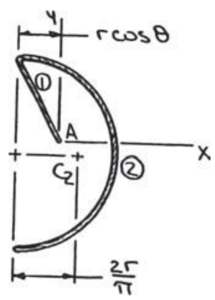
PROBLEM 5.31



The homogeneous wire *ABC* is bent into a semicircular arc and a straight section as shown and is attached to a hinge at *A*. Determine the value of θ for which the wire is in equilibrium for the indicated position.

SOLUTION

First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through *A*. Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line. Thus,



$$\overline{X} = 0$$

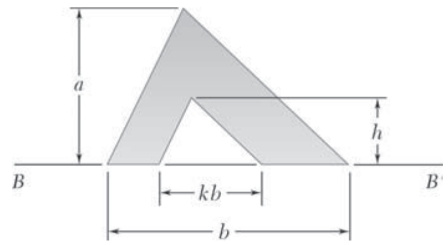
So that $\quad \Sigma \overline{x} L = 0$

Then $\quad \left(-\frac{1}{2} r\cos\theta\right)(r) + \left(\frac{2r}{\pi} - r\cos\theta\right)(\pi r) = 0$

or $\quad \cos\theta = \frac{4}{1+2\pi}$

$$= 0.54921$$

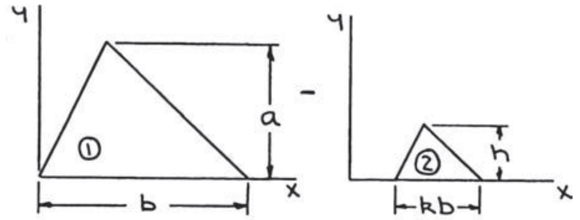
or $\quad \theta = 56.7° \blacktriangleleft$



PROBLEM 5.32

Determine the distance h for which the centroid of the shaded area is as far above line BB' as possible when (a) $k = 0.10$, (b) $k = 0.80$.

SOLUTION



	A	$\bar{y}$	$\bar{y}A$
1	$\frac{1}{2}ba$	$\frac{1}{3}a$	$\frac{1}{6}a^2b$
2	$-\frac{1}{2}(kb)h$	$\frac{1}{3}h$	$-\frac{1}{6}kbh^2$
Σ	$\frac{b}{2}(a-kh)$		$\frac{b}{6}(a^2-kh^2)$

Then $$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

$$\bar{Y}\left[\frac{b}{2}(a-kh)\right] = \frac{b}{6}(a^2-kh^2)$$

or $$\bar{Y} = \frac{a^2-kh^2}{3(a-kh)} \qquad (1)$$

and $$\frac{d\bar{Y}}{dh} = \frac{1}{3}\frac{-2kh(a-kh)-(a^2-kh^2)(-k)}{(a-kh)^2} = 0$$

or $$2h(a-kh) - a^2 + kh^2 = 0 \qquad (2)$$

Simplifying Eq. (2) yields

$$kh^2 - 2ah + a^2 = 0$$

Then $$h = \frac{2a \pm \sqrt{(-2a)^2 - 4(k)(a^2)}}{2k}$$

$$= \frac{a}{k}\left[1 \pm \sqrt{1-k}\right]$$

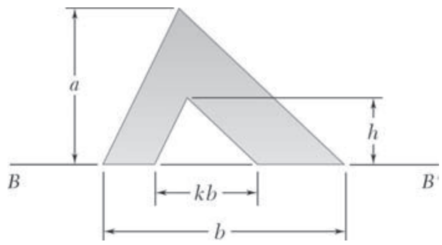
Note that only the negative root is acceptable since $h < a$. Then

(*a*) $$k = 0.10$$

$$h = \frac{a}{0.10}\left[1 - \sqrt{1-0.10}\right]$$ or $h = 0.513a$ ◀

(*b*) $$k = 0.80$$

$$h = \frac{a}{0.80}\left[1 - \sqrt{1-0.80}\right]$$ or $h = 0.691a$ ◀



PROBLEM 5.33

Knowing that the distance h has been selected to maximize the distance $\bar{y}$ from line BB' to the centroid of the shaded area, show that $\bar{y} = 2h/3$.

SOLUTION

See solution to Problem 5.32 for analysis leading to the following equations:

$$\bar{Y} = \frac{a^2 - kh^2}{3(a - kh)} \tag{1}$$

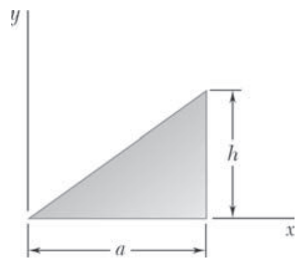
$$2h(a - kh) - a^2 + kh^2 = 0 \tag{2}$$

Rearranging Eq. (2) (which defines the value of h which maximizes $\bar{Y}$) yields

$$a^2 - kh^2 = 2h(a - kh)$$

Then substituting into Eq. (1) (which defines $\bar{Y}$)

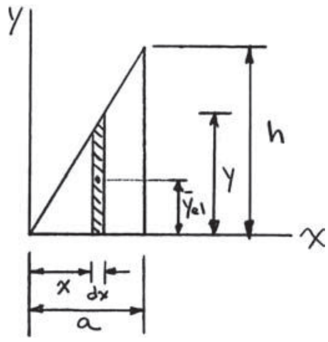
$$\bar{Y} = \frac{1}{3(a - kh)} \times 2h(a - kh) \qquad \text{or} \quad \bar{Y} = \frac{2}{3}h \blacktriangleleft$$



PROBLEM 5.34

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION



$$\frac{y}{x} = \frac{h}{a}$$

$$y = \frac{h}{a}x$$

$$\overline{x}_{EL} = x$$

$$\overline{y}_{EL} = \frac{1}{2}y$$

$$dA = ydx$$

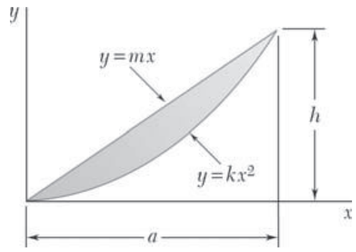
$$A = \int_0^a ydx = \int_0^a \left(\frac{h}{a}x\right)dx = \frac{1}{2}ah$$

$$\int \overline{x}_{EL}dA = \int xydx = \int_0^a x\left(\frac{h}{a}x\right)dx = \frac{h}{a}\left[\frac{x^3}{3}\right]_0^a = \frac{1}{3}ha^2$$

$$\int \overline{y}_{EL}dA = \int_0^a \left(\frac{1}{2}y\right)ydx = \frac{1}{2}\int_0^a \left(\frac{h}{a}x\right)^2 dx = \frac{1}{2}\frac{h^2}{b^2}\left[\frac{x^3}{3}\right]_0^a = \frac{1}{6}h^2a$$

$$\overline{x}A = \int \overline{x}_{EL}dA: \quad \overline{x}\left(\frac{1}{2}ah\right) = \frac{1}{3}ha^2 \qquad \overline{x} = \frac{2}{3}a \blacktriangleleft$$

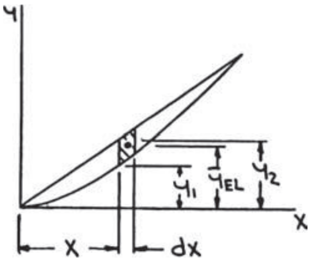
$$\overline{y}A = \int \overline{y}_{EL}dA: \quad \overline{y}\left(\frac{1}{2}ah\right) = \frac{1}{6}h^2a \qquad \overline{y} = \frac{1}{3}h \blacktriangleleft$$



PROBLEM 5.35

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION



At (a, h) $\quad y_1: \quad h = ka^2$

or $\quad k = \dfrac{h}{a^2}$

$y_2: \quad h = ma$

or $\quad m = \dfrac{h}{a}$

Now $\quad \overline{x}_{EL} = \overline{x}$

$$\overline{y}_{EL} = \frac{1}{2}(y_1 + y_2)$$

and $\quad dA = (y_2 - y_1)dx = \left[\dfrac{h}{a}x - \dfrac{h}{a^2}x^2\right]dx$

$$= \frac{h}{a^2}(ax - x^2)dx$$

Then $\quad A = \displaystyle\int dA = \int_0^a \frac{h}{a^2}(ax - x^2)dx = \frac{h}{a^2}\left[\frac{a}{2}x^2 - \frac{1}{3}x^3\right]_0^a = \frac{1}{6}ah$

and $\quad \displaystyle\int \overline{x}_{EL}\,dA = \int_0^a x\left[\frac{h}{a^2}(ax - x^2)dx\right] = \frac{h}{a^2}\left[\frac{a}{3}x^3 - \frac{1}{4}x^4\right]_0^a = \frac{1}{12}a^2h$

$$\int \overline{y}_{EL}\,dA = \int \frac{1}{2}(y_1 + y_2)[(y_2 - y_1)dx] = \int \frac{1}{2}\left(y_2^2 - y_1^2\right)dx$$

$$= \frac{1}{2}\int_0^a \left(\frac{h^2}{a^2}x^2 - \frac{h^2}{a^4}x^4\right)dx$$

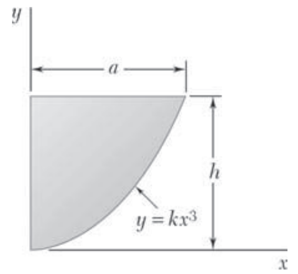
$$= \frac{1}{2}\frac{h^2}{a^4}\left[\frac{a^2}{3}x^3 - \frac{1}{5}x^5\right]_0^a$$

$$= \frac{1}{15}ah^2$$

PROBLEM 5.35 (Continued)

$$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{1}{6}ah\right) = \frac{1}{12}a^2h \qquad \bar{x} = \frac{1}{2}a \blacktriangleleft$$

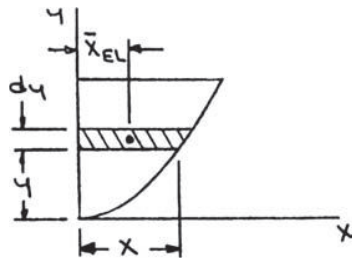
$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{1}{6}ah\right) = \frac{1}{15}ah^2 \qquad \bar{y} = \frac{2}{5}h \blacktriangleleft$$



PROBLEM 5.36

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION



For the element (EL) shown

At $\quad x = a, \quad y = h: \quad h = ka^3 \quad \text{or} \quad k = \dfrac{h}{a^3}$

Then $\quad x = \dfrac{a}{h^{1/3}} y^{1/3}$

Now $\quad dA = xdy = \dfrac{a}{h^{1/3}} y^{1/3}\, dy$

$$\bar{x}_{EL} = \frac{1}{2}x = \frac{1}{2}\frac{a}{h^{1/3}} y^{1/3}$$

$$\bar{y}_{EL} = y$$

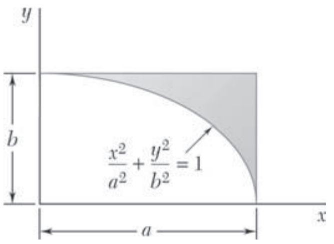
Then $\quad A = \int dA = \int_0^h \dfrac{a}{h^{1/3}} y^{1/3} dy = \dfrac{3}{4}\dfrac{a}{h^{1/3}}\left(y^{4/3}\right)\Big|_0^h = \dfrac{3}{4}ah$

and $\quad \int \bar{x}_{EL}\, dA = \int_0^h \dfrac{1}{2}\dfrac{a}{h^{1/3}} y^{1/3}\left(\dfrac{a}{h^{1/3}} y^{1/3} dy\right) = \dfrac{1}{2}\dfrac{a}{h^{2/3}}\left(\dfrac{3}{5} y^{5/3}\right)\Big|_0^h = \dfrac{3}{10}a^2 h$

$$\int \bar{y}_{EL}\, dA = \int_0^h y\left(\frac{a}{h^{1/3}} y^{1/3} dy\right) = \frac{a}{h^{1/3}}\left(\frac{3}{7} y^{7/3}\right)\Big|_0^h = \frac{3}{7}ah^2$$

Hence $\quad \bar{x}A = \int \bar{x}_{EL}\, dA: \quad \bar{x}\left(\dfrac{3}{4}ah\right) = \dfrac{3}{10}a^2 h \qquad \bar{x} = \dfrac{2}{5}a$ ◀

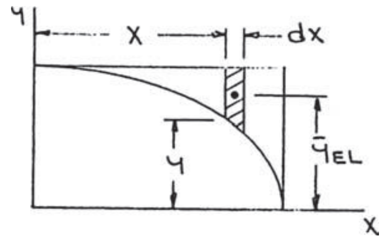
$$\bar{y}A = \int \bar{y}_{EL}\, dA: \quad \bar{y}\left(\frac{3}{4}ah\right) = \frac{3}{7}ah^2 \qquad \bar{y} = \frac{4}{7}h$$ ◀



PROBLEM 5.37

Determine by direct integration the centroid of the area shown.

SOLUTION



For the element (EL) shown $\quad y = \frac{b}{a}\sqrt{a^2 - x^2}$

and

$$dA = (b - y)dx$$

$$= \frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx$$

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}(y + b)$$

$$= \frac{b}{2a}\left(a + \sqrt{a^2 - x^2}\right)$$

Then

$$A = \int dA = \int_0^a \frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx$$

To integrate, let $\quad x = a\sin\theta: \quad \sqrt{a^2 - x^2} = a\cos\theta, \; dx = a\cos\theta\, d\theta$

Then

$$A = \int_0^{\pi/2} \frac{b}{a}(a - a\cos\theta)(a\cos\theta d\theta)$$

$$= \frac{b}{a}\left[a^2\sin\theta - a^2\left(\frac{\theta}{2} + \sin\frac{2\theta}{4}\right)\right]_0^{\pi/2}$$

$$= ab\left(1 - \frac{\pi}{4}\right)$$

and

$$\int \bar{x}_{EL}\, dA = \int_0^a x\left[\frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx\right]$$

$$= \frac{b}{a}\left[\left(\frac{a}{2}x^2 + \frac{1}{3}(a^2 - x^2)^{3/2}\right)\right]_0^{\pi/2}$$

$$= \frac{1}{6}a^3 b$$

PROBLEM 5.37 (Continued)

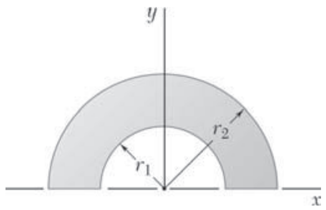
$$\int \overline{y}_{EL}\,dA = \int_0^a \frac{b}{2a}\left(a+\sqrt{a^2-x^2}\right)\left[\frac{b}{a}\left(a-\sqrt{a^2-x^2}\right)dx\right]$$

$$= \frac{b^2}{2a^2}\int_0^a (x^2)\;dx = \frac{b^2}{2a^2}\left(\frac{x^3}{3}\right)\Bigg|_0^a$$

$$= \frac{1}{6}ab^2$$

$$\overline{x}A = \int \overline{x}_{EL}\,dA: \quad \overline{x}\left[ab\left(1-\frac{\pi}{4}\right)\right] = \frac{1}{6}a^2b \qquad \text{or } \overline{x} = \frac{2a}{3(4-\pi)} \blacktriangleleft$$

$$\overline{y}A = \int \overline{y}_{EL}\,dA: \quad \overline{y}\left[ab\left(1-\frac{\pi}{4}\right)\right] = \frac{1}{6}ab^2 \qquad \text{or } \overline{y} = \frac{2b}{3(4-\pi)} \blacktriangleleft$$



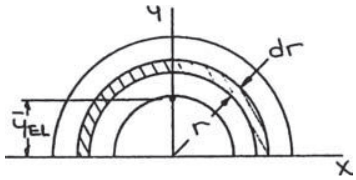
PROBLEM 5.38

Determine by direct integration the centroid of the area shown.

SOLUTION

First note that symmetry implies $\bar{x} = 0$ ◀

For the element (EL) shown



$$\bar{y}_{EL} = \frac{2r}{\pi} \quad \text{(Figure 5.8B)}$$

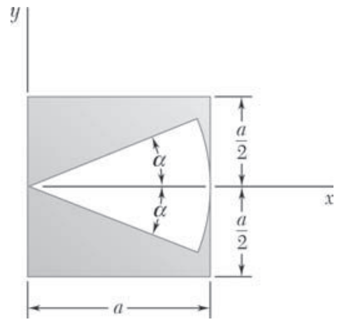
$$dA = \pi r dr$$

Then $$A = \int dA = \int_{r_1}^{r_2} \pi r\, dr = \pi\left(\frac{r^2}{2}\right)\Bigg|_{r_1}^{r_2} = \frac{\pi}{2}\left(r_2^2 - r_1^2\right)$$

and $$\int \bar{y}_{EL}\, dA = \int_{r_1}^{r_2} \frac{2r}{\pi}(\pi r dr) = 2\left(\frac{1}{3}r^3\right)\Bigg|_{r_1}^{r_2} = \frac{2}{3}\left(r_2^3 - r_1^3\right)$$

So $$\bar{y}A = \int \bar{y}_{EL}\, dA: \quad \bar{y}\left[\frac{\pi}{2}\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)$$

or $\bar{y} = \dfrac{4}{3\pi}\dfrac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$ ◀

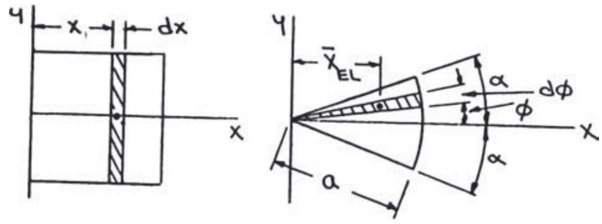


PROBLEM 5.39

Determine by direct integration the centroid of the area shown.

SOLUTION

First note that symmetry implies $\bar{y} = 0$ ◀



$$dA = adx \qquad dA = \frac{1}{2}a(ad\phi)$$

$$\bar{x}_{EL} = x \qquad \bar{x}_{EL} = \frac{2}{3}a\cos\phi$$

Then
$$A = \int dA = \int_0^a adx - \int_{-\alpha}^{\alpha} \frac{1}{2}a^2 d\phi$$

$$= a[x]_0^a - \frac{a^2}{2}[\phi]_{\alpha}^{\alpha} = a^2(1-\alpha)$$

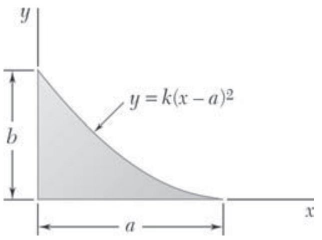
and
$$\int \bar{x}_{EL} dA = \int_0^a x(adx) - \int_{-\alpha}^{\alpha} \frac{2}{3}a\cos\phi\left(\frac{1}{2}a^2 d\phi\right)$$

$$= a\left[\frac{x^2}{2}\right]_0^a - \frac{1}{3}a^3[\sin\phi]_{-\alpha}^{\alpha}$$

$$= a^3\left(\frac{1}{2} - \frac{2}{3}\sin\alpha\right)$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x}[a^2(1-\alpha)] = a^3\left(\frac{1}{2} - \frac{2}{3}\sin\alpha\right)$$

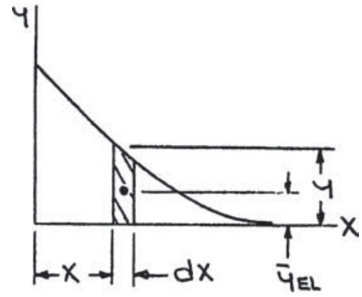
or $\bar{x} = \dfrac{3 - 4\sin\alpha}{6(1-\alpha)}a$ ◀



PROBLEM 5.40

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION



At $$x = 0, \quad y = b$$

$$b = k(0-a)^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

Then $$y = \frac{b}{a^2}(x-a)^2$$

Now $$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{y}{2} = \frac{b}{2a^2}(x-a)^2$$

and $$dA = ydx = \frac{b}{a^2}(x-a)^2\,dx$$

Then $$A = \int dA = \int_0^a \frac{b}{a^2}(x-a)^2\,dx = \frac{b}{3a^2}\left[(x-a)^3\right]_0^a = \frac{1}{3}ab$$

and $$\int \bar{x}_{EL}\,dA = \int_0^a x\left[\frac{b}{a^2}(x-a)^2\,dx\right] = \frac{b}{a^2}\int_0^a\left(x^3 - 2ax^2 + a^2x\right)dx$$

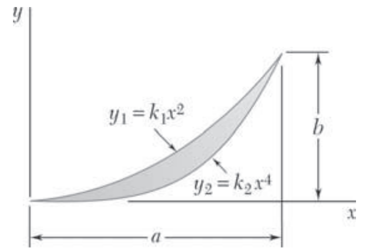
$$= \frac{b}{a^2}\left(\frac{x^4}{4} - \frac{2}{3}ax^3 + \frac{a^2}{2}x^2\right) = \frac{1}{12}a^2b$$

$$\int \bar{y}_{EL}\,dA = \int_0^a \frac{b}{2a^2}(x-a)^2\left[\frac{b}{a^2}(x-a)^2\,dx\right] = \frac{b^2}{2a^4}\left[\frac{1}{5}(x-a)^5\right]_0^a$$

$$= \frac{1}{10}ab^2$$

Hence $$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{1}{3}ab\right) = \frac{1}{12}a^2b$$ $$\bar{x} = \frac{1}{4}a \quad ◀$$

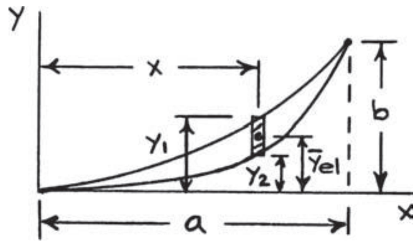
$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{1}{3}ab\right) = \frac{1}{10}ab^2$$ $$\bar{y} = \frac{3}{10}b \quad ◀$$



PROBLEM 5.41

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION



$$y_1 = k_1x^2 \quad \text{but} \quad b = k_1a^2 \quad y_1 = \frac{b}{a^2}x^2$$

$$y_2 = k_2x^4 \quad \text{but} \quad b = k_2a^4 \quad y_2 = \frac{b}{a^4}x^4$$

$$dA = (y_2 - y_1)dx = \frac{b}{a^2}\left(x^2 - \frac{x^4}{a^2}\right)dx$$

$$\overline{x}_{EL} = x$$

$$\overline{y}_{EL} = \frac{1}{2}(y_1 + y_2)$$

$$= \frac{b}{2a^2}\left(x^2 + \frac{x^4}{a^2}\right)$$

$$A = \int dA = \frac{b}{a^2}\int_0^a \left(x^2 - \frac{x^4}{a^2}\right)dx$$

$$= \frac{b}{a^2}\left[\frac{x^3}{3} - \frac{x^5}{5a^2}\right]_0^a$$

$$= \frac{2}{15}ba$$

$$\int \overline{x}_{EL}\,dA = \int_0^a \times\frac{b}{a^2}\left(x^2 - \frac{x^4}{a^2}\right)dx$$

$$= \frac{b}{a^2}\int_0^a \left(x^3 - \frac{x^5}{a^2}\right)dx$$

$$= \frac{b}{a^2}\left[\frac{x^4}{4} - \frac{x^6}{6a^2}\right]_0^a$$

$$= \frac{1}{12}a^2b$$

PROBLEM 5.41 (Continued)

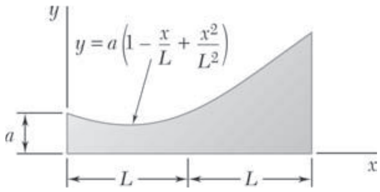
$$\int \overline{y}_{EL}\,dA = \int_0^a \frac{b}{2a^2}\left(x^2 + \frac{x^4}{a^2}\right)\frac{b}{a^2}\left(x^2 - \frac{x^4}{a^2}\right)dx$$

$$= \frac{b^2}{2a^4}\int_0^a \left(x^4 - \frac{x^8}{a^4}\right)dx$$

$$= \frac{b^2}{2a^4}\left[\frac{x^5}{5} - \frac{x^9}{9a^4}\right]_0^a = \frac{2}{45}ab^2$$

$$\overline{x}A = \int \overline{x}_{EL}\,dA: \quad \overline{x}\left(\frac{2}{15}ba\right) = \frac{1}{12}a^2 b \qquad \overline{x} = \frac{5}{8}a \blacktriangleleft$$

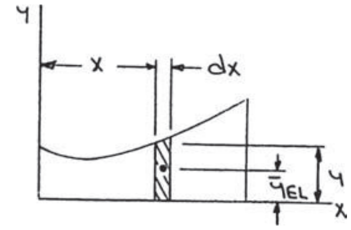
$$\overline{y}A = \int \overline{y}_{EL}\,dA: \quad \overline{y}\left(\frac{2}{15}ba\right) = \frac{2}{45}ab^2 \qquad \overline{y} = \frac{1}{3}b \blacktriangleleft$$



PROBLEM 5.42

Determine by direct integration the centroid of the area shown.

SOLUTION



We have $\quad \bar{x}_{EL} = x$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{a}{2}\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)$$

$$dA = ydx = a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx$$

Then

$$A = \int dA = \int_0^{2L} a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx = a\left[x - \frac{x^2}{2L} + \frac{x^3}{3L^2}\right]_0^{2L}$$

$$= \frac{8}{3}aL$$

and

$$\int \bar{x}_{EL}dA = \int_0^{2L} x\left[a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx\right] = a\left[\frac{x^2}{2} - \frac{x^3}{3L} + \frac{x^4}{4L^2}\right]_0^{2L}$$

$$= \frac{10}{3}aL^2$$

$$\int \bar{y}_{EL}dA = \int_0^{2L} \frac{a}{2}\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)\left[a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx\right]$$

$$= \frac{a^2}{2}\int_0^{EL}\left(1 - 2\frac{x}{L} + 3\frac{x^2}{L^2} - 2\frac{x^3}{L^3} + \frac{x^4}{L^4}\right)dx$$

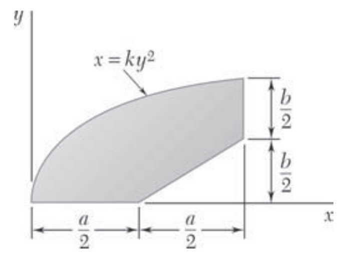
$$= \frac{a^2}{2}\left[x - \frac{x^2}{L} + \frac{x^3}{L^2} - \frac{x^4}{2L^3} + \frac{x^5}{5L^4}\right]_0^{2L}$$

$$= \frac{11}{5}a^2L$$

Hence

$$\bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\frac{8}{3}aL\right) = \frac{10}{3}aL^2 \qquad \bar{x} = \frac{5}{4}L \blacktriangleleft$$

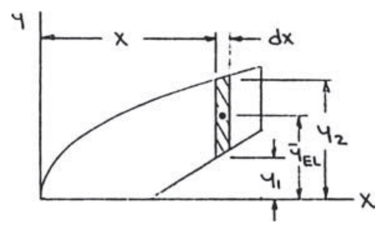
$$\bar{y}A = \int \bar{y}_{EL}dA: \quad \bar{y}\left(\frac{1}{8}a\right) = \frac{11}{5}a^2 \qquad \bar{y} = \frac{33}{40}a \blacktriangleleft$$



PROBLEM 5.43

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b.

SOLUTION



For y_2 at $\quad x = a,\ y = b:\quad a = kb^2 \quad \text{or} \quad k = \dfrac{a}{b^2}$

Then $\quad y_2 = \dfrac{b}{\sqrt{a}} x^{1/2}$

Now $\quad \bar{x}_{EL} = x$

and for $\quad 0 \le x \le \dfrac{a}{2}:\quad \bar{y}_{EL} = \dfrac{y_2}{2} = \dfrac{b}{2}\dfrac{x^{1/2}}{\sqrt{a}}$

$$dA = y_2 dx = b\frac{x^{1/2}}{\sqrt{a}} dx$$

For $\quad \dfrac{a}{2} \le x \le a:\quad \bar{y}_{EL} = \dfrac{1}{2}(y_1 + y_2) = \dfrac{b}{2}\left(\dfrac{x}{a} - \dfrac{1}{2} + \dfrac{x^{1/2}}{\sqrt{a}}\right)$

$$dA = (y_2 - y_1)dx = b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

Then

$$A = \int dA = \int_0^{a/2} b\frac{x^{1/2}}{\sqrt{a}} dx + \int_{a/2}^{a} b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{3}x^{3/2}\right]_0^{a/2} + b\left[\frac{2}{3}\frac{x^{3/2}}{\sqrt{a}} - \frac{x^2}{2a} + \frac{1}{2}x\right]_{a/2}^{a}$$

$$= \frac{2}{3}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{3/2} + (a)^{3/2} - \left(\frac{a}{2}\right)^{3/2}\right]$$

$$+ b\left\{-\frac{1}{2a}\left[(a^2) - \left(\frac{a}{2}\right)^2\right] + \frac{1}{2}\left[(a) - \left(\frac{a}{2}\right)\right]\right\}$$

$$= \frac{13}{24}ab$$

PROBLEM 5.43 (Continued)

and $$\int \bar{x}_{EL}\,dA = \int_0^{a/2} x\left(b\frac{x^{1/2}}{\sqrt{a}}dx\right) + \int_{a/2}^{a} x\left[b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)\right]dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{5}x^{5/2}\right]_0^{a/2} + b\left[\frac{2}{5}\frac{x^{5/2}}{\sqrt{a}} - \frac{x^3}{3a} + \frac{x^4}{4}\right]_{a/2}^{a}$$

$$= \frac{2}{5}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{5/2} + (a)^{5/2} - \left(\frac{a}{2}\right)^{5/2}\right]$$

$$+ b\left\{-\frac{1}{3a}\left[(a)^3 - \left(\frac{a}{2}\right)^3\right] + \frac{1}{4}\left[(a)^2 - \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{71}{240}a^2 b$$

$$\int \bar{y}_{EL}\,dA = \int_0^{a/2} \frac{b}{2}\frac{x^{1/2}}{\sqrt{a}}\left[b\frac{x^{1/2}}{\sqrt{a}}dx\right]$$

$$+ \int_{a/2}^{a} \frac{b}{2}\left(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}}\right)\left[b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx\right]$$

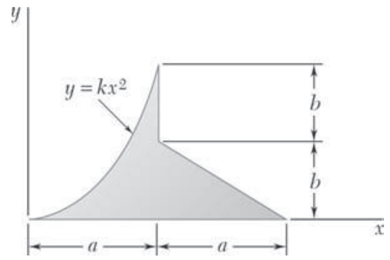
$$= \frac{b^2}{2a}\left[\frac{1}{2}x^2\right]_0^{a/2} + \frac{b^2}{2}\left[\left(\frac{x^2}{2a} - \frac{1}{3a}\left(\frac{x}{a} - \frac{1}{2}\right)^3\right)\right]_{a/2}^{a}$$

$$= \frac{b}{4a}\left[\left(\frac{a}{2}\right)^2 + (a)^2 - \left(\frac{a}{2}\right)^2\right] - \frac{b^2}{6a}\left(\frac{a}{2} - \frac{1}{2}\right)^3$$

$$= \frac{11}{48}ab^2$$

Hence $$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{13}{24}ab\right) = \frac{71}{240}a^2 b \qquad \bar{x} = \frac{17}{130}a = 0.546a \blacktriangleleft$$

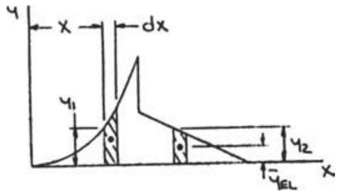
$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{13}{24}ab\right) = \frac{11}{48}ab^2 \qquad \bar{y} = \frac{11}{26}b = 0.423b \blacktriangleleft$$



PROBLEM 5.44

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b.

SOLUTION



For y_1 at $\quad x = a, \quad y = 2b \quad 2b = ka^2 \quad$ or $\quad k = \dfrac{2b}{a^2}$

Then $$y_1 = \frac{2b}{a^2}x^2$$

By observation $$y_2 = -\frac{b}{a}(x + 2b) = b\left(2 - \frac{x}{a}\right)$$

Now $$\overline{x}_{EL} = x$$

and for $0 \le x \le a$: $$\overline{y}_{EL} = \frac{1}{2}y_1 = \frac{b}{a^2}x^2 \quad \text{and} \quad dA = y_1 dx = \frac{2b}{a^2}x^2 dx$$

For $a \le x \le 2a$: $$\overline{y}_{EL} = \frac{1}{2}y_2 = \frac{b}{2}\left(2 - \frac{x}{a}\right) \quad \text{and} \quad dA = y_2 dx = b\left(2 - \frac{x}{a}\right)dx$$

Then $$A = \int dA = \int_0^a \frac{2b}{a^2}x^2 dx + \int_a^{2a} b\left(2 - \frac{x}{a}\right)dx$$

$$= \frac{2b}{a^2}\left[\frac{x^3}{3}\right]_0^a + b\left[-\frac{a}{2}\left(2 - \frac{x}{a}\right)^2\right]_0^{2a} = \frac{7}{6}ab$$

and $$\int \overline{x}_{EL} dA = \int_0^a x\left(\frac{2b}{a^2}x^2 dx\right) + \int_a^{2a} x\left[b\left(2 - \frac{x}{a}\right)dx\right]$$

$$= \frac{2b}{a^2}\left[\frac{x^4}{4}\right]_0^a + b\left[x^2 - \frac{x^3}{3a}\right]_0^{2a}$$

$$= \frac{1}{2}a^2 b + b\left\{\left[(2a)^2 - (a)^2\right] + \frac{1}{3a}\left[(2a^2) - (a)^3\right]\right\}$$

$$= \frac{7}{6}a^2 b$$

PROBLEM 5.44 (Continued)

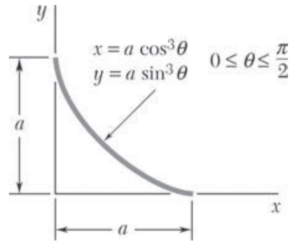
$$\int \overline{y}_{EL}\,dA = \int_0^a \frac{b}{a^2}x^2\left[\frac{2b}{a^2}x^2dx\right] + \int_0^{2a}\frac{b}{2}\left(2-\frac{x}{a}\right)\left[b\left(2-\frac{x}{a}\right)dx\right]$$

$$= \frac{2b^2}{a^4}\left[\frac{x^5}{5}\right]_0^a + \frac{b^2}{2}\left[-\frac{a}{3}\left(2-\frac{x}{a}\right)^3\right]_a^{2a}$$

$$= \frac{17}{30}ab^2$$

Hence $\quad \overline{x}A = \int \overline{x}_{EL}\,dA: \quad \overline{x}\left(\frac{7}{6}ab\right) = \frac{7}{6}a^2b \qquad \overline{x} = a$ ◀

$$\overline{y}A = \int \overline{y}_{EL}\,dA: \quad \overline{y}\left(\frac{7}{6}ab\right) = \frac{17}{30}ab^2 \qquad \overline{y} = \frac{17}{35}b \text{ ◀}$$

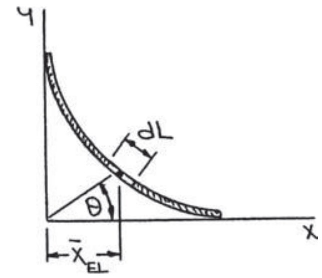


PROBLEM 5.45

A homogeneous wire is bent into the shape shown. Determine by direct integration the *x* coordinate of its centroid.

SOLUTION

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line



Now $\qquad \bar{x}_{EL} = a\cos^3\theta \quad \text{and} \quad dL = \sqrt{dx^2 + dy^2}$

Where $\qquad x = a\cos^3\theta: \quad dx = -3a\cos^2\theta\sin\theta d\theta$

$$y = a\sin^3\theta: \quad dy = 3a\sin^2\theta\cos\theta d\theta$$

Then $\qquad dL = [(-3a\cos^2\theta\sin\theta d\theta)^2 + (3a\sin^2\theta\cos\theta d\theta)^2]^{1/2}$

$$= 3a\cos\theta\sin\theta(\cos^2\theta + \sin^2\theta)^{1/2}d\theta$$

$$= 3a\cos\theta\sin\theta d\theta$$

$$L = \int dL = \int_0^{\pi/2} 3a\cos\theta\sin\theta d\theta = 3a\left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/2}$$

$$= \frac{3}{2}a$$

and $\qquad \int \bar{x}_{EL} dL = \int_0^{\pi/2} a\cos^3\theta(3a\cos\theta\sin\theta d\theta)$

$$= 3a^2\left[-\frac{1}{5}\cos^5\theta\right]_0^{\pi/2} = \frac{3}{5}a^2$$

Hence $\qquad \bar{x}L = \int \bar{x}_{EL} dL: \quad \bar{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2 \qquad\qquad \bar{x} = \frac{2}{5}a$ ◀

Alternative Solution

$$x = a\cos^3\theta \Rightarrow \cos^2\theta = \left(\frac{x}{a}\right)^{2/3}$$

$$y = a\sin^3\theta \Rightarrow \sin^2\theta = \left(\frac{y}{a}\right)^{2/3}$$

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1 \quad \text{or} \quad y = (a^{2/3} - x^{2/3})^{3/2}$$

PROBLEM 5.45 (Continued)

Then $$\frac{dy}{dx} = (a^{2/3} - x^{2/3})^{1/2}(-x^{-1/3})$$

Now $$\overline{x}_{EL} = x$$

and $$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

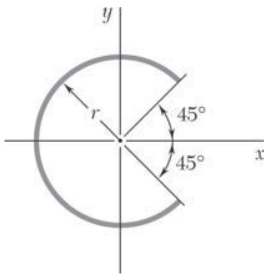
$$dx = \left\{1 + \left[(a^{2/3} - x^{2/3})^{1/2}(-x^{-1/3})\right]^2\right\}^{1/2} dx$$

Then $$L = \int dL = \int_0^a \frac{a^{1/3}}{x^{1/3}} dx = a^{1/3}\left[\frac{3}{2}x^{2/3}\right]_0^a = \frac{3}{2}a$$

and $$\int \overline{x}_{EL}\, dL = \int_0^a x\left(\frac{a^{1/3}}{x^{1/3}} dx\right) = a^{1/3}\left[\frac{3}{5}x^{5/3}\right]_0^a = \frac{3}{5}a^2$$

Hence $$\overline{x}L = \int \overline{x}_{EL}\, dL: \quad \overline{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2$$

$$\overline{x} = \frac{2}{5}a \blacktriangleleft$$

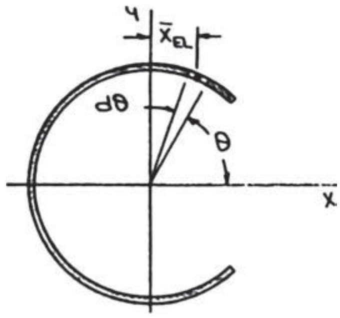


PROBLEM 5.46

A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid.

SOLUTION

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line



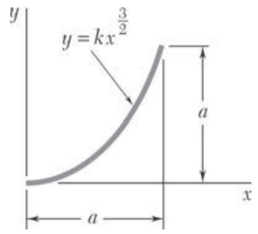
Now $\quad \bar{x}_{EL} = r\cos\theta \quad \text{and} \quad dL = rd\theta$

Then $\quad L = \int dL = \int_{\pi/4}^{7\pi/4} rd\theta = r[\theta]_{\pi/4}^{7\pi/4} = \frac{3}{2}\pi r$

and
$$\int \bar{x}_{EL} dL = \int_{\pi/4}^{7\pi/4} r\cos\theta(rd\theta)$$
$$= r^2[\sin\theta]_{\pi/4}^{7\pi/4}$$
$$= r^2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$
$$= -r^2\sqrt{2}$$

Thus $\quad \bar{x}L = \int \bar{x} dL: \quad \bar{x}\left(\frac{3}{2}\pi r\right) = -r^2\sqrt{2}$

$$\bar{x} = -\frac{2\sqrt{2}}{3\pi} r \quad \blacktriangleleft$$

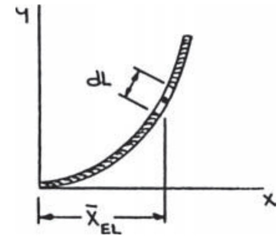


PROBLEM 5.47*

A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid. Express your answer in terms of a.

SOLUTION

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.



We have at $\quad x = a, \quad y = a$

$$a = ka^{3/2} \quad \text{or} \quad k = \frac{1}{\sqrt{a}}$$

Then
$$y = \frac{1}{\sqrt{a}} x^{3/2}$$

and
$$\frac{dy}{dx} = \frac{3}{2\sqrt{a}} x^{1/2}$$

Now
$$\bar{x}_{EL} = x$$

and
$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$
$$= \left[1 + \left(\frac{3}{2\sqrt{a}} x^{1/2}\right)^2\right]^{1/2} dx$$
$$= \frac{1}{2\sqrt{a}} \sqrt{4a + 9x}\; dx$$

Then
$$L = \int dL = \int_0^a \frac{1}{2\sqrt{a}} \sqrt{4a + 9x}\, dx$$
$$= \frac{1}{2\sqrt{a}} \left[\frac{2}{3} \times \frac{1}{9} (4a + 9x)^{3/2}\right]_0^a$$
$$= \frac{a}{27}[(13)^{3/2} - 8]$$
$$= 1.43971a$$

and
$$\int \bar{x}_{EL}\, dL = \int_0^a x \left[\frac{1}{2\sqrt{a}} \sqrt{4a + 9x}\, dx\right]$$

PROBLEM 5.47* (Continued)

Use integration by parts with

$$u = x \qquad dv = \sqrt{4a + 9x}\, dx$$

$$du = dx \qquad v = \frac{2}{27}(4a + 9x)^{3/2}$$

Then

$$\int \overline{x}_{EL}\, dL = \frac{1}{2\sqrt{a}} \left\{ \left[x \times \frac{2}{27}(4a + 9x)^{3/2} \right]_0^a - \int_0^a \frac{2}{27}(4a + 9x)^{3/2}\, dx \right\}$$

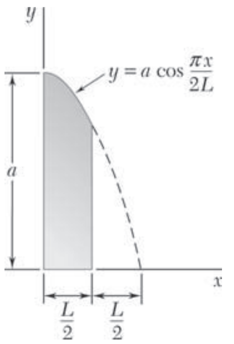
$$= \frac{(13)^{3/2}}{27} a^2 - \frac{1}{27\sqrt{a}} \left[\frac{2}{45}(4a + 9x)^{5/2} \right]_0^a$$

$$= \frac{a^2}{27} \left\{ (13)^{3/2} - \frac{2}{45}[(13)^{5/2} - 32] \right\}$$

$$= 0.78566a^2$$

$$\overline{x}L = \int x_{EL}\, dL: \quad \overline{x}(1.43971a) = 0.78566a^2$$

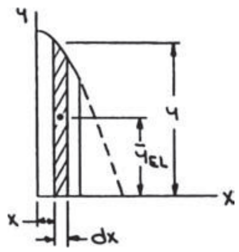
or $\overline{x} = 0.546a$ ◀



PROBLEM 5.48*

Determine by direct integration the centroid of the area shown.

SOLUTION



We have $\qquad \bar{x}_{EL} = x$

$$y_{EL} = \frac{1}{2}y = \frac{a}{2}\cos\frac{\pi x}{2L}$$

and $\qquad dA = ydx = a\cos\frac{\pi x}{2L}dx$

Then $\qquad A = \int dA = \int_0^{L/2} a\cos\frac{\pi x}{2L}dx$

$$= a\left[\frac{2L}{\pi}\sin\frac{\pi x}{2L}\right]_0^{L/2}$$

$$= \frac{\sqrt{2}}{\pi}aL$$

and $\qquad \int x_{EL}dA = \int x\left(a\cos\frac{\pi x}{2L}dx\right)$

Use integration by parts with $\qquad u = x \qquad dv = \cos\frac{\pi x}{2L}dx$

$$du = dx \qquad v = \frac{2L}{\pi}\sin\frac{\pi x}{2L}$$

Then $\qquad \int \bar{x}\cos\frac{\pi x}{2L}dx = \frac{2L}{\pi}\times\sin\frac{\pi x}{2L} - \int\frac{2L}{\pi}\sin\frac{\pi x}{2L}dx$

$$= \frac{2L}{\pi}\left(x\sin\frac{\pi x}{2L} + \frac{2L}{\pi}\cos\frac{\pi x}{2L}\right)$$

$$\int x_{EL}dA = a\frac{2L}{\pi}\left[x\ \sin\frac{\pi x}{2L} + \frac{2L}{\pi}\cos\frac{\pi x}{2L}\right]_0^{L/2}$$

$$= a\frac{2L}{\pi}\left[\left(\frac{L}{2\sqrt{2}} + \frac{\sqrt{2}}{\pi}L\right) - \frac{2L}{\pi}\right]$$

$$= 0.106374aL^2$$

PROBLEM 5.48* (Continued)

Also $$\int \overline{y}_{EL}\, dA = \int_0^{L/2} \frac{a}{2}\cos\frac{\pi x}{2L}\left(a\cos\frac{\pi x}{2L}dx\right)$$

$$= \frac{a^2}{2}\left[\frac{x}{2}+\frac{\sin\frac{\pi x}{L}}{\frac{2\pi}{L}}\right]_0^{L/2} = \frac{a^{-2}}{2}\left(\frac{L}{4}+\frac{L}{2\pi}\right)$$

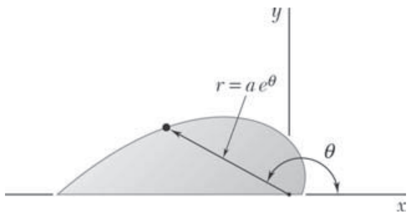
$$= 0.20458a^2L$$

$$\overline{x}A = \int \overline{x}_{EL}dA: \quad \overline{x}\left(\frac{\sqrt{2}}{\pi}aL\right) = 0.106374aL^2$$

or $\overline{x} = 0.236L$ ◀

$$\overline{y}A = \int \overline{y}_{EL}dA: \quad \overline{y}\left(\frac{\sqrt{2}}{\pi}aL\right) = 0.20458a^2L$$

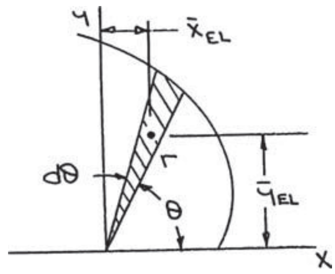
or $\overline{y} = 0.454a$ ◀



PROBLEM 5.49*

Determine by direct integration the centroid of the area shown.

SOLUTION



We have $$\bar{x}_{EL} = \frac{2}{3} r \cos\theta = \frac{2}{3} a e^{\theta} \cos\theta$$

$$\bar{y}_{EL} = \frac{2}{3} r \sin\theta = \frac{2}{3} a e^{\theta} \sin\theta$$

and $$dA = \frac{1}{2}(r)(r d\theta) = \frac{1}{2} a^2 e^{2\theta} d\theta$$

Then $$A = \int dA = \int_0^{\pi} \frac{1}{2} a^2 e^{2\theta} d\theta = \frac{1}{2} a^2 \left[\frac{1}{2} e^{2\theta}\right]_0^{\pi}$$

$$= \frac{1}{4} a^2 (e^{2\pi} - 1)$$

$$= 133.623 a^2$$

and $$\int \bar{x}_{EL} dA = \int_0^{\pi} \frac{2}{3} a e^{\theta} \cos\theta \left(\frac{1}{2} a^2 e^{2\theta} d\theta\right)$$

$$= \frac{1}{3} a^3 \int_0^{\pi} e^{3\theta} \cos\theta d\theta$$

To proceed, use integration by parts, with

$$u = e^{3\theta} \quad \text{and} \quad du = 3e^{3\theta} d\theta$$

$$dv = \cos\theta\, d\theta \quad \text{and} \quad v = \sin\theta$$

Then $$\int e^{3\theta} \cos\theta d\theta = e^{3\theta} \sin\theta - \int \sin\theta (3e^{3\theta} d\theta)$$

Now let $$u = e^{3\theta} \quad \text{then} \quad du = 3e^{3\theta} d\theta$$

$$dv = \sin\theta d\theta, \quad \text{then} \quad v = -\cos\theta$$

Then $$\int e^{3\theta} \sin\theta d\theta = e^{3\theta} \sin\theta - 3\left[-e^{3\theta} \cos\theta - \int (-\cos\theta)(3e^{3\theta} d\theta)\right]$$

PROBLEM 5.49* (Continued)

So that
$$\int e^{3\theta}\cos\theta d\theta = \frac{e^{3\theta}}{10}(\sin\theta + 3\cos\theta)$$

$$\int x_{EL}dA = \frac{1}{3}a^3\left[\frac{e^{3\theta}}{10}(\sin\theta + 3\cos\theta)\right]_0^{\pi}$$

$$= \frac{a^3}{30}(-3e^{3\pi} - 3) = -1239.26a^3$$

Also
$$\int \overline{y}_{EL}dA = \int_0^{\pi}\frac{2}{3}ae^{\theta}\sin\theta\left(\frac{1}{2}a^2e^{2\theta}d\theta\right)$$

$$= \frac{1}{3}a^3\int_0^{\pi}e^{3\theta}\sin\theta d\theta$$

Using integration by parts, as above, with

$$u = e^{3\theta} \quad \text{and} \quad du = 3e^{3\theta}d\theta$$

$$dv = \int \sin\theta\, d\theta \quad \text{and} \quad v = -\cos\theta$$

Then
$$\int e^{3\theta}\sin\theta d\theta = -e^{3\theta}\cos\theta - \int(-\cos\theta)(3e^{3\theta}d\theta)$$

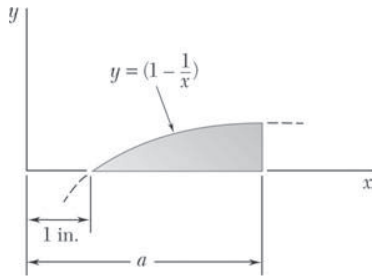
So that
$$\int e^{3\theta}\sin\theta d\theta = \frac{e^{3\theta}}{10}(-\cos\theta + 3\sin\theta)$$

$$\int \overline{y}_{EL}dA = \frac{1}{3}a^3\left[\frac{e^{3\theta}}{10}(-\cos\theta + 3\sin\theta)\right]_0^{\pi}$$

$$= \frac{a^3}{30}(e^{3\pi} + 1) = 413.09a^3$$

Hence
$$\overline{x}A = \int x_{EL}dA: \quad \overline{x}(133.623a^2) = -1239.26a^3 \qquad \text{or } \overline{x} = -9.27a \blacktriangleleft$$

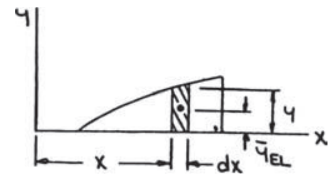
$$\overline{y}A = \int \overline{y}_{EL}dA: \quad \overline{y}(133.623a^2) = 413.09a^3 \qquad \text{or } \overline{y} = 3.09a \blacktriangleleft$$



PROBLEM 5.50

Determine the centroid of the area shown when $a = 2$ in.

SOLUTION



We have $\quad \bar{x}_{EL} = x$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2}\left(1 - \frac{1}{x}\right)$$

and $\quad dA = ydx = \left(1 - \frac{1}{x}\right)dx$

Then $\quad A = \int dA = \int_1^a \left(1 - \frac{1}{x}\right)\frac{dx}{2} = [x - \ln x]_1^a = (a - \ln a - 1)\,\text{in.}^2$

and $\quad \int \bar{x}_{EL}\,dA = \int_1^a x\left[\left(1 - \frac{1}{x}\right)dx\right] = \left[\frac{x^2}{2} - x\right]_1^a = \left(\frac{a^2}{2} - a + \frac{1}{2}\right)\text{in.}^3$

$$\int \bar{y}_{EL}\,dA = \int_1^a \frac{1}{2}\left(1 - \frac{1}{x}\right)\left[\left(1 - \frac{1}{x}\right)dx\right] = \frac{1}{2}\int_1^a \left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$$

$$= \frac{1}{2}\left[x - 2\ln x - \frac{1}{x}\right]_1^a = \frac{1}{2}\left(a - 2\ln a - \frac{1}{a}\right)\text{in.}^3$$

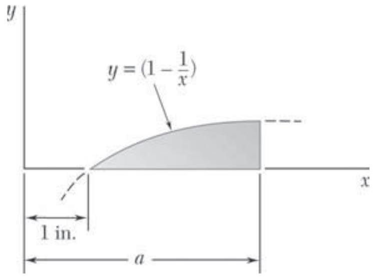
$$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x} = \frac{\frac{a^2}{2} - a + \frac{1}{2}}{a - \ln a - 1}\,\text{in.}$$

$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y} = \frac{a - 2\ln a - \frac{1}{a}}{2(a - \ln a - 1)}\,\text{in.}$$

Find: $\bar{x}$ and $\bar{y}$ when $a = 2$ in.

We have $\quad \bar{x} = \dfrac{\frac{1}{2}(2)^2 - 2 + \frac{1}{2}}{2 - \ln 2 - 1} \quad$ or $\quad \bar{x} = 1.629$ in. ◀

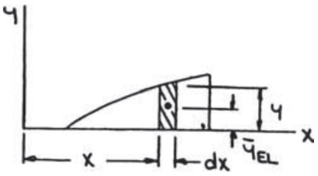
and $\quad \bar{y} = \dfrac{2 - 2\ln 2 - \frac{1}{2}}{2(2 - \ln 2 - 1)} \quad$ or $\quad \bar{y} = 0.1853$ in. ◀



PROBLEM 5.51

Determine the value of a for which the ratio $\bar{x}/\bar{y}$ is 9.

SOLUTION



We have
$$\bar{x}_{EL} = x$$
$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2}\left(1 - \frac{1}{x}\right)$$

and
$$dA = ydx = \left(1 - \frac{1}{x}\right)dx$$

Then
$$A = \int dA = \int_1^a \left(1 - \frac{1}{x}\right)\frac{dx}{2} = [x - \ln x]_1^a$$
$$= (a - \ln a - 1)\,\text{in.}^2$$

and
$$\int \bar{x}_{EL}\,dA = \int_1^a x\left[\left(1 - \frac{1}{x}\right)dx\right] = \left[\frac{x^2}{2} - x\right]_1^a$$
$$= \left(\frac{a^2}{2} - a + \frac{1}{2}\right)\text{in.}^3$$

$$\int \bar{y}_{EL}\,dA = \int_1^a \frac{1}{2}\left(1 - \frac{1}{x}\right)\left[\left(1 - \frac{1}{x}\right)dx\right] = \frac{1}{2}\int_1^a \left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$$
$$= \frac{1}{2}\left[x - 2\ln x - \frac{1}{x}\right]_1^a$$
$$= \frac{1}{2}\left(a - 2\ln a - \frac{1}{a}\right)\text{in.}^3$$

$$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x} = \frac{\frac{a^2}{2} - a + \frac{1}{2}}{a - \ln a - 1}\,\text{in.}$$
$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y} = \frac{a - 2\ln a - \frac{1}{a}}{2(a - \ln a - 1)}\,\text{in.}$$

PROBLEM 5.51 (Continued)

Find: a so that $\dfrac{\bar{x}}{\bar{y}} = 9$

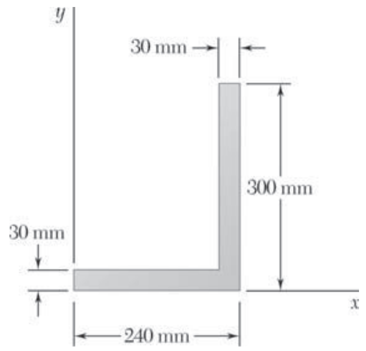
We have
$$\frac{\bar{x}}{\bar{y}} = \frac{\bar{x}A}{\bar{y}A} = \frac{\int \bar{x}_{EL}\,dA}{\int \bar{y}_{EL}\,dA}$$

Then
$$\frac{\frac{1}{2}a^2 - a + \frac{1}{2}}{\frac{1}{2}\left(a - 2\ln a - \frac{1}{a}\right)} = 9$$

or
$$a^3 - 11a^2 + a + 18a\ln a + 9 = 0$$

Using trial and error or numerical methods and ignoring the trivial solution $a = 1$ in., find

$a = 1.901$ in. and $a = 3.74$ in. ◀

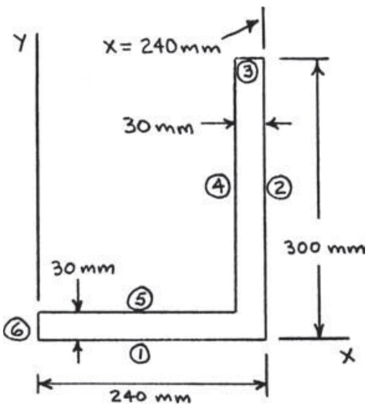


PROBLEM 5.52

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.1 about (*a*) the line $x = 240$ mm, (*b*) the y axis.

PROBLEM 5.1 Locate the centroid of the plane area shown.

SOLUTION



From the solution to Problem 5.1 we have

$$A = 15.3 \times 10^3 \text{ mm}^2$$

$$\Sigma \bar{x}A = 2.6865 \times 10^6 \text{ mm}^3$$

$$\Sigma \bar{y}A = 1.4445 \times 10^6 \text{ mm}^3$$

Applying the theorems of Pappus-Guldinus we have

(*a*) Rotation about the line $x = 240$ mm

$$\text{Volume} = 2\pi(240 - \bar{x})A$$
$$= 2\pi(240A - \Sigma\bar{x}A)$$
$$= 2\pi[240(15.3 \times 10^3) - 2.6865 \times 10^6]$$

$\text{Volume} = 6.19 \times 10^6 \text{ mm}^3$ ◀

$$\text{Area} = 2\pi \bar{X}_{\text{line}} L = 2\pi \Sigma(\bar{x}_{\text{line}})L$$
$$= 2\pi(\bar{x}_1 L_1 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5 + \bar{x}_6 L_6)$$

Where $\bar{x}_1, \ldots, \bar{x}_6$ are measured with respect to line $x = 240$ mm.

$$\text{Area} = 2\pi[(120)(240) + (15)(30) + (30)(270) + (135)(210) + (240)(30)]$$

$\text{Area} = 458 \times 10^3 \text{ mm}^2$ ◀

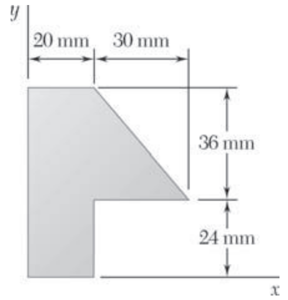
(*b*) Rotation about the y axis

$$\text{Volume} = 2\pi \bar{X}_{\text{area}} A = 2\pi(\Sigma\bar{x}A)$$
$$= 2\pi(2.6865 \times 10^6 \text{ mm}^3)$$

$\text{Volume} = 16.88 \times 10^6 \text{ mm}^3$ ◀

$$\text{Area} = 2\pi \bar{X}_{\text{line}} L = 2\pi \Sigma(\bar{x}_{\text{line}})L$$
$$= 2\pi(\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5)$$
$$= 2\pi[(120)(240) + (240)(300) + (225)(30) + (210)(270) + (105)(210)]$$

$\text{Area} = 1.171 \times 10^6 \text{ mm}^2$ ◀

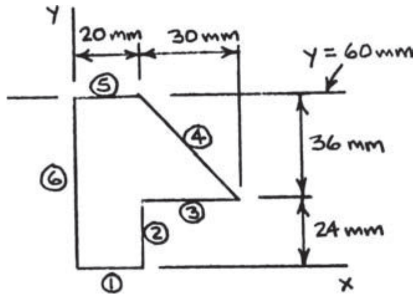


PROBLEM 5.53

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.2 about (*a*) the line $y = 60$ mm, (*b*) the y axis.

PROBLEM 5.2 Locate the centroid of the plane area shown.

SOLUTION



From the solution to Problem 5.2 we have

$$A = 1740 \text{ mm}^2$$

$$\Sigma \bar{x}A = 28200 \text{ mm}^3$$

$$\Sigma \bar{y}A = 55440 \text{ mm}^3$$

Applying the theorems of Pappus-Guldinus we have

(*a*) Rotation about the line $y = 60$ mm

$$\text{Volume} = 2\pi(60 - \bar{y})A$$

$$= 2\pi(60A - \Sigma \bar{y}A)$$

$$= 2\pi[60(1740) - 55440]$$

$\text{Volume} = 308 \times 10^3 \text{ mm}^3$ ◀

$$\text{Area} = 2\pi \bar{Y}_{\text{line}}$$

$$= 2\pi \Sigma(\bar{y}_{\text{line}})L$$

$$= 2\pi(\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4 + \bar{y}_6 L_6)$$

Where $\bar{y}_1, \ldots, \bar{y}_6$ are measured with respect to line $y = 60$ mm.

$$\text{Area} = 2\pi\left[(60)(20) + (48)(24) + (36)(30) + (18)\sqrt{(30)^2 + (36)^2} + (30)(60)\right]$$

$\text{Area} = 38.2 \times 10^3 \text{ mm}^2$ ◀

(*b*) Rotation about the y axis

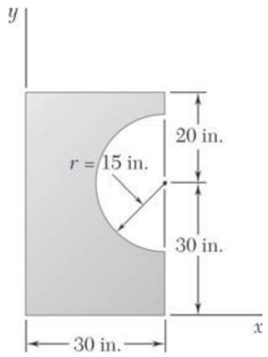
$$\text{Volume} = 2\pi \bar{X}_{\text{area}} A = 2\pi(\Sigma \bar{x}A) = 2\pi(28200 \text{ mm}^3)$$

$\text{Volume} = 177.2 \times 10^6 \text{ mm}^3$ ◀

$$\text{Area} = 2\pi \bar{X}_{\text{line}} L = 2\pi \Sigma(\bar{x}_{\text{line}})L = 2\pi(\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5)$$

$$= 2\pi\left[(10)(20) + (20)(24) + (35)(30) + (35)\sqrt{(30)^2 + (36)^2} + (10)(20)\right]$$

$\text{Area} = 22.4 \times 10^3 \text{ mm}^2$ ◀

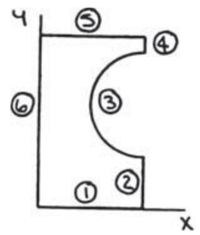


PROBLEM 5.54

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.8 about (*a*) the *x* axis, (*b*) the *y* axis.

PROBLEM 5.8 Locate the centroid of the plane area shown.

SOLUTION



From the solution to Problem 5.8 we have

$$A = 1146.57 \text{ in.}^2$$
$$\Sigma \bar{x}A = 14147.0 \text{ in.}^3$$
$$\Sigma \bar{y}A = 26897 \text{ in.}^3$$

Applying the theorems of Pappus-Guldinus we have

(*a*) Rotation about the *x* axis:

$$\text{Volume} = 2\pi \overline{Y}_{area} A = 2\pi \Sigma \bar{y} A$$
$$= 2\pi(26897 \text{ in.}^3) \qquad \text{or} \quad \text{Volume} = 169.0 \times 10^3 \text{ in.}^3 \blacktriangleleft$$

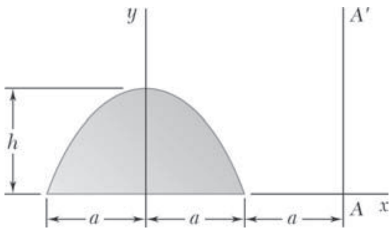
$$\text{Area} = 2\pi \overline{Y}_{line} A$$
$$= 2\pi \Sigma(\bar{y}_{line})A$$
$$= 2\pi(\bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4 + \bar{y}_5 L_5 + \bar{y}_6 L_6)$$
$$= 2\pi[(7.5)(15) + (30)(\pi \times 15) + (47.5)(5)$$
$$+ (50)(30) + (25)(50)] \qquad \text{or} \quad \text{Area} = 28.4 \times 10^3 \text{ in.}^2 \blacktriangleleft$$

(*b*) Rotation about the *y* axis

$$\text{Volume} = 2\pi \overline{X}_{area} A = 2\pi \Sigma \bar{x} A$$
$$= 2\pi(14147.0 \text{ in.}^3) \qquad \text{or} \quad \text{Volume} = 88.9 \times 10^3 \text{ in.}^3 \blacktriangleleft$$

$$\text{Area} = 2\pi \overline{X}_{line} L = 2\pi \Sigma(\bar{x}_{line})L$$
$$= 2\pi(\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5)$$
$$= 2\pi\left[(15)(30) + (30)(15) + \left(30 - \frac{2\times 15}{\pi}\right)(\pi \times 15) + (30)(5) + (15)(30)\right]$$

or $$\text{Area} = 15.48 \times 10^3 \text{ in.}^2 \blacktriangleleft$$



PROBLEM 5.55

Determine the volume of the solid generated by rotating the parabolic area shown about (*a*) the *x* axis, (*b*) the axis *AA′*.

SOLUTION

First, from Figure 5.8a we have

$$A = \frac{4}{3}ah$$

$$\overline{y} = \frac{2}{5}h$$

Applying the second theorem of Pappus-Guldinus we have

(*a*) Rotation about the *x* axis:

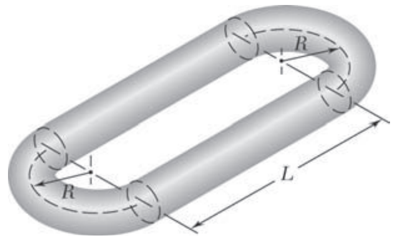
$$\text{Volume} = 2\pi\overline{y}A$$

$$= 2\pi\left(\frac{2}{5}h\right)\left(\frac{4}{3}ah\right) \qquad \text{or} \quad \text{Volume} = \frac{16}{15}\pi ah^2 \blacktriangleleft$$

(*b*) Rotation about the line *AA′*:

$$\text{Volume} = 2\pi(2a)A$$

$$= 2\pi(2a)\left(\frac{4}{3}ah\right) \qquad \text{or} \quad \text{Volume} = \frac{16}{3}\pi a^2 h \blacktriangleleft$$



PROBLEM 5.56

Determine the volume and the surface area of the chain link shown, which is made from a 6-mm-diameter bar, if $R = 10$ mm and $L = 30$ mm.

SOLUTION

The area A and circumference C of the cross section of the bar are

$$A = \frac{\pi}{4}d^2 \quad \text{and} \quad C = \pi d.$$

Also, the semicircular ends of the link can be obtained by rotating the cross section through a horizontal semicircular arc of radius R. Now, applying the theorems of Pappus-Guldinus, we have for the volume V:

$$\begin{aligned} V &= 2(V_{\text{side}}) + 2(V_{\text{end}}) \\ &= 2(AL) + 2(\pi RA) \\ &= 2(L + \pi R)A \end{aligned}$$

or

$$V = 2[30 \text{ mm} + \pi(10 \text{ mm})]\left[\frac{\pi}{4}(6 \text{ mm})^2\right]$$

$$= 3470 \text{ mm}^3$$

or $V = 3470 \text{ mm}^3$ ◀

For the area A:

$$\begin{aligned} A &= 2(A_{\text{side}}) + 2(A_{\text{end}}) \\ &= 2(CL) + 2(\pi RC) \\ &= 2(L + \pi R)C \end{aligned}$$

or

$$A = 2[30 \text{ mm} + \pi(10 \text{ mm})][\pi(6 \text{ mm})]$$

$$= 2320 \text{ mm}^2$$

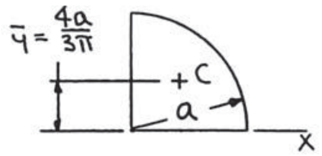
or $A = 2320 \text{ mm}^2$ ◀

PROBLEM 5.57

Verify that the expressions for the volumes of the first four shapes in Figure 5.21 on Page 253 are correct.

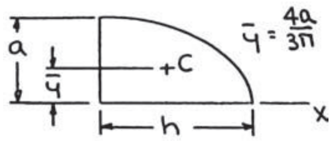
SOLUTION

Following the second theorem of Pappus-Guldinus, in each case a specific generating area A will be rotated about the x axis to produce the given shape. Values of $\bar{y}$ are from Figure 5.8a.



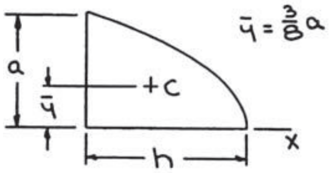
(1) Hemisphere: the generating area is a quarter circle

We have $V = 2\pi \bar{y} A = 2\pi \left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}a^2\right)$ or $V = \frac{2}{3}\pi a^3$ ◀



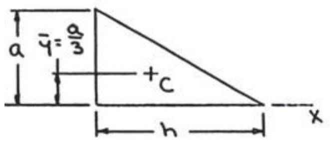
(2) Semiellipsoid of revolution: the generating area is a quarter ellipse

We have $V = 2\pi \bar{y} A = 2\pi \left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}ha\right)$ or $V = \frac{2}{3}\pi a^2 h$ ◀



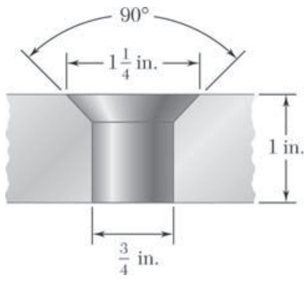
(3) Paraboloid of revolution: the generating area is a quarter parabola

We have $V = 2\pi \bar{y} A = 2\pi \left(\frac{3}{8}a\right)\left(\frac{2}{3}ah\right)$ or $V = \frac{1}{2}\pi a^2 h$ ◀



(4) Cone: the generating area is a triangle

We have $V = 2\pi \bar{y} A = 2\pi \left(\frac{a}{3}\right)\left(\frac{1}{2}ha\right)$ or $V = \frac{1}{3}\pi a^2 h$ ◀

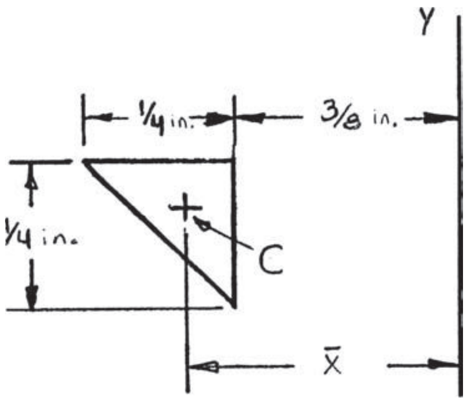


PROBLEM 5.58

A $\frac{3}{4}$-in.-diameter hole is drilled in a piece of 1-in.-thick steel; the hole is then countersunk as shown. Determine the volume of steel removed during the countersinking process.

SOLUTION

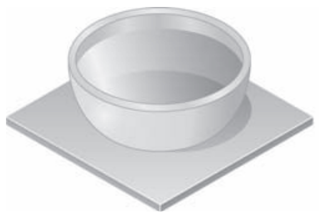
The required volume can be generated by rotating the area shown about the *y* axis. Applying the second theorem of Pappus-Guldinus, we have

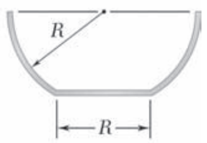


$$V = 2\pi \bar{x} A$$

$$= 2\pi\left[\frac{3}{8}+\frac{1}{3}\left(\frac{1}{4}\right)\text{in.}\right]\times\left[\frac{1}{2}\times\frac{1}{4}\text{ in.}\times\frac{1}{4}\text{ in.}\right]$$

$V = 0.0900\text{ in.}^3$ ◀



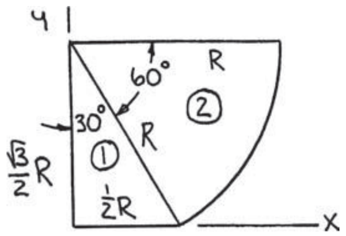


PROBLEM 5.59

Determine the capacity, in liters, of the punch bowl shown if $R = 250$ mm.

SOLUTION

The volume can be generated by rotating the triangle and circular sector shown about the y axis. Applying the second theorem of Pappus-Guldinus and using Figure 5.8a, we have

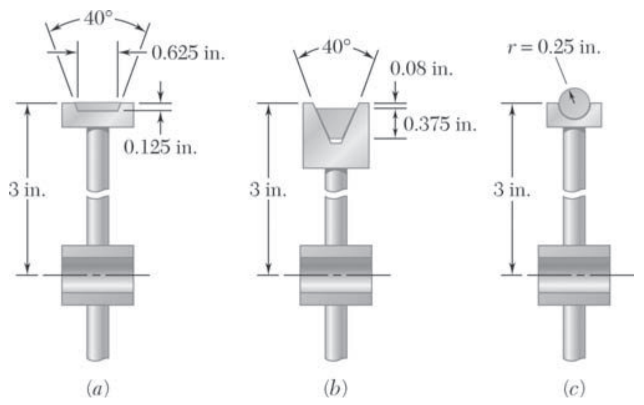


$$
\begin{aligned}
V &= 2\pi\bar{x}A = 2\pi\Sigma\bar{x}A \\
&= 2\pi(\bar{x}_1A_1 + \bar{x}_2A_2) \\
&= 2\pi\left[\left(\frac{1}{3}\times\frac{1}{2}R\right)\left(\frac{1}{2}\times\frac{1}{2}R\times\frac{\sqrt{3}}{2}R\right) + \left(\frac{2R\sin 30^\circ}{3\times\frac{\pi}{6}}\right)\left(\frac{\pi}{6}R^2\right)\right] \\
&= 2\pi\left(\frac{R^3}{16\sqrt{3}} + \frac{R^3}{2\sqrt{3}}\right) \\
&= \frac{3\sqrt{3}}{8}\pi R^3 \\
&= \frac{3\sqrt{3}}{8}\pi(0.25\text{ m})^3 \\
&= 0.031883\text{ m}^3
\end{aligned}
$$

Since $10^3\, l = 1\text{ m}^3$

$$V = 0.031883\text{ m}^3 \times \frac{10^3\, l}{1\text{ m}^3}$$

$V = 31.9\, l$ ◀



PROBLEM 5.60

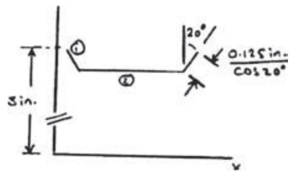
Three different drive belt profiles are to be studied. If at any given time each belt makes contact with one-half of the circumference of its pulley, determine the *contact area* between the belt and the pulley for each design.

SOLUTION

Applying the first theorem of Pappus-Guldinus, the contact area A_C of a belt is given by:

$$A_C = \pi \bar{y} L = \pi \Sigma \bar{y} L$$

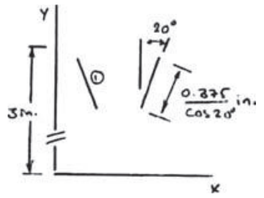
where the individual lengths are the lengths of the belt cross section that are in contact with the pulley.



(*a*) $A_C = \pi[2(\bar{y}_1 L_1) + \bar{y}_2 L_2]$

$$= \pi \left\{ 2\left[\left(3 - \frac{0.125}{2} \right) \text{in.} \right] \left[\frac{0.125 \text{ in.}}{\cos 20°} \right] + [(3 - 0.125)\text{ in.}](0.625 \text{ in.}) \right\}$$

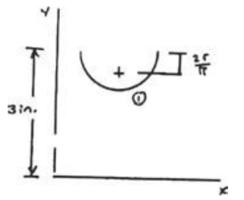
or $A_C = 8.10 \text{ in.}^2$ ◀



(*b*) $A_C = \pi[2(\bar{y}_1 L_1)]$

$$= 2\pi \left[\left(3 - 0.08 - \frac{0.375}{2} \right) \text{in.} \right] \left(\frac{0.375 \text{ in.}}{\cos 20°} \right)$$

or $A_C = 6.85 \text{ in.}^2$ ◀



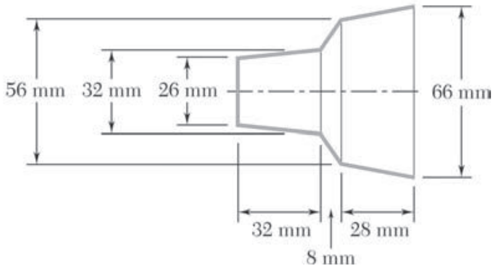
(*c*) $A_C = \pi[2(\bar{y}_1 L_1)]$

$$= \pi \left[\left(3 - \frac{2(0.25)}{\pi} \right) \text{in.} \right] [\pi(0.25 \text{ in.})]$$

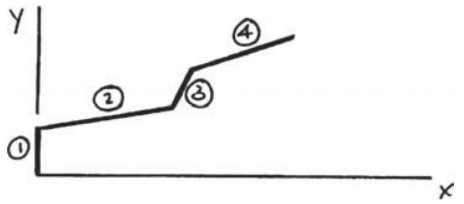
or $A_C = 7.01 \text{ in.}^2$ ◀

PROBLEM 5.61

The aluminum shade for the small high-intensity lamp shown has a uniform thickness of 1 mm. Knowing that the density of aluminum is 2800 kg/m^3, determine the mass of the shade.



SOLUTION



The mass of the lamp shade is given by

$$m = \rho V = \rho A t$$

Where A is the surface area and t is the thickness of the shade. The area can be generated by rotating the line shown about the x axis. Applying the first theorem of Pappus Guldinus we have

$$A = 2\pi \bar{y} L = 2\pi \Sigma \bar{y} L$$
$$= 2\pi(\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4)$$

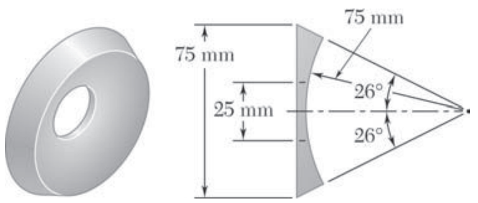
or

$$A = 2\pi\left[\frac{13 \text{ mm}}{2}(13 \text{ mm}) + \left(\frac{13+16}{2}\right)\text{mm} \times \sqrt{(32 \text{ mm})^2 + (3 \text{ mm})^2}\right.$$
$$+\left(\frac{16+28}{2}\right)\text{mm} \times \sqrt{(8 \text{ mm})^2 + (12 \text{ mm})^2}$$
$$\left.+\left(\frac{28+33}{2}\right)\text{mm} \times \sqrt{(28 \text{ mm})^2 + (5 \text{ mm})^2}\right]$$
$$= 2\pi(84.5 + 466.03 + 317.29 + 867.51)$$
$$= 10903.4 \text{ mm}^2$$

Then

$$m = \rho A t$$
$$= (2800 \text{ kg/m}^3)(10.9034 \times 10^{-3} \text{ m}^2)(0.001 \text{ m})$$

or $$m = 0.0305 \text{ kg} \blacktriangleleft$$

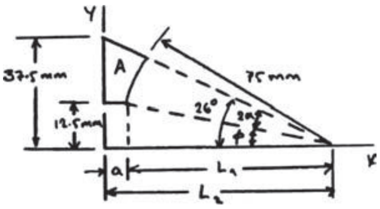


PROBLEM 5.62

The escutcheon (a decorative plate placed on a pipe where the pipe exits from a wall) shown is cast from brass. Knowing that the density of brass is 8470 kg/m^3, determine the mass of the escutcheon.

SOLUTION

The mass of the escutcheon is given by $m = (\text{density})V$, where V is the volume. V can be generated by rotating the area A about the x-axis.



From the figure:

$$L_1 = \sqrt{75^2 - 12.5^2} = 73.9510 \text{ m}$$

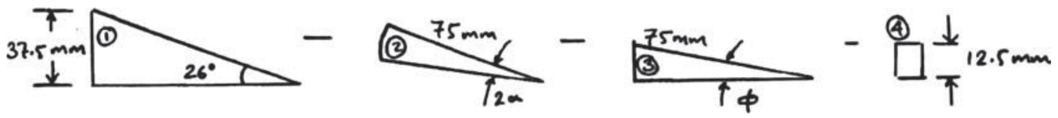
$$L_2 = \frac{37.5}{\tan 26°} = 76.8864 \text{ mm}$$

$$a = L_2 - L_1 = 2.9324 \text{ mm}$$

$$\phi = \sin^{-1}\frac{12.5}{75} = 9.5941°$$

$$\alpha = \frac{26° - 9.5941°}{2} = 8.2030° = 0.143168 \text{ rad}$$

Area A can be obtained by combining the following four areas:



Applying the second theorem of Pappus-Guldinus and using Figure 5.8a, we have

$$V = 2\pi\bar{y}A = 2\pi\Sigma\bar{y}A$$

Seg.	A, mm^2	$\bar{y}$, mm	$\bar{y}_A$, mm^3
1	$\frac{1}{2}(76.886)(37.5) = 1441.61$	$\frac{1}{3}(37.5) = 12.5$	18020.1
2	$-\alpha(75)^2 = -805.32$	$\frac{2(75)\sin\alpha}{3\alpha}\sin(\alpha+\phi) = 15.2303$	−12265.3
3	$-\frac{1}{2}(73.951)(12.5) = -462.19$	$\frac{1}{3}(12.5) = 4.1667$	−1925.81
4	$-(2.9354)(12.5) = -36.693$	$\frac{1}{2}(12.5) = 6.25$	−229.33
Σ			3599.7

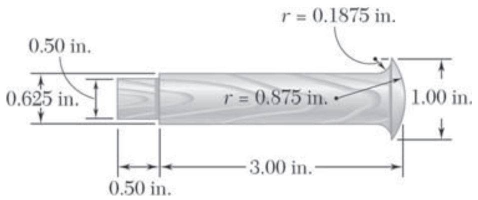
PROBLEM 5.62 (Continued)

Then

$$V = 2\pi\Sigma\overline{y}A$$
$$= 2\pi(3599.7\text{ mm}^3)$$
$$= 22618\text{ mm}^3$$
$$m = (\text{density})V$$
$$= (8470\text{ kg/m}^3)(22.618\times10^{-6}\text{ m}^3)$$
$$= 0.191574\text{ kg}$$

or $m = 0.1916\text{ kg}$ ◀

PROBLEM 5.63



A manufacturer is planning to produce 20,000 wooden pegs having the shape shown. Determine how many gallons of paint should be ordered, knowing that each peg will be given two coats of paint and that one gallon of paint covers 100 ft^2.

SOLUTION

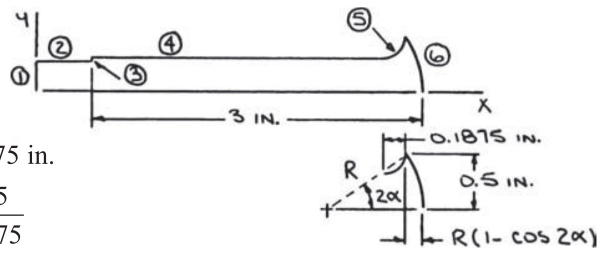
The number of gallons of paint needed is given by

$$\text{Number of gallons} = (\text{Number of pegs})(\text{Surface area of 1 peg})\left(\frac{1 \text{ gallon}}{100 \text{ ft}^2}\right)(2 \text{ coats})$$

or $$\text{Number of gallons} = 400\, A_s \quad (A_s \sim \text{ft}^2)$$

where A_s is the surface area of one peg. A_s can be generated by rotating the line shown about the x axis. Using the first theorem of Pappus-Guldinus and Figures 5.8b,

We have



$$R = 0.875 \text{ in.}$$

$$\sin 2\alpha = \frac{0.5}{0.875}$$

or $$2\alpha = 34.850° \quad \alpha = 17.425°$$

$$A_s = 2\pi \overline{Y} L = 2\pi \Sigma \overline{y} L$$

	L, in.	$\overline{y}$, in.	$\overline{y}L$, in.^{2}
1	0.25	0.125	0.03125
2	0.5	0.25	0.125
3	0.0625	$\frac{0.25+0.3125}{2} = 0.28125$	0.0175781
4	$3 - 0.875(1-\cos 34.850) - 0.1875 = 2.6556$	0.3125	0.82988
5	$\frac{\pi}{2} \times 0.1875 = 0.29452$	$0.5 - \frac{2 \times 0.1875}{\pi} = 0.38063$	0.112103
6	$2\alpha(0.875)$	$\frac{0.875 \sin 17.425°}{\alpha} \times \sin 17.425°$	0.137314

$$\Sigma \overline{y} L = 1.25312 \text{ in.}^2$$

PROBLEM 5.63 (Continued)

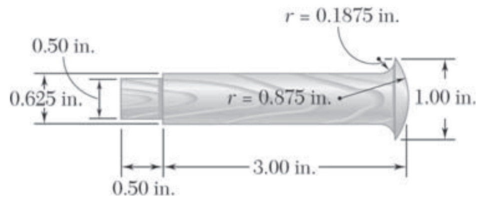
Then $$A_s = 2\pi(1.25312 \text{ in.}^2) \times \frac{1 \text{ ft}^2}{144 \text{ in.}^2}$$

$$= 0.054678 \text{ ft}^2$$

Finally $$\text{Number of gallons} = 400 \times 0.054678$$

$$= 21.87 \text{ gallons}$$

Order 22 gallons ◀

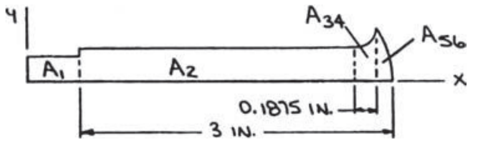


PROBLEM 5.64

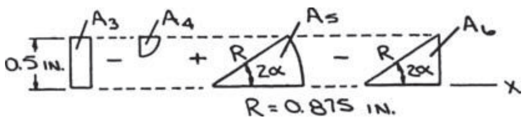
The wooden peg shown is turned from a dowel 1 in. in diameter and 4 in. long. Determine the percentage of the initial volume of the dowel that becomes waste.

SOLUTION

To obtain the solution it is first necessary to determine the volume of the peg. That volume can be generated by rotating the area shown about the *x* axis.



The generating area is next divided into six components as indicated



$$\sin 2\alpha = \frac{0.5}{0.875}$$

or $$2\alpha = 34.850° \qquad \alpha = 17.425°$$

Applying the second theorem of Pappus-Guldinus and then using Figure 5.8a, we have

$$V_{PEG} = 2\pi \overline{Y} A = 2\pi \Sigma \overline{y} A$$

	A, in.^{2}	$\overline{y}$, in.	$\overline{y}A$, in.^{3}
1	$0.5 \times 0.25 \times 0.125$	0.125	0.015625
2	$[3 - 0.875(1-\cos 34.850°) - 0.1875]$ $\times(0.3125) = 0.82987$	0.15625	0.129667
3	$0.1875 \times 0.5 \times 0.9375$	0.25	0.023438
4	$-\frac{\pi}{4}(0.1875)^2 = -0.027612$	$0.5 - \frac{4\times 0.1875}{3\pi} = 0.42042$	−0.011609
5	$\alpha(0.875)^2$	$\frac{2\times 0.875 \sin 17.425°}{3\alpha} \times \sin 17.425°$	0.04005
6	$-\frac{1}{2}(0.875\cos 34.850°)(0.5) = -0.179517$	$\frac{1}{3}(0.5) = 0.166667$	−0.029920

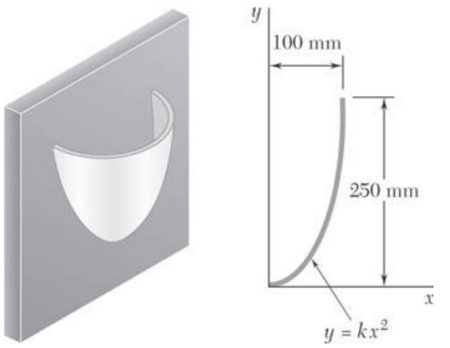
$$\Sigma \overline{y} L = 0.167252 \text{ in.}^3$$

PROBLEM 5.64 (Continued)

Then
$$V_{peg} = 2\pi(0.167252 \text{ in.}^3)$$
$$= 1.05088 \text{ in.}^3$$

Now
$$V_{dowel} = \frac{\pi}{4}(\text{diameter})^2 \text{ (length)}$$
$$= \frac{\pi}{4}(1 \text{ in.})^2 \text{ (4 in.)}$$
$$= 3.14159 \text{ in.}^3$$

Then
$$\% \text{ Waste} = \frac{V_{waste}}{V_{dowel}} \times 100\%$$
$$= \frac{V_{dowel} - V_{peg}}{V_{dowel}} \times 100\%$$
$$= \left(1 - \frac{1.05088}{3.14159}\right) \times 100\% \qquad \text{or} \quad \% \text{ Waste} = 66.5\% \blacktriangleleft$$



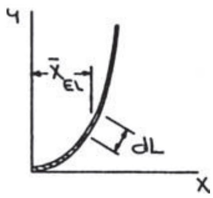
PROBLEM 5.65*

The shade for a wall-mounted light is formed from a thin sheet of translucent plastic. Determine the surface area of the outside of the shade, knowing that it has the parabolic cross section shown.

SOLUTION

First note that the required surface area A can be generated by rotating the parabolic cross section through π radians about the y axis. Applying the first theorem of Pappus-Guldinus we have

$$A = \pi \bar{x} L$$



Now at $\quad x = 100 \text{ mm}, \quad y = 250 \text{ mm}$

$$250 = k(100)^2 \quad \text{or} \quad k = 0.025 \text{ mm}^{-1}$$

and $\quad \bar{x}_{EL} = x$

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

where $\quad \dfrac{dy}{dx} = 2kx$

Then $\quad dL = \sqrt{1 + 4k^2x^2}\, dx$

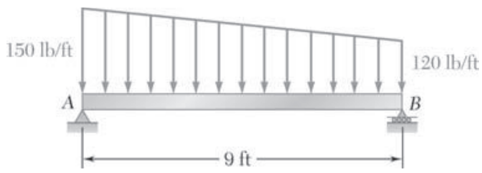
We have $\quad \bar{x}L = \int \bar{x}_{EL}\, dL = \int_0^{100} x\left(\sqrt{1+4k^2x^2}\,dx\right)$

$$\bar{x}L = \left[\frac{1}{3}\,\frac{1}{4k^2}(1+4k^2x^2)^{3/2}\right]_0^{100}$$

$$= \frac{1}{12}\,\frac{1}{(0.025)^2}\left\{[1+4(0.025)^2(100)^2]^{3/2} - (1)^{3/2}\right\}$$

$$= 17543.3 \text{ mm}^2$$

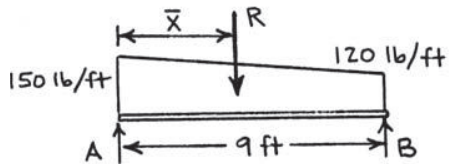
Finally $\quad A = \pi(17543.3 \text{ mm}^2) \qquad$ or $\quad A = 55.1 \times 10^3 \text{ mm}^2$ ◀

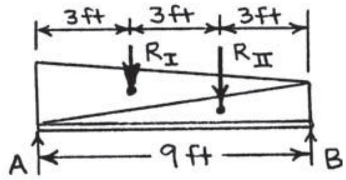


PROBLEM 5.66

For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION





$$R_{\mathrm{I}} = \frac{1}{2}(150\,\text{lb/ft})(9\text{ ft}) = 675\text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(120\text{ lb/ft})(9\,\text{ft}) = 540\,\text{lb}$$

$$R = R_{\mathrm{I}} + R_{\mathrm{II}} = 675 + 540 = 1215\text{ lb}$$

$$\overline{X}R = \Sigma\overline{X}R: \quad \overline{X}(1215) = (3)675 + (6)(540) \quad \overline{X} = 4.3333\text{ ft}$$

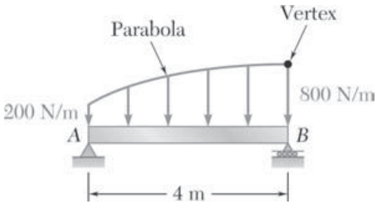
(*a*) $\mathbf{R} = 1215\text{ lb}\downarrow \quad \overline{X} = 4.33\text{ ft}$ ◀

(*b*) Reactions: $+\circlearrowleft \Sigma M_A = 0: \quad B(9\text{ ft}) - (1215\text{ lb})(4.3333\text{ ft}) = 0$

$B = 585.00\text{ lb}$ $\qquad\mathbf{B} = 585\text{ lb}\uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad A + 585.00\text{ lb} - 1215\text{ lb} = 0$

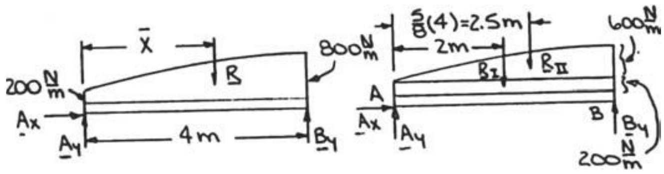
$A = 630.00\text{ lb}$ $\qquad\mathbf{A} = 630\text{ lb}\uparrow$ ◀



PROBLEM 5.67

For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION



(*a*) We have $R_{\text{I}} = (4\text{ m})(200\text{ N/m}) = 800\text{ N}$

$$R_{\text{II}} = \frac{2}{3}(4\text{ m})(600\text{ N/m}) = 1600\text{ N}$$

Then ΣF_y: $-R = -R_{\text{I}} - R_{\text{II}}$

or $R = 800 + 1600 = 2400\text{ N}$

and ΣM_A: $-\bar{X}(2400) = -2(800) - 2.5(1600)$

or $\bar{X} = \dfrac{7}{3}\text{ m}$

$\mathbf{R} = 2400\text{ N}\downarrow \quad \bar{X} = 2.33\text{ m}$ ◀

(*b*) Reactions $\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

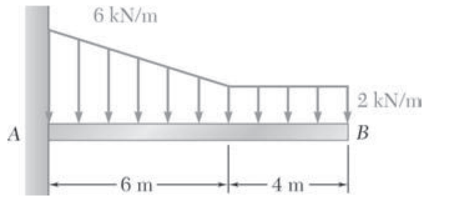
$$+\circlearrowleft \Sigma M_A = 0:\quad (4\text{ m})B_y - \left(\frac{7}{3}\text{ m}\right)(2400\text{ N}) = 0$$

or $B_y = 1400\text{ N}$

$$+\uparrow \Sigma F_y = 0:\quad A_y + 1400\text{ N} - 2400\text{ N} = 0$$

or $A_y = 1000\text{ N}$

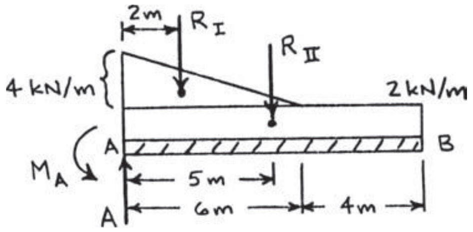
$\mathbf{A} = 1000\text{ N}\uparrow \quad \mathbf{B} = 1400\text{ N}\uparrow$ ◀



PROBLEM 5.68

Determine the reactions at the beam supports for the given loading.

SOLUTION



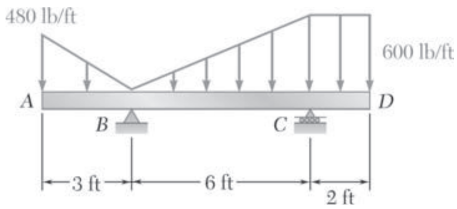
$$R_{\mathrm{I}} = \frac{1}{2}(4\text{ kN/m})(6\text{ m})$$
$$= 12\text{ kN}$$
$$R_{\mathrm{II}} = (2\text{ kN/m})(10\text{ m})$$
$$= 20\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A - 12\text{ kN} - 20\text{ kN} = 0$$

$$\mathbf{A} = 32.0\text{ kN} \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (12\text{ kN})(2\text{ m}) - (20\text{ kN})(5\text{ m}) = 0$$

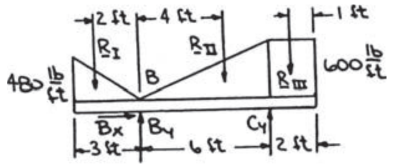
$$\mathbf{M}_A = 124.0\text{ kN}\cdot\text{m} \circlearrowleft \blacktriangleleft$$



PROBLEM 5.69

Determine the reactions at the beam supports for the given loading.

SOLUTION



We have

$$R_{\mathrm{I}} = \frac{1}{2}(3\text{ ft})(480\text{ lb/ft}) = 720\text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(6\text{ ft})(600\text{ lb/ft}) = 1800\text{ lb}$$

$$R_{\mathrm{III}} = (2\text{ ft})(600\text{ lb/ft}) = 1200\text{ lb}$$

Then

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (2\text{ ft})(720\text{ lb}) - (4\text{ ft})(1800\text{ lb}) + (6\text{ ft})C_y - (7\text{ ft})(1200\text{ lb}) = 0$$

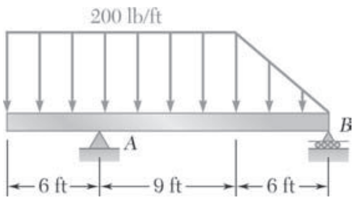
or

$$C_y = 2360\text{ lb} \qquad \mathbf{C} = 2360\text{ lb}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -720\text{ lb} + B_y - 1800\text{ lb} + 2360\text{ lb} - 1200\text{ lb} = 0$$

or

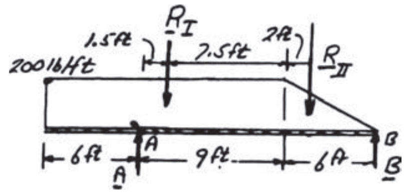
$$B_y = 1360\text{ lb} \qquad \mathbf{B} = 1360\text{ lb}\uparrow \blacktriangleleft$$



PROBLEM 5.70

Determine the reactions at the beam supports for the given loading.

SOLUTION



$$R_{\mathrm{I}} = (200 \text{ lb/ft})(15 \text{ ft})$$

$$R_{\mathrm{I}} = 3000 \text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(200 \text{ lb/ft})(6 \text{ ft})$$

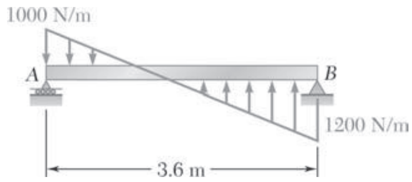
$$R_{\mathrm{II}} = 600 \text{ lb}$$

$+\circlearrowleft \Sigma M_A = 0$: $-(3000 \text{ lb})(1.5 \text{ ft}) - (600 \text{ lb})(9 \text{ ft} + 2 \text{ ft}) + B(15 \text{ ft}) = 0$

$B = 740 \text{ lb}$ $\qquad$ $\mathbf{B} = 740 \text{ lb} \uparrow$ ◀

$+\uparrow \Sigma F_y = 0$: $A + 740 \text{ lb} - 3000 \text{ lb} - 600 \text{ lb} = 0$

$A = 2860 \text{ lb}$ $\qquad$ $\mathbf{A} = 2860 \text{ lb} \uparrow$ ◀

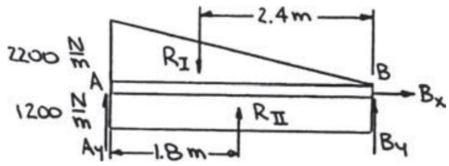


PROBLEM 5.71

Determine the reactions at the beam supports for the given loading.

SOLUTION

First replace the given loading with the loading shown below. The two loadings are equivalent because both are defined by a linear relation between load and distance and the values at the end points are the same.



We have $R_{\text{I}} = \frac{1}{2}(3.6 \text{ m})(2200 \text{ N/m}) = 3960 \text{ N}$

$$R_{\text{II}} = (3.6 \text{ m})(1200 \text{ N/m}) = 4320 \text{ N}$$

Then $\xrightarrow{+} \Sigma F_x = 0$: $B_x = 0$

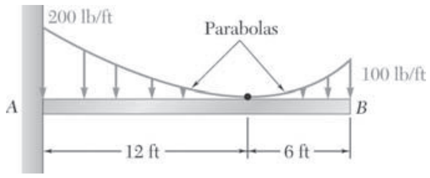
$+\circlearrowleft \Sigma M_B = 0$: $-(3.6 \text{ m})A_y + (2.4 \text{ m})(3960 \text{ N})$

$$-(1.8 \text{ m})(4320 \text{ N}) = 0$$

or $A_y = 480 \text{ N}$ $\qquad$ $\mathbf{A} = 480 \text{ N}\uparrow$ ◀

$+\uparrow \Sigma F_y = 0$: $480 \text{ N} - 3960 \text{ N} + 4320 + B_y = 0$

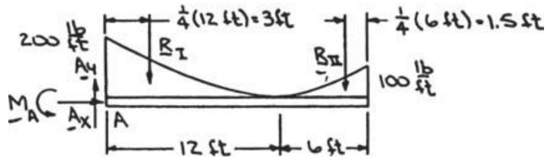
or $B_y = -840 \text{ N}$ $\qquad$ $\mathbf{B} = 840 \text{ N}\downarrow$ ◀



PROBLEM 5.72

Determine the reactions at the beam supports for the given loading.

SOLUTION



We have $$R_I = \frac{1}{3}(12\text{ ft})(200\text{ lb/ft}) = 800\text{ lb}$$

$$R_{II} = \frac{1}{3}(6\text{ ft})(100\text{ lb/ft}) = 200\text{ lb}$$

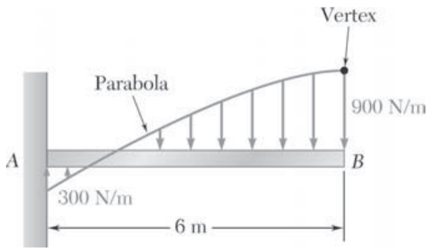
Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 800\text{ lb} - 200\text{ lb} = 0$$

or $$A_y = 1000\text{ lb} \qquad \mathbf{A} = 1000\text{ lb} \uparrow ◀$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (3\text{ ft})(800\text{ lb}) - (16.5\text{ ft})(200\text{ lb}) = 0$$

or $$M_A = 5700\text{ lb}\cdot\text{ft} \qquad \mathbf{M}_A = 5700\text{ lb}\cdot\text{ft} \circlearrowleft ◀$$

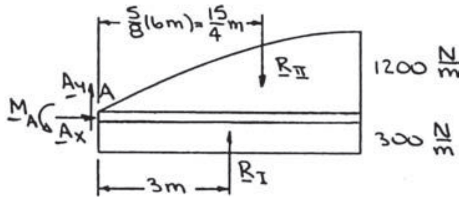


PROBLEM 5.73

Determine the reactions at the beam supports for the given loading.

SOLUTION

First replace the given loading with the loading shown below. The two loadings are equivalent because both are defined by a parabolic relation between load and distance and the values at the end points are the same.



We have $\qquad R_{\text{I}} = (6\text{ m})(300\text{ N/m}) = 1800\text{ N}$

$$R_{\text{II}} = \frac{2}{3}(6\text{ m})(1200\text{ N/m}) = 4800\text{ N}$$

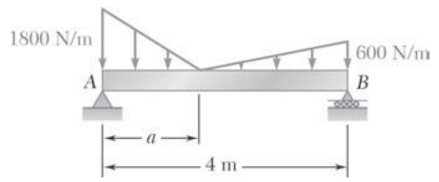
Then $\qquad \xrightarrow{+} \Sigma F_x = 0{:}\quad A_x = 0$

$$+\!\uparrow \Sigma F_y = 0{:}\quad A_y + 1800\text{ N} - 4800\text{ N} = 0$$

or $\qquad A_y = 3000\text{ N} \qquad \mathbf{A} = 3000\text{ N}\uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0{:}\quad M_A + (3\text{ m})(1800\text{ N}) - \left(\frac{15}{4}\text{ m}\right)(4800\text{ N}) = 0$$

or $\qquad M_A = 12.6\text{ kN}\cdot\text{m} \qquad \mathbf{M}_A = 12.6\text{ kN}\cdot\text{m} \circlearrowleft$ ◀

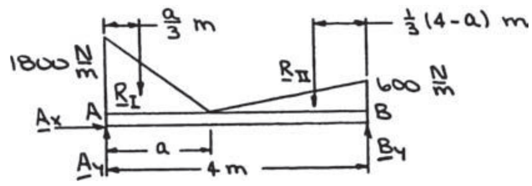


PROBLEM 5.74

Determine (*a*) the distance *a* so that the vertical reactions at supports *A* and *B* are equal, (*b*) the corresponding reactions at the supports.

SOLUTION

(*a*)



We have $R_{\mathrm{I}} = \frac{1}{2}(a\text{ m})(1800\text{ N/m}) = 900a\text{ N}$

$$R_{\mathrm{II}} = \frac{1}{2}[(4-a)\text{m}](600\text{ N/m}) = 300(4-a)\text{N}$$

Then $+\uparrow \Sigma F_y = 0$: $A_y - 900a - 300(4-a) + B_y = 0$

or $A_y + B_y = 1200 + 600a$

Now $A_y = B_y \Rightarrow A_y = B_y = 600 + 300a\,(\text{N})$ (1)

Also $+\circlearrowleft \Sigma M_B = 0$: $-(4\text{ m})A_y + \left[\left(4 - \frac{a}{3}\right)\text{m}\right][(900a)\text{N}]$

$$+\left[\frac{1}{3}(4-a)\text{m}\right][300(4-a)\text{N}] = 0$$

or $A_y = 400 + 700a - 50a^2$ (2)

Equating Eqs. (1) and (2) $600 + 300a = 400 + 700a - 50a^2$

or $a^2 - 8a + 4 = 0$

Then $$a = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2}$$

or $a = 0.53590\text{ m}$ $a = 7.4641\text{ m}$

Now $a \le 4\text{ m} \Rightarrow$ $a = 0.536\text{ m}$ ◀

PROBLEM 5.74 (Continued)

(*b*) We have $\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

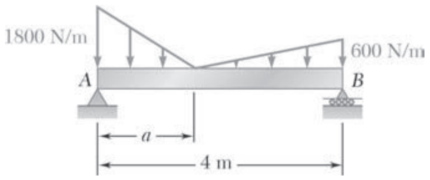
Eq. (1)

$$A_y = B_y$$

$$= 600 + 300(0.53590)$$

$$= 761 \text{ N}$$

$\mathbf{A} = \mathbf{B} = 761 \text{ N} \uparrow$ ◀

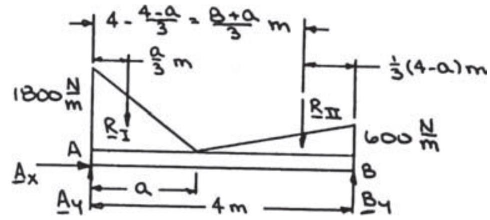


PROBLEM 5.75

Determine (*a*) the distance *a* so that the reaction at support *B* is minimum, (*b*) the corresponding reactions at the supports.

SOLUTION

(*a*)



We have $$R_{\text{I}} = \frac{1}{2}(a \text{ m})(1800 \text{ N/m}) = 900a \text{ N}$$

$$R_{\text{II}} = \frac{1}{2}[(4-a)\text{m}](600 \text{ N/m}) = 300(4-a) \text{ N}$$

Then $$+\circlearrowleft \Sigma M_A = 0: \quad -\left(\frac{a}{3}\text{m}\right)(900a \text{ N}) - \left(\frac{8+a}{3}\text{m}\right)\left[300(4-a)\text{N}\right] + (4 \text{ m})B_y = 0$$

or $$B_y = 50a^2 - 100a + 800 \tag{1}$$

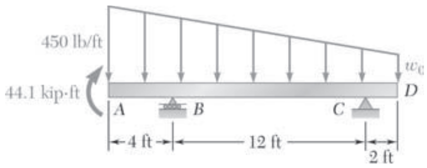
Then $$\frac{dB_y}{da} = 100a - 100 = 0$$ or $a = 1.000$ m ◀

(*b*) Eq. (1) $$B_y = 50(1)^2 - 100(1) + 800 = 750 \text{ N}$$ $\mathbf{B} = 750 \text{ N} \uparrow$ ◀

and $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 900(1)\text{N} - 300(4-1)\text{N} + 750 \text{ N} = 0$$

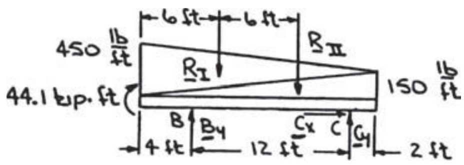
or $$A_y = 1050 \text{ N}$$ $\mathbf{A} = 1050 \text{ N} \uparrow$ ◀



PROBLEM 5.76

Determine the reactions at the beam supports for the given loading when $\omega_0 = 150$ lb/ft.

SOLUTION



We have $$R_{\text{I}} = \frac{1}{2}(18 \text{ ft})(450 \text{ lb/ft}) = 4050 \text{ lb}$$

$$R_{\text{II}} = \frac{1}{2}(18 \text{ ft})(150 \text{ lb/ft}) = 1350 \text{ lb}$$

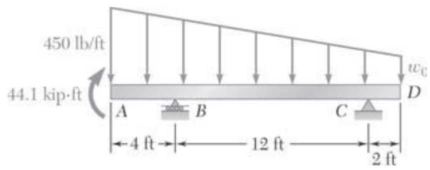
Then $$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -(44{,}100 \text{ kip} \cdot \text{ft}) - (2 \text{ ft}) - (4050 \text{ lb}) - (8 \text{ ft})(1350 \text{ lb}) + (12 \text{ ft})C_y = 0$$

or $$C_y = 5250 \text{ lb} \qquad \mathbf{C} = 5250 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 4050 \text{ lb} - 1350 \text{ lb} + 5250 \text{ lb} = 0$$

or $$B_y = 150 \text{ lb} \qquad \mathbf{B} = 150.0 \text{ lb} \uparrow \blacktriangleleft$$

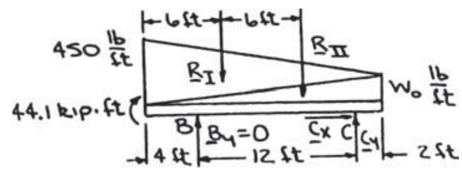


PROBLEM 5.77

Determine (*a*) the distributed load ω_0 at the end *D* of the beam *ABCD* for which the reaction at *B* is zero, (*b*) the corresponding reaction at *C*.

SOLUTION

(*a*)



We have $$R_{\text{I}} = \frac{1}{2}(18\text{ ft})(450\text{ lb/ft}) = 4050\text{ lb}$$

$$R_{\text{II}} = \frac{1}{2}(18\text{ ft})(\omega_0\text{ lb/ft}) = 9\,\omega_0\text{ lb}$$

Then $$+\circlearrowleft \Sigma M_C = 0: \quad -(44{,}100\text{ lb}\cdot\text{ft}) + (10\text{ ft})(4050\text{ lb}) + (4\text{ ft})(9\omega_0\text{ lb}) = 0$$

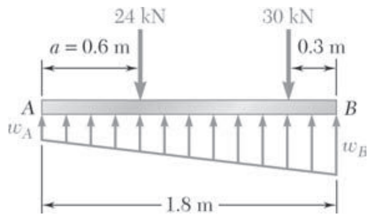
or $$\omega_0 = 100.0\text{ lb/ft} \blacktriangleleft$$

(*b*) $$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad -4050\text{ lb} - (9\times 100)\text{ lb} + C_y = 0$$

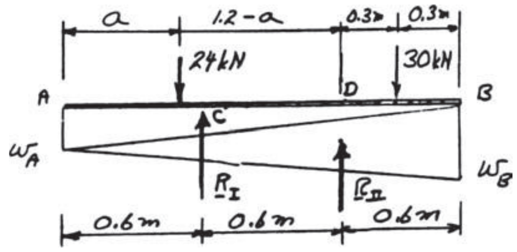
or $$C_y = 4950\text{ lb} \qquad \mathbf{C} = 4950\text{ lb}\uparrow \blacktriangleleft$$

PROBLEM 5.78



The beam *AB* supports two concentrated loads and rests on soil that exerts a linearly distributed upward load as shown. Determine the values of ω_A and ω_B corresponding to equilibrium.

SOLUTION



$$R_{\mathrm{I}} = \frac{1}{2}\omega_A(1.8\text{ m}) = 0.9\omega_A$$

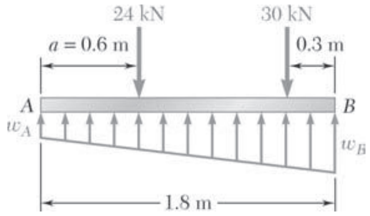
$$R_{\mathrm{II}} = \frac{1}{2}\omega_B(1.8\text{ m}) = 0.9\omega_B$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (24\text{ kN})(1.2 - a) - (30\text{ kN})(0.3\text{ m}) - (0.9\omega_A)(0.6\text{ m}) = 0 \qquad (1)$$

For $\qquad a = 0.6\text{ m}: \quad 24(1.2 - 0.6) - (30)(0.3) - 0.54\,\omega_a = 0$

$$14.4 - 9 - 0.54\omega_A = 0 \qquad \omega_A = 10.00\text{ kN/m} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -24\text{ kN} - 30\text{ kN} + 0.9(10\text{ kN/m}) + 0.9\omega_B = 0 \qquad \omega_B = 50.0\text{ kN/m} \blacktriangleleft$$

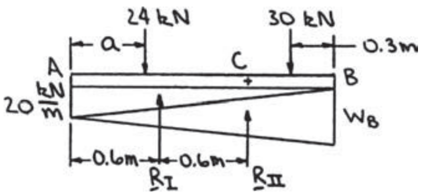


PROBLEM 5.79

For the beam and loading of Problem 5.78, determine (*a*) the distance *a* for which $\omega_A = 20$ kN/m, (*b*) the corresponding value of ω_B.

PROBLEM 5.78 The beam *AB* supports two concentrated loads and rests on soil that exerts a linearly distributed upward load as shown. Determine the values of ω_A and ω_B corresponding to equilibrium.

SOLUTION



We have $$R_{\text{I}} = \frac{1}{2}(1.8 \text{ m})(20 \text{ kN/m}) = 18 \text{ kN}$$

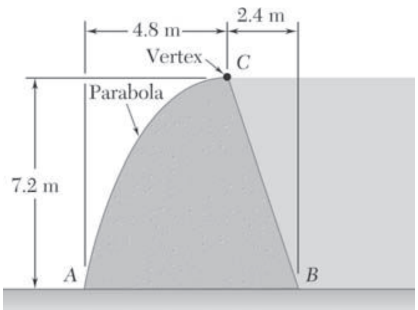
$$R_{\text{II}} = \frac{1}{2}(1.8 \text{ m})(\omega_B \text{ kN/m}) = 0.9\omega_B \text{ kN}$$

(*a*) $$+\circlearrowleft \Sigma M_C = 0: \quad (1.2 - a)\text{m} \times 24 \text{ kN} - 0.6 \text{ m} \times 18 \text{ kN} - 0.3 \text{ m} \times 30 \text{ kN} = 0$$

or $$a = 0.375 \text{ m} \blacktriangleleft$$

(*b*) $$+\uparrow \Sigma F_y = 0: \quad -24 \text{ kN} + 18 \text{ kN} + (0.9\omega_B) \text{ kN} - 30 \text{ kN} = 0$$

or $$\omega_B = 40.0 \text{ kN/m} \blacktriangleleft$$

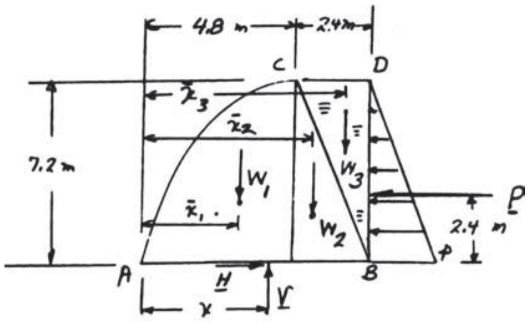


PROBLEM 5.80

The cross section of a concrete dam is as shown. For a 1-m-wide dam section determine (*a*) the resultant of the reaction forces exerted by the ground on the base *AB* of the dam, (*b*) the point of application of the resultant of Part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face *BC* of the dam.

SOLUTION

(*a*) Consider free body made of dam and triangular section of water shown. (Thickness = 1 m)



$$p = (7.2\,\text{m})(10^3\,\text{kg/m}^3)(9.81\,\text{m/s}^2)$$

$$W_1 = \frac{2}{3}(4.8\text{ m})(7.2\text{ m})(1\text{ m})(2.4\times 10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)$$
$$= 542.5\text{ kN}$$

$$W_2 = \frac{1}{2}(2.4\text{ m})(7.2\text{ m})(1\text{ m})(2.4\times 10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)$$
$$= 203.4\text{ kN}$$

$$W_3 = \frac{1}{2}(2.4\text{ m})(7.2\text{ m})(1\text{ m})(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)$$
$$= 84.8\text{ kN}$$

$$P = \frac{1}{2}Ap = \frac{1}{2}(7.2\text{ m})(1\text{ m})(7.2\text{ m})(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)$$
$$= 254.3\text{ kN}$$

$\xrightarrow{+} \Sigma F_x = 0\text{:}\quad H - 254.3\text{ kN} = 0$ $\qquad$ $\mathbf{H} = 254\text{ kN} \longrightarrow$ ◀

$+\!\uparrow \Sigma F_y = 0\text{:}\quad V - 542.5 - 203.4 - 84.8 = 0$

$V = 830.7\text{ kN}$ $\qquad$ $\mathbf{V} = 831\text{ kN}\uparrow$ ◀

PROBLEM 5.80 (Continued)

(*b*) $$\bar{x}_1 = \frac{5}{8}(4.8 \text{ m}) = 3 \text{ m}$$

$$\bar{x}_2 = 4.8 + \frac{1}{3}(2.4) = 5.6 \text{ m}$$

$$\bar{x}_3 = 4.8 + \frac{2}{3}(2.4) = 6.4 \text{ m}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad xV - \Sigma \bar{x}W + P(2.4 \text{ m}) = 0$$

$$x(830.7 \text{ kN}) - (3 \text{ m})(542.5 \text{ kN}) - (5.6 \text{ m})(203.4 \text{ kN})$$
$$- (6.4 \text{ m})(84.8 \text{ kN}) + (2.4 \text{ m})(254.3 \text{ kN}) = 0$$

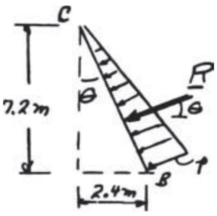
$$x(830.7) - 1627.5 - 1139.0 - 542.7 + 610.3 = 0$$

$$x(830.7) - 2698.9 = 0$$

$x = 3.25$ m (To right of A) ◀

(*c*) Resultant on face BC

Direct computation:



$$P = \rho g h = (10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(7.2 \text{ m})$$

$$P = 70.63 \text{ kN/m}^2$$

$$BC = \sqrt{(2.4)^2 + (7.2)^2}$$
$$= 7.589 \text{ m}$$

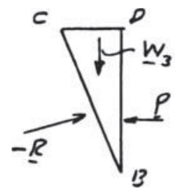
$$\theta = 18.43°$$

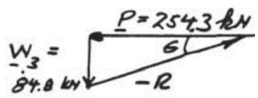
$$R = \frac{1}{2}PA$$

$$= \frac{1}{2}(70.63 \text{ kN/m}^2)(7.589 \text{ m})(1 \text{ m})$$

$\mathbf{R} = 268$ kN 18.43° ◀

Alternate computation: Use free body of water section BCD.

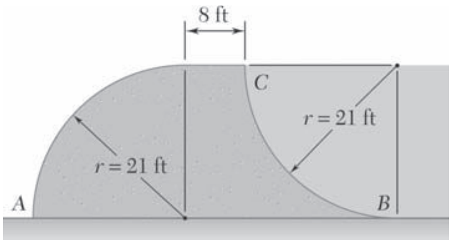




$-\mathbf{R} = 268$ kN 18.43°

$\mathbf{R} = 268$ kN 18.43° ◀

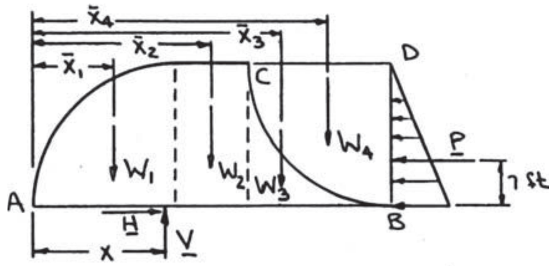
PROBLEM 5.81



The cross section of a concrete dam is as shown. For a 1-ft-wide dam section determine (*a*) the resultant of the reaction forces exerted by the ground on the base *AB* of the dam, (*b*) the point of application of the resultant of Part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face *BC* of the dam.

SOLUTION

The free body shown consists of a 1-ft thick section of the dam and the quarter circular section of water above the dam.



Note:

$$\bar{x}_1 = \left(21 - \frac{4\times 21}{3\pi}\right)\text{ft}$$

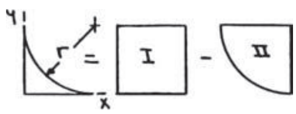
$$= 12.0873 \text{ ft}$$

$$\bar{x}_2 = (21+4)\,\text{ft} = 25 \text{ ft}$$

$$\bar{x}_4 = \left(50 - \frac{4\times 21}{3\pi}\right)\text{ft}$$

$$= 41.087 \text{ ft}$$

For area 3 first note.



	a	$\bar{x}$
I	r^2	$\frac{1}{2}r$
II	$-\frac{\pi}{4}r^2$	$r - \frac{4r}{3\pi}$

Then

$$\bar{x}_3 = 29 \text{ ft} + \left[\frac{\frac{1}{2}(21)(21)^2 + \left(21 - \frac{4\times 21}{3\pi}\right)\left(-\frac{\pi}{4}\times 21^2\right)}{(21)^2 - \frac{\pi}{4}(21)^2}\right]\text{ft}$$

$$= (29 + 4.6907)\text{ft} = 33.691 \text{ ft}$$

PROBLEM 5.81 (Continued)

(*a*) Now $W = \gamma V$

So that $W_1 = (150 \text{ lb/ft}^3)\left[\frac{\pi}{4}(21 \text{ ft})^2(1 \text{ ft})\right] = 51{,}954 \text{ lb}$

$$W_2 = (150 \text{ lb/ft}^3)[(8 \text{ ft})(21 \text{ ft})(1 \text{ ft})] = 25{,}200 \text{ lb}$$

$$W_3 = (150 \text{ lb/ft}^3)\left[\left(21^2 - \frac{\pi}{4}\times 21^2\right)\text{ft}^2 \times (1 \text{ ft})\right] = 14{,}196 \text{ lb}$$

$$W_4 = (62.4 \text{ lb/ft}^3)\left[\frac{\pi}{4}(21 \text{ ft})^2(1 \text{ ft})\right] = 21{,}613 \text{ lb}$$

Also $P = \frac{1}{2}Ap = \frac{1}{2}[(21 \text{ ft})(1 \text{ ft})][(62.4 \text{ lb/ft}^3)(21 \text{ ft})] = 13{,}759 \text{ lb}$

Then $\xrightarrow{+} \Sigma F_x = 0$: $H - 13{,}759 \text{ lb} = 0$ or $\mathbf{H} = 13.76 \text{ kips} \longrightarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $V - 51{,}954 \text{ lb} - 25{,}200 \text{ lb} - 14{,}196 \text{ lb} - 21{,}613 \text{ lb} = 0$

or $V = 112{,}963 \text{ lb}$ $\mathbf{V} = 113.0 \text{ kips} \uparrow$ ◀

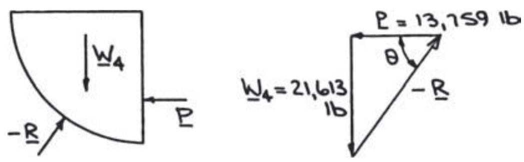
(*b*) We have $+\circlearrowleft \Sigma M_A = 0$: $x(112{,}963 \text{ lb}) - (12.0873 \text{ ft})(51{,}954 \text{ lb}) - (25 \text{ ft})(25{,}200 \text{ lb})$
$-(33.691 \text{ ft})(14{,}196 \text{ lb}) - (41.087 \text{ ft})(21{,}613 \text{ lb})$
$+(7 \text{ ft})(13{,}759 \text{ lb}) = 0$

or $112{,}963x - 627{,}980 - 630{,}000 - 478{,}280 - 888{,}010 + 96{,}313 = 0$

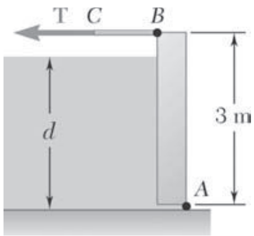
or $x = 22.4 \text{ ft}$ ◀

(*c*) Consider water section *BCD* as the free body

We have $\Sigma\mathbf{F} = 0$



Then $-\mathbf{R} = 25.6 \text{ kips} \measuredangle 57.5°$ or $\mathbf{R} = 25.6 \text{ kips} \measuredangle 57.5°$ ◀

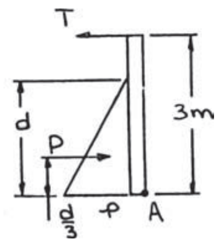


PROBLEM 5.82

The 3 × 4-m side *AB* of a tank is hinged at its bottom *A* and is held in place by a thin rod *BC*. The maximum tensile force the rod can withstand without breaking is 200 kN, and the design specifications require the force in the rod not to exceed 20 percent of this value. If the tank is slowly filled with water, determine the maximum allowable depth of water *d* in the tank.

SOLUTION

Consider the free-body diagram of the side.



We have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g d)$$

Now $$+\circlearrowleft \Sigma M_A = 0: \quad hT - \frac{d}{3}P = 0$$

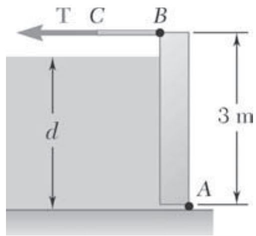
Where $$h = 3 \text{ m}$$

Then for d_{max}.

$$(3 \text{ m})(0.2 \times 200 \times 10^3 \text{ N}) - \frac{d_{max}}{3}\left[\frac{1}{2}(4 \text{ m} \times d_{max}) \times (10^3 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times d_{max})\right] = 0$$

or $$120 \text{ N}\cdot\text{m} - 6.54 d_{max}^3 \text{ N/m}^2 = 0$$

or $$d_{max} = 2.64 \text{ m} \blacktriangleleft$$

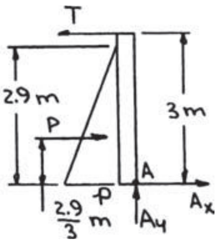


PROBLEM 5.83

The 3 × 4-m side of an open tank is hinged at its bottom A and is held in place by a thin rod BC. The tank is to be filled with glycerine, whose density is 1263 kg/m^3. Determine the force **T** in the rod and the reactions at the hinge after the tank is filled to a depth of 2.9 m.

SOLUTION

Consider the free-body diagram of the side.



We have
$$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g d)$$
$$= \frac{1}{2}[(2.9 \text{ m})(4 \text{ m})]\,[(1263 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.9 \text{ m})]$$
$$= 208.40 \text{ kN}$$

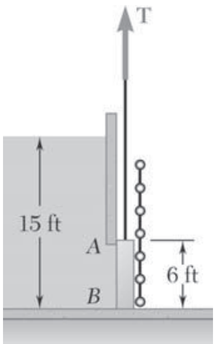
Then
$$+\!\uparrow \Sigma F_y = 0: \quad A_y = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (3 \text{ m})T - \left(\frac{2.9}{3}\text{ m}\right)(208.4 \text{ kN}) = 0$$

or $\quad T = 67.151 \text{ kN} \qquad \mathbf{T} = 67.2 \text{ kN} \longleftarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 208.40 \text{ kN} - 67.151 \text{ kN} = 0$$

or $\quad A_x = -141.249 \text{ kN} \qquad \mathbf{A} = 141.2 \text{ kN} \longleftarrow$ ◀

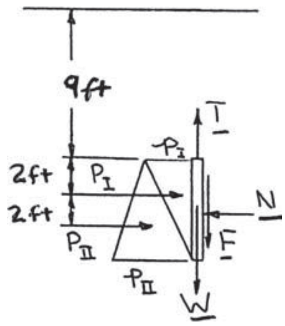


PROBLEM 5.84

The friction force between a 6 × 6-ft square sluice gate *AB* and its guides is equal to 10 percent of the resultant of the pressure forces exerted by the water on the face of the gate. Determine the initial force needed to lift the gate if it weighs 1000 lb.

SOLUTION

Consider the free-body diagram of the gate.

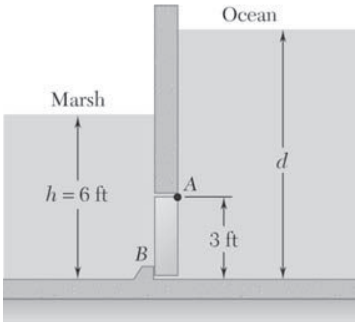


Now
$$P_{\text{I}} = \frac{1}{2}Ap_{\text{I}} = \frac{1}{2}[(6\times 6)\ \text{ft}^2][(62.4\ \text{lb/ft}^3)(9\ \text{ft})]$$
$$= 10{,}108.8\ \text{lb}$$

$$P_{\text{II}} = \frac{1}{2}Ap_{\text{II}} = \frac{1}{2}[(6\times 6)\,\text{ft}^2][(62.4\ \text{lb/ft}^3)(15\ \text{ft})]$$
$$= 16848\ \text{lb}$$

Then
$$F = 0.1P = 0.1(P_{\text{I}} + P_{\text{II}})$$
$$= 0.1(10108.8 + 16848)\text{lb}$$
$$= 2695.7\ \text{lb}$$

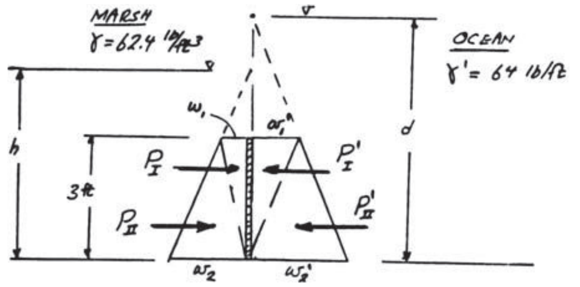
Finally $+\!\uparrow \Sigma F_y = 0\text{:}\quad T - 2695.7\ \text{lb} - 1000\ \text{lb} = 0$ or $\mathbf{T} = 3.70\ \text{kips}\uparrow$ ◀



PROBLEM 5.85

A freshwater marsh is drained to the ocean through an automatic tide gate that is 4 ft wide and 3 ft high. The gate is held by hinges located along its top edge at A and bears on a sill at B. If the water level in the marsh is $h = 6$ ft, determine the ocean level d for which the gate will open. (Specific weight of salt water = 64 lb/ft^3.)

SOLUTION

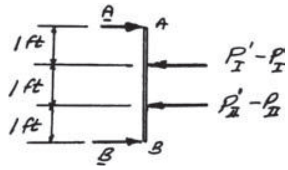


Since gate is 4 ft wide $\quad w = (4 \text{ ft})p = 4\gamma(\text{depth})$

Thus:

$$w_1 = 4\gamma(h-3)$$
$$w_2 = 4\gamma h$$
$$w_1' = 4\gamma'(d-3)$$
$$w_2' = 4\gamma' d$$

$$P_I' - P_I = \frac{1}{2}(3 \text{ ft})(w_1' - w_1)$$
$$= \frac{1}{2}(3 \text{ ft})[4\gamma'(d-3) - 4\lambda(h-3)] = 6\gamma'(d-3) - 6\gamma(h-3)$$



$$P_{II}' - P_{II} = \frac{1}{2}(3 \text{ ft})(w_2' - w_2)$$
$$= \frac{1}{2}(3 \text{ ft})[4\gamma' d - 4\gamma h] = 6\gamma' d - 6\gamma h$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (3 \text{ ft})B - (1 \text{ ft})(P_I' - P_I) - (2 \text{ ft})(P_{II}' - P_{II}) = 0$$

$$B = \frac{1}{3}(P_I' - P_I) - \frac{2}{3}(P_{II}' - P_{II})$$
$$= \frac{1}{3}[6\gamma'(d-3) - 6\gamma(h-3)] - \frac{2}{3}[6\gamma' d - 6\gamma h]$$
$$= 2\gamma(d-3) - 2\gamma(h-3) + 4\gamma' d - 4\gamma h$$
$$B = 6\gamma'(d-1) - 6\gamma(h-1)$$

PROBLEM 5.85 (Continued)

With $B = 0$ and $h = 6$ ft: $0 = 6\gamma'(d-1) - 6\gamma(h-1)$

$$d - 1 = 5\frac{\gamma}{\gamma'}$$

Data:

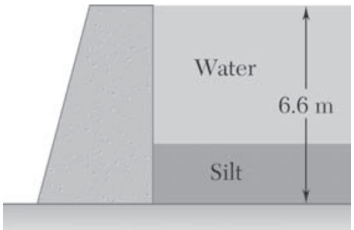
$$\gamma' = 64 \text{ lb/ft}^3$$

$$\gamma = 62.4 \text{ lb/ft}^3$$

$$d - 1 = 5\frac{62.4 \text{ lb/ft}^3}{64 \text{ lb/ft}^3}$$

$$= 4.875 \text{ ft}$$

$d = 5.88$ ft ◀



PROBLEM 5.86

The dam for a lake is designed to withstand the additional force caused by silt that has settled on the lake bottom. Assuming that silt is equivalent to a liquid of density $\rho_s = 1.76\times10^3\ \text{kg/m}^3$ and considering a 1-m-wide section of dam, determine the percentage increase in the force acting on the dam face for a silt accumulation of depth 2 m.

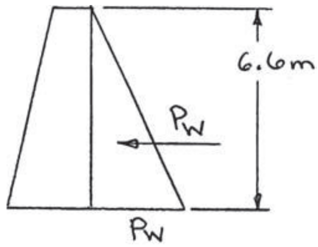
SOLUTION

First, determine the force on the dam face without the silt.

We have $$P_w = \frac{1}{2}Ap_w = \frac{1}{2}A(\rho gh)$$

$$= \frac{1}{2}[(6.6\text{ m})(1\text{ m})][(10^3\ \text{kg/m}^3)(9.81\text{ m/s}^2)(6.6\text{ m})]$$

$$= 213.66\text{ kN}$$



Next, determine the force on the dam face with silt

We have $$P'_w = \frac{1}{2}[(4.6\text{ m})(1\text{ m})][(10^3\ \text{kg/m}^3)(9.81\text{ m/s}^2)(4.6\text{ m})]$$

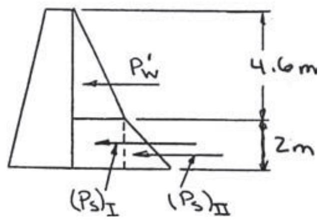
$$= 103.790\text{ kN}$$

$$(P_s)_{\text{I}} = [(2.0\text{ m})(1\text{ m})][(10^3\ \text{kg/m}^3)(9.81\text{ m/s}^2)(4.6\text{ m})]$$

$$= 90.252\text{ kN}$$

$$(P_s)_{\text{II}} = \frac{1}{2}[(2.0\text{ m})(1\text{ m})][(1.76\times10^3\ \text{kg/m}^3)(9.81\text{ m/s}^2)(2.0\text{ m})]$$

$$= 34.531\text{ kN}$$



Then $$P' = P'_w + (P_s)_{\text{I}} + (P_s)_{\text{II}} = 228.57\text{ kN}$$

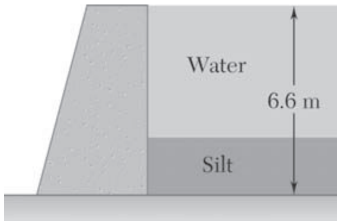
The percentage increase, % inc., is then given by

$$\%\text{ inc.} = \frac{P' - P_w}{P_w}\times100\%$$

$$= \frac{(228.57 - 213.66)}{213.66}\times100\%$$

$$= 6.9874\%$$

$$\%\text{ inc.} = 6.98\% \blacktriangleleft$$



PROBLEM 5.87

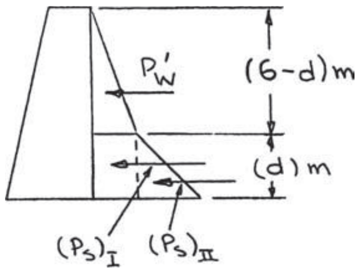
The base of a dam for a lake is designed to resist up to 120 percent of the horizontal force of the water. After construction, it is found that silt (that is equivalent to a liquid of density $\rho_s = 1.76 \times 10^3\ \text{kg/m}^3$) is settling on the lake bottom at the rate of 12 mm/year. Considering a 1-m-wide section of dam, determine the number of years until the dam becomes unsafe.

SOLUTION

From Problem 5.86, the force on the dam face before the silt is deposited, is $P_w = 213.66\ \text{kN}$. The maximum allowable force P_{allow} on the dam is then:

$$P_{\text{allow}} = 1.2P_w = (1.5)(213.66\ \text{kN}) = 256.39\ \text{kN}$$

Next determine the force P' on the dam face after *a* depth *d* of silt has settled



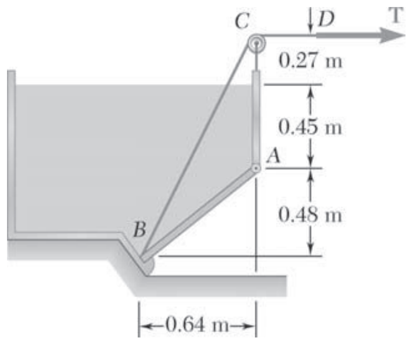
We have
$$P_w' = \frac{1}{2}[(6.6-d)\text{m} \times (1\ \text{m})][(10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6.6-d)\text{m}]$$
$$= 4.905(6.6-d)^2\ \text{kN}$$
$$(P_s)_{\text{I}} = [d(1\ \text{m})][(10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6.6-d)\text{m}]$$
$$= 9.81\,(6.6d - d^2)\text{kN}$$
$$(P_s)_{\text{II}} = \frac{1}{2}[d(1\ \text{m})][(1.76\times 10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(d)\text{m}]$$
$$= 8.6328d^2\ \text{kN}$$
$$P' = P_w' + (P_s)_{\text{I}} + (P_s)_{\text{II}} = [4.905(43.560 - 13.2000d + d^2) + 9.81(6.6d - d^2) + 8.6328d^2]\text{kN}$$
$$= [3.7278d^2 + 213.66]\text{kN}$$

Now required that $P' = P_{\text{allow}}$ to determine the maximum value of *d*.

$$(3.7278d^2 + 213.66)\text{kN} = 256.39\ \text{kN}$$

or
$$d = 3.3856\ \text{m}$$

Finally
$$3.3856\ \text{m} = 12\times 10^{-3}\ \frac{\text{m}}{\text{year}} \times \text{N}$$
or $N = 282$ years ◀

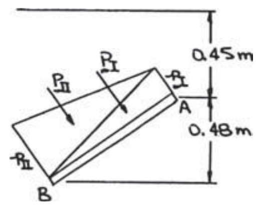


PROBLEM 5.88

A 0.5 × 0.8-m gate *AB* is located at the bottom of a tank filled with water. The gate is hinged along its top edge *A* and rests on a frictionless stop at *B*. Determine the reactions at *A* and *B* when cable *BCD* is slack.

SOLUTION

First consider the force of the water on the gate.



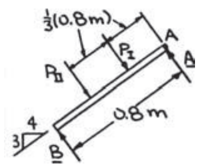
We have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g h)$$

so that $$P_{\mathrm{I}} = \frac{1}{2}[(0.5 \text{ m})(0.8 \text{ m})] \times [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.45 \text{ m})]$$
$$= 882.9 \text{ N}$$

$$P_{\mathrm{II}} = \frac{1}{2}[(0.5 \text{ m})(0.8 \text{ m})] \times [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.93 \text{ m})]$$
$$= 1824.66 \text{ N}$$

Reactions at *A* and *B* when $T = 0$

We have



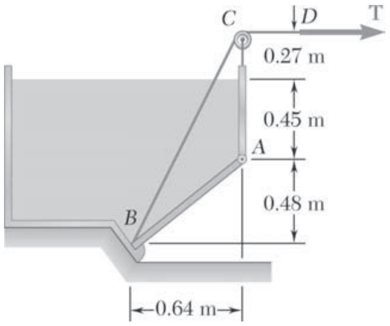
$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{1}{3}(0.8 \text{ m})(882.9 \text{ N}) + \frac{2}{3}(0.8 \text{ m})(1824.66 \text{ N}) - (0.8 \text{ m})B = 0$$

or $$B = 1510.74 \text{ N}$$

or $$\mathbf{B} = 1511 \text{ N} \measuredangle 53.1° \blacktriangleleft$$

$$+\nwarrow \Sigma F = 0: \quad A + 1510.74 \text{ N} - 882.9 \text{ N} - 1824.66 \text{ N} = 0$$

or $$\mathbf{A} = 1197 \text{ N} \measuredangle 53.1° \blacktriangleleft$$



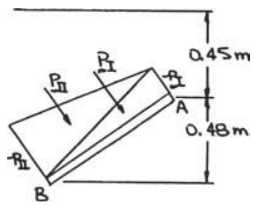
PROBLEM 5.89

A 0.5 × 0.8-m gate *AB* is located at the bottom of a tank filled with water. The gate is hinged along its top edge *A* and rests on a frictionless stop at *B*. Determine the minimum tension required in cable *BCD* to open the gate.

SOLUTION

First consider the force of the water on the gate.

We have $P = \frac{1}{2}Ap = \frac{1}{2}A(\rho gh)$



so that $P_{\text{I}} = \frac{1}{2}[(0.5\text{ m})(0.8\text{ m})]\times[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.45\text{ m})]$

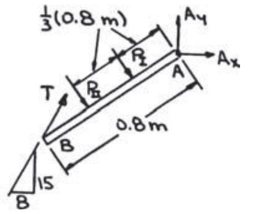
$= 882.9\text{ N}$

$P_{\text{II}} = \frac{1}{2}[(0.5\text{ m})(0.8\text{ m})]\times[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.93\text{ m})]$

$= 1824.66\text{ N}$

T to open gate

First note that when the gate begins to open, the reaction at $B \longrightarrow 0$.

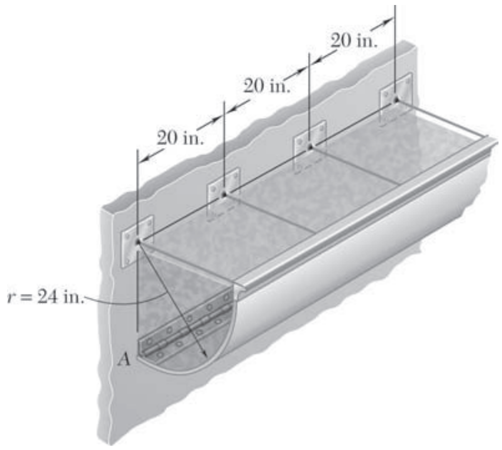


Then $+\circlearrowleft \Sigma M_A = 0$: $\frac{1}{3}(0.8\text{ m})(882.9\text{ N}) + \frac{2}{3}(0.8\text{ m})(1824.66\text{ N})$

$-(0.45+0.27)\text{m}\times\left(\frac{8}{17}T\right) = 0$

or $235.44 + 973.152 - 0.33882T = 0$

or **T** $= 3570$ N ◀



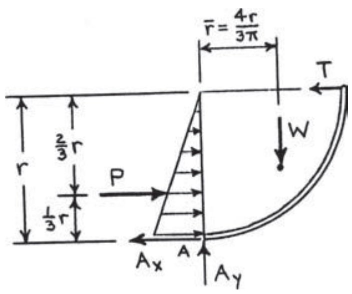
PROBLEM 5.90

A long trough is supported by a continuous hinge along its lower edge and by a series of horizontal cables attached to its upper edge. Determine the tension in each of the cables, at a time when the trough is completely full of water.

SOLUTION

Consider free body consisting of 20-in. length of the trough and water

$$l = 20\text{-in. length of free body}$$



$$W = \gamma v = \gamma\left[\frac{\pi}{4}r^2 l\right]$$

$$P_A = \gamma r$$

$$P = \frac{1}{2}P_A r l = \frac{1}{2}(\gamma r) r l = \frac{1}{2}\gamma r^2 l$$

$$+\circlearrowleft \Sigma M_A = 0: \quad Tr - Wr - P\left(\frac{1}{3}r\right) = 0$$

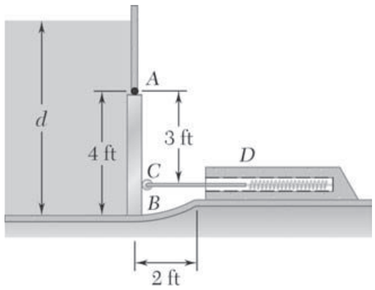
$$Tr - \left(\gamma\frac{\pi}{4}r^2 l\right)\left(\frac{4r}{3\pi}\right) - \left(\frac{1}{2}\gamma r^2 l\right)\left(\frac{1}{3}r\right) = 0$$

$$T = \frac{1}{3}\gamma r^2 l + \frac{1}{6}\gamma r^2 l = \frac{1}{2}\gamma r^2 l$$

Data: $\gamma = 62.4 \text{ lb/ft}^3 \quad r = \frac{24}{12}\text{ft} = 2\text{ft} \quad l = \frac{20}{12}\text{ft}$

Then $T = \frac{1}{2}(62.4 \text{ lb/ft}^3)(2 \text{ ft})^2\left(\frac{20}{12}\text{ft}\right)$

$= 208.00 \text{ lb}$ $\quad T = 208 \text{ lb}$ ◀

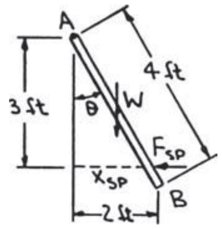


PROBLEM 5.91

A 4×2-ft gate is hinged at *A* and is held in position by rod *CD*. End *D* rests against a spring whose constant is 828 lb/ft. The spring is undeformed when the gate is vertical. Assuming that the force exerted by rod *CD* on the gate remains horizontal, determine the minimum depth of water *d* for which the bottom *B* of the gate will move to the end of the cylindrical portion of the floor.

SOLUTION

First determine the forces exerted on the gate by the spring and the water when *B* is at the end of the cylindrical portion of the floor

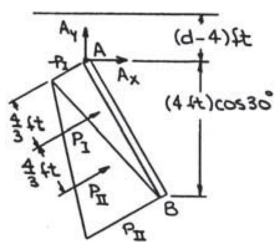


We have $\qquad \sin\theta = \dfrac{2}{4} \quad \theta = 30°$

Then $\qquad x_{SP} = (3\text{ ft})\tan 30°$

and
$$F_{SP} = kx_{SP}$$
$$= 828\text{ lb/ft} \times 3\text{ ft} \times \tan 30°$$
$$= 1434.14\text{ lb}$$

Assume $\qquad d \geq 4\text{ ft}$



We have $\qquad P = \dfrac{1}{2}Ap = \dfrac{1}{2}A(\gamma h)$

Then
$$P_{\mathrm{I}} = \frac{1}{2}[(4\text{ ft})(2\text{ ft})] \times [(62.4\text{ lb/ft}^3)(d-4)\text{ft}]$$
$$= 249.6(d-4)\text{lb}$$
$$P_{\mathrm{II}} = \frac{1}{2}[(4\text{ ft})(2\text{ ft})] \times [(62.4\text{ lb/ft}^3)(d - 4 + 4\cos 30°)]$$
$$= 249.6(d - 0.53590°)\text{lb}$$

For $d_{\min}$ so that gate opens, $\qquad W = 0$

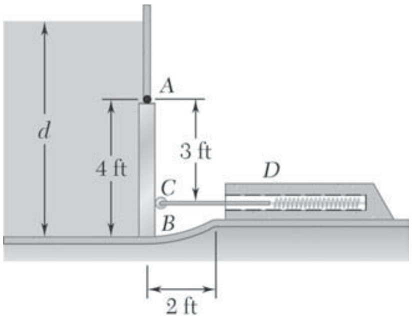
Using the above free-body diagrams of the gate, we have

$$+\circlearrowleft \Sigma M_A = 0: \quad \left(\frac{4}{3}\text{ ft}\right)[249.6(d-4)\text{lb}] + \left(\frac{8}{3}\text{ ft}\right)[249.6(d - 0.53590)\text{lb}]$$
$$-(3\text{ ft})(1434.14\text{ lb}) = 0$$

or
$$(332.8d - 1331.2) + (665.6d - 356.70) - 4302.4 = 0$$

or
$$d = 6.00\text{ ft}$$

$d \geq 4\text{ ft} \Rightarrow$ assumption correct $\qquad d = 6.00\text{ ft}$ ◀



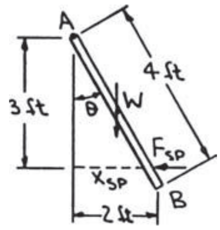
PROBLEM 5.92

Solve Problem 5.91 if the gate weighs 1000 lb.

PROBLEM 5.91 A 4×2-ft gate is hinged at A and is held in position by rod CD. End D rests against a spring whose constant is 828 lb/ft. The spring is undeformed when the gate is vertical. Assuming that the force exerted by rod CD on the gate remains horizontal, determine the minimum depth of water d for which the bottom B of the gate will move to the end of the cylindrical portion of the floor.

SOLUTION

First determine the forces exerted on the gate by the spring and the water when B is at the end of the cylindrical portion of the floor



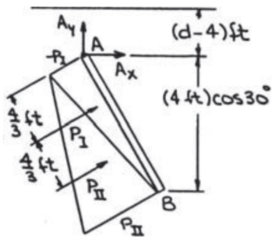
We have $\qquad \sin\theta = \dfrac{2}{4} \qquad \theta = 30°$

Then $\qquad x_{SP} = (3\text{ ft})\tan 30°$

and $\qquad F_{SP} = kx_{SP} = 828\text{ lb/ft}\times 3\text{ ft}\times\tan 30°$

$$= 1434.14\text{ lb}$$

Assume $\qquad d \geq 4\text{ ft}$



We have $\qquad P = \dfrac{1}{2}Ap = \dfrac{1}{2}A(\gamma h)$

Then $\qquad P_{\text{I}} = \dfrac{1}{2}[(4\text{ ft})(2\text{ ft})]\times[(62.4\text{ lb/ft}^3)(d-4)\text{ft}]$

$$= 249.6(d-4)\text{lb}$$

$$P_{\text{II}} = \frac{1}{2}[(4\text{ ft})(2\text{ ft})]\times[(62.4\text{ lb/ft}^3)(d-4+4\cos 30°)]$$

$$= 249.6(d-0.53590°)\text{lb}$$

For $d_{\min}$ so that gate opens, $\qquad W = 1000\text{ lb}$

Using the above free-body diagrams of the gate, we have

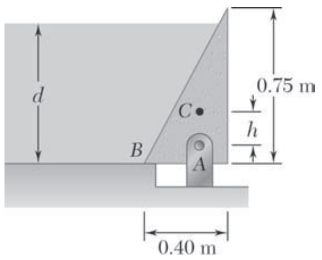
$$+\circlearrowleft \Sigma M_A = 0: \quad \left(\frac{4}{3}\text{ ft}\right)[249.6(d-4)\text{ lb}] + \left(\frac{8}{3}\text{ ft}\right)[249.6(d-0.53590)\text{ lb}]$$

$$-(3\text{ ft})(1434.14\text{ lb}) - (1\text{ ft})(1000\text{ lb}) = 0$$

or $\qquad (332.8d - 1331.2) + (665.6d - 356.70) - 4302.4 - 1000 = 0$

or $\qquad d = 7.00\text{ ft}$

$d \geq 4\text{ ft} \Rightarrow$ assumption correct $\qquad d = 7.00\text{ ft}$ ◀



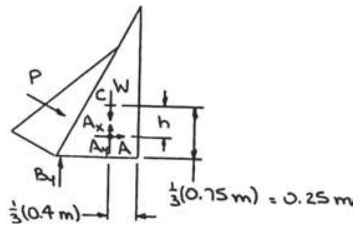
PROBLEM 5.93

A prismatically shaped gate placed at the end of a freshwater channel is supported by a pin and bracket at *A* and rests on a frictionless support at *B*. The pin is located at a distance $h = 0.10$ m below the center of gravity *C* of the gate. Determine the depth of water *d* for which the gate will open.

SOLUTION

First note that when the gate is about to open (clockwise rotation is impending), $B_y \longrightarrow 0$ and the line of action of the resultant **P** of the pressure forces passes through the pin at *A*. In addition, if it is assumed that the gate is homogeneous, then its center of gravity *C* coincides with the centroid of the triangular area. Then

$$a = \frac{d}{3} - (0.25 - h)$$

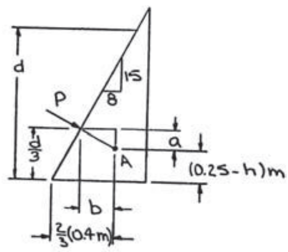


and

$$b = \frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)$$

Now

$$\frac{a}{b} = \frac{8}{15}$$



so that

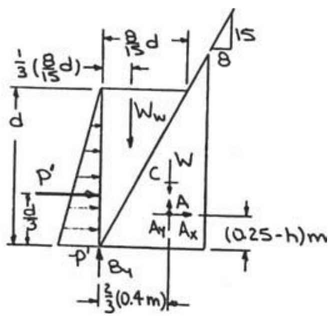
$$\frac{\frac{d}{3} - (0.25 - h)}{\frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)} = \frac{8}{15}$$

Simplifying yields

$$\frac{289}{45}d + 15h = \frac{70.6}{12} \tag{1}$$

<u>Alternative solution</u>

Consider a free body consisting of a 1-m thick section of the gate and the triangular section *BDE* of water above the gate.



Now

$$P' = \frac{1}{2}Ap' = \frac{1}{2}(d \times 1\text{ m})(\rho g d)$$

$$= \frac{1}{2}\rho g d^2 \quad \text{(N)}$$

$$W' = \rho g V = \rho g\left(\frac{1}{2} \times \frac{8}{15}d \times d \times 1\text{ m}\right)$$

$$= \frac{4}{15}\rho g d^2 \quad \text{(N)}$$

PROBLEM 5.93 (Continued)

Then with $B_y = 0$ (as explained above), we have

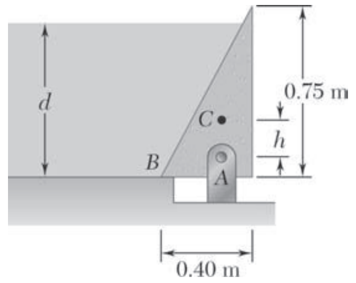
$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\frac{2}{3}(0.4) - \frac{1}{3}\left(\frac{8}{15}d\right)\right]\left(\frac{4}{15}\rho g d^2\right) - \left[\frac{d}{3} - (0.25 - h)\right]\left(\frac{1}{2}\rho g d^2\right) = 0$$

Simplifying yields $\qquad \dfrac{289}{45}d + 15h = \dfrac{70.6}{12}$

as above.

Find d, $\qquad h = 0.10\text{ m}$

Substituting into Eq. (1) $\qquad \dfrac{289}{45}d + 15(0.10) = \dfrac{70.6}{12}$ $\qquad$ or $d = 0.683\text{ m}$ ◀



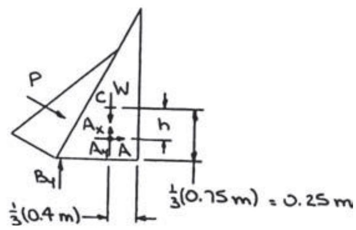
PROBLEM 5.94

A prismatically shaped gate placed at the end of a freshwater channel is supported by a pin and bracket at A and rests on a frictionless support at B. Determine the distance h if the gate is to open when $d = 0.75$ m.

SOLUTION

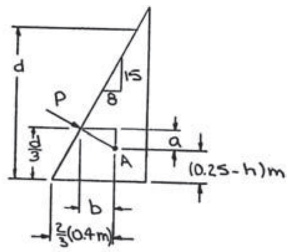
First note that when the gate is about to open (clockwise rotation is impending), $B_y \longrightarrow 0$ and the line of action of the resultant **P** of the pressure forces passes through the pin at A. In addition, if it is assumed that the gate is homogeneous, then its center of gravity C coincides with the centroid of the triangular area. Then

$$a = \frac{d}{3} - (0.25 - h)$$



and
$$b = \frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)$$

Now
$$\frac{a}{b} = \frac{8}{15}$$

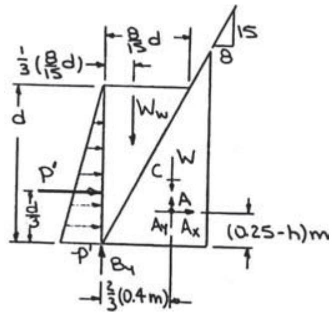


so that
$$\frac{\frac{d}{3} - (0.25 - h)}{\frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)} = \frac{8}{15}$$

Simplifying yields
$$\frac{289}{45}d + 15h = \frac{70.6}{12} \tag{1}$$

Alternative solution

Consider a free body consisting of a 1-m thick section of the gate and the triangular section BDE of water above the gate.



Now
$$P' = \frac{1}{2}Ap' = \frac{1}{2}(d \times 1\text{ m})(\rho g d)$$
$$= \frac{1}{2}\rho g d^2 \quad \text{(N)}$$
$$W' = \rho g V = \rho g\left(\frac{1}{2} \times \frac{8}{15}d \times d \times 1\text{ m}\right)$$
$$= \frac{4}{15}\rho g d^2 \quad \text{(N)}$$

PROBLEM 5.94 (Continued)

Then with $B_y = 0$ (as explained above), we have

$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\frac{2}{3}(0.4) - \frac{1}{3}\left(\frac{8}{15}d\right)\right]\left(\frac{4}{15}\rho g d^2\right) - \left[\frac{d}{3} - (0.25 - h)\right]\left(\frac{1}{2}\rho g d^2\right) = 0$$

Simplifying yields $$\frac{289}{45}d + 15h = \frac{70.6}{12}$$

as above.

Find h, $$d = 0.75 \text{ m}$$

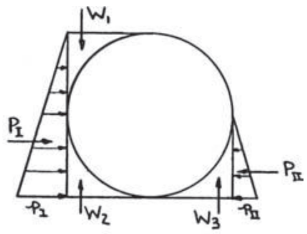
Substituting into Eq. (1) $$\frac{289}{45}(0.75) + 15h = \frac{70.6}{12}$$ or $h = 0.0711 \text{ m}$ ◀

PROBLEM 5.95

A 55-gallon 23-in.-diameter drum is placed on its side to act as a dam in a 30-in.-wide freshwater channel. Knowing that the drum is anchored to the sides of the channel, determine the resultant of the pressure forces acting on the drum.

SOLUTION

Consider the elements of water shown. The resultant of the weights of water above each section of the drum and the resultants of the pressure forces acting on the vertical surfaces of the elements is equal to the resultant hydrostatic force acting on the drum. Then



$$P_{\mathrm{I}} = \frac{1}{2}Ap_{\mathrm{I}} = \frac{1}{2}A(\gamma h)$$

$$= \frac{1}{2}\left[\left(\frac{30}{12}\right)\text{ft} \times \left(\frac{23}{12}\right)\text{ft}\right] \times \left[(62.4\ \text{lb/ft}^3)\left(\frac{23}{12}\ \text{ft}\right)\right]$$

$$= 286.542\ \text{lb}$$

$$P_{\mathrm{II}} = \frac{1}{2}Ap_{\mathrm{II}} = \frac{1}{2}A(\gamma h)$$

$$= \frac{1}{2}\left[\left(\frac{30}{12}\right)\text{ft} \times \left(\frac{11.5}{12}\right)\text{ft}\right] \times \left[(62.4\ \text{lb/ft}^3)\left(\frac{11.5}{12}\ \text{ft}\right)\right]$$

$$= 71.635\ \text{lb}$$

$$W_1 = \gamma V_1 = (62.4\ \text{lb/ft}^3)\left[\left(\frac{11.5}{12}\right)^2 \text{ft}^2 - \frac{\pi}{4}\left(\frac{11.5}{12}\right)^2 \text{ft}^2\right]\left(\frac{30}{12}\text{ft}\right) = 30.746\ \text{lb}$$

$$W_2 = \gamma V_2 = (62.4\ \text{lb/ft}^3)\left[\left(\frac{11.5}{12}\right)^2 \text{ft}^2 + \frac{\pi}{4}\left(\frac{11.5}{12}\right)^2 \text{ft}^2\right]\left(\frac{30}{12}\text{ft}\right) = 255.80\ \text{lb}$$

$$W_3 = \gamma V_3 = (62.4\ \text{lb/ft}^3)\left[\frac{\pi}{4}\left(\frac{11.5}{12}\right)^2 \text{ft}^2\right]\left(\frac{30}{12}\ \text{ft}\right) = 112.525\ \text{lb}$$

Then $\xrightarrow{+} \Sigma F_x$: $R_x = (286.542 - 71.635)\ \text{lb} = 214.91\ \text{lb}$

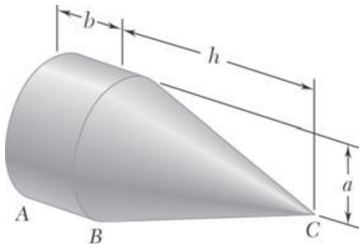
$+\uparrow \Sigma F_y$: $R_y = (-30.746 + 255.80 + 112.525)\ \text{lb} = 337.58\ \text{lb}$

Finally $R = \sqrt{R_x^2 + R_y^2} = 400.18\ \text{lb}$

$$\tan\theta = \frac{R_y}{R_x}$$

$\theta = 57.5°$

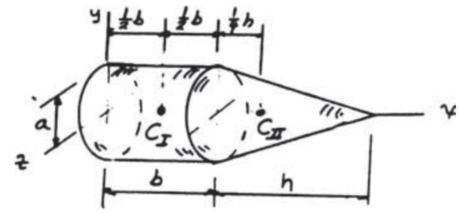
R $= 400\ \text{lb} \measuredangle 57.5°$ ◀



PROBLEM 5.96

Determine the location of the centroid of the composite body shown when (a) $h = 2b$, (b) $h = 2.5b$.

SOLUTION



	V	$\bar{x}$	$\bar{x}V$
Cylinder I	$\pi a^2 b$	$\frac{1}{2}b$	$\frac{1}{2}\pi a^2 b^2$
Cone II	$\frac{1}{3}\pi a^2 h$	$b + \frac{1}{4}h$	$\frac{1}{3}\pi a^2 h\left(b + \frac{1}{4}h\right)$

$$V = \pi a^2\left(b + \frac{1}{3}h\right)$$

$$\Sigma\bar{x}V = \pi a^2\left(\frac{1}{2}b^2 + \frac{1}{3}hb + \frac{1}{12}h^2\right)$$

(a) For $h = 2b$:

$$V = \pi a^2\left[b + \frac{1}{3}(2b)\right] = \frac{5}{3}\pi a^2 b$$

$$\Sigma\bar{x}V = \pi a^2\left[\frac{1}{2}b^2 + \frac{1}{3}(2b)b + \frac{1}{12}(2b)^2\right]$$

$$= \pi a^2 b^2\left[\frac{1}{2} + \frac{2}{3} + \frac{1}{3}\right] = \frac{3}{2}\pi a^2 b^2$$

$$\bar{X}V = \Sigma\bar{x}V: \quad \bar{X}\left(\frac{5}{3}\pi a^2 b\right) = \frac{3}{2}\pi a^2 b^2 \quad \bar{X} = \frac{9}{10}b$$

Centroid is $\frac{1}{10}b$ to left of base of cone ◀

PROBLEM 5.96 (Continued)

(*b*) For $h = 2.5b$:

$$V = \pi a^2\left[b + \frac{1}{3}(2.5b)\right] = 1.8333\pi a^2 b$$

$$\Sigma \bar{x} V = \pi a^2\left[\frac{1}{2}b^2 + \frac{1}{3}(2.5b)b + \frac{1}{12}(2.5b)^2\right]$$

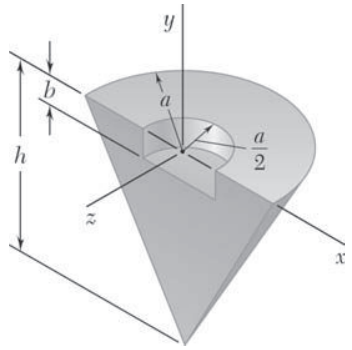
$$= \pi a^2 b^2[0.5 + 0.8333 + 0.52083]$$

$$= 1.85416\pi a^2 b^2$$

$$\bar{X}V = \Sigma \bar{x} V: \quad \bar{X}(1.8333\pi a^2 b) = 1.85416\pi a^2 b^2 \quad \bar{X} = 1.01136b$$

Centroid is $0.01136b$ to right of base of cone ◀

Note: Centroid is at base of cone for $h = \sqrt{6}b = 2.449b$

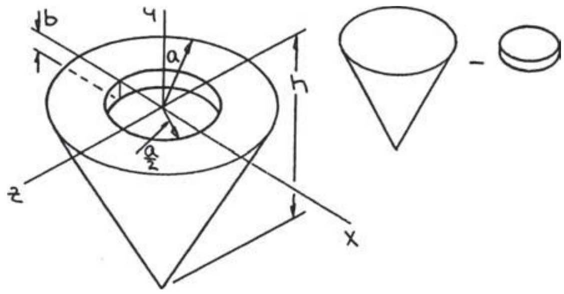


PROBLEM 5.97

Determine the y coordinate of the centroid of the body shown.

SOLUTION

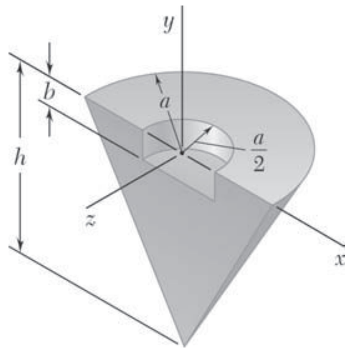
First note that the values of $\overline{Y}$ will be the same for the given body and the body shown below. Then



	V	$\bar{y}$	$\bar{y}V$
Cone	$\frac{1}{3}\pi a^2 h$	$-\frac{1}{4}h$	$-\frac{1}{12}\pi a^2 h^2$
Cylinder	$-\pi\left(\frac{a}{2}\right)^2 b = -\frac{1}{4}\pi a^2 b$	$-\frac{1}{2}b$	$\frac{1}{8}\pi a^2 b^2$
Σ	$\frac{\pi}{12}a^2(4h-3b)$		$-\frac{\pi}{24}a^2(2h^2-3b^2)$

We have $$\overline{Y}\Sigma V = \Sigma \bar{y} V$$

Then $$\overline{Y}\left[\frac{\pi}{12}a^2(4h-3b)\right] = -\frac{\pi}{24}a^2(2h^2-3b^2)$$ or $$\overline{Y} = -\frac{2h^2-3b^2}{2(4h-3b)}$$ ◀

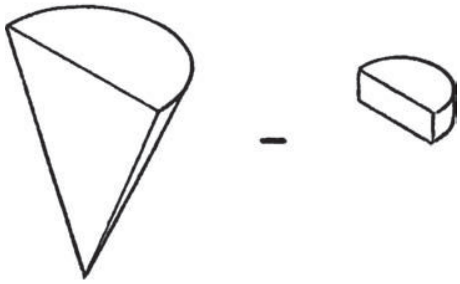


PROBLEM 5.98

Determine the z coordinate of the centroid of the body shown. (*Hint:* Use the result of Sample Problem 5.13.)

SOLUTION

First note that the body can be formed by removing a "half-cylinder" from a "half-cone," as shown.

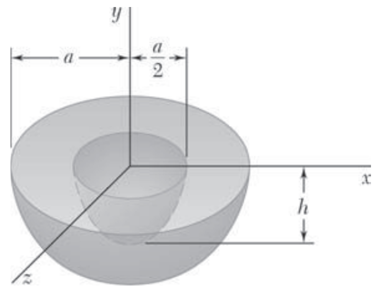


	V	$\bar{z}$	$\bar{z}V$
Half-Cone	$\frac{1}{6}\pi a^2 h$	$-\frac{a}{\pi}$ *	$-\frac{1}{6}a^3 h$
Half-Cylinder	$-\frac{\pi}{2}\left(\frac{a}{2}\right)^2 b = -\frac{\pi}{8}a^2 b$	$-\frac{4}{3\pi}\left(\frac{a}{2}\right) = -\frac{2a}{3\pi}$	$\frac{1}{12}a^3 b$
Σ	$\frac{\pi}{24}a^2(4h-3b)$		$-\frac{1}{12}a^3(2h-b)$

From Sample Problem 5.13

We have $$\bar{Z}\Sigma V = \Sigma \bar{z}V$$

Then $$\bar{Z}\left[\frac{\pi}{24}a^2(4h-3b)\right] = -\frac{1}{12}a^3(2h-b)$$ or $$\bar{Z} = -\frac{a}{\pi}\left(\frac{4h-2b}{4h-3b}\right) \blacktriangleleft$$



PROBLEM 5.99

The composite body shown is formed by removing a semiellipsoid of revolution of semimajor axis h and semiminor axis $a/2$ from a hemisphere of radius a. Determine (a) the y coordinate of the centroid when $h = a/2$, (b) the ratio h/a for which $\bar{y} = -0.4a$.

SOLUTION

	V	$\bar{y}$	$\bar{y}V$
Hemisphere	$\frac{2}{3}\pi a^3$	$-\frac{3}{8}a$	$-\frac{1}{4}\pi a^4$
Semiellipsoid	$-\frac{2}{3}\pi\left(\frac{a}{2}\right)^2 h = -\frac{1}{6}\pi a^2 h$	$-\frac{3}{8}h$	$+\frac{1}{16}\pi a^2 h^2$

Then $$\Sigma V = \frac{\pi}{6}a^2(4a - h)$$

$$\Sigma\bar{y}V = -\frac{\pi}{16}a^2(4a^2 - h^2)$$

Now $$\bar{Y}\Sigma V = \Sigma\bar{y}V$$

So that $$\bar{Y}\left[\frac{\pi}{6}a^2(4a - h)\right] = -\frac{\pi}{16}a^2(4a^2 - h^2)$$

or $$\bar{Y}\left(4 - \frac{h}{a}\right) = -\frac{3}{8}a\left[4 - \left(\frac{h}{a}\right)^2\right] \qquad (1)$$

(a) $$\bar{Y} = ? \quad \text{when} \quad h = \frac{a}{2}$$

Substituting $\frac{h}{a} = \frac{1}{2}$ into Eq. (1)

$$\bar{Y}\left(4 - \frac{1}{2}\right) = -\frac{3}{8}a\left[4 - \left(\frac{1}{2}\right)^2\right]$$

or $$\bar{Y} = -\frac{45}{112}a$$ $\bar{Y} = -0.402a$ ◀

PROBLEM 5.99 (Continued)

(*b*) $$\frac{h}{a} = ? \quad \text{when} \quad \overline{Y} = -0.4a$$

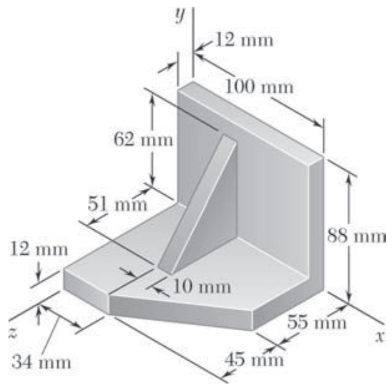
Substituting into Eq. (1)

$$(-0.4a)\left(4 - \frac{h}{a}\right) = -\frac{3}{8}a\left[4 - \left(\frac{h}{a}\right)^2\right]$$

or $$3\left(\frac{h}{a}\right)^2 - 3.2\left(\frac{h}{a}\right) + 0.8 = 0$$

Then $$\frac{h}{a} = \frac{3.2 \pm \sqrt{(-3.2)^2 - 4(3)(0.8)}}{2(3)}$$

$$= \frac{3.2 \pm 0.8}{6} \qquad \text{or} \quad \frac{h}{a} = \frac{2}{5} \quad \text{and} \quad \frac{h}{a} = \frac{2}{3} \blacktriangleleft$$

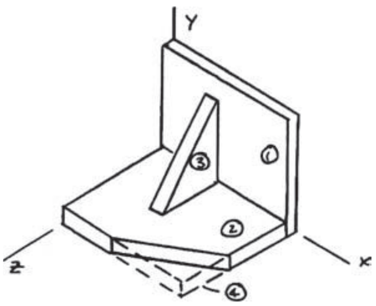


PROBLEM 5.100

For the stop bracket shown, locate the x coordinate of the center of gravity.

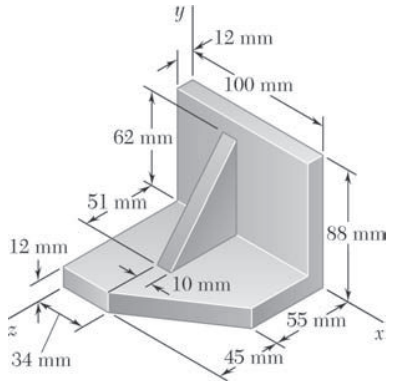
SOLUTION

Assume that the bracket is homogeneous so that its center of gravity coincides with the centroid of the volume.



	V, mm^3	$\bar{x}$, mm	$\bar{x}V$, mm^4
1	$(100)(88)(12) = 105600$	50	5280000
2	$(100)(12)(88) = 105600$	50	5280000
3	$\frac{1}{2}(62)(51)(10) = 15810$	39	616590
4	$-\frac{1}{2}(66)(45)(12) = -17820$	$34 + \frac{2}{3}(66) = 78$	-1389960
Σ	209190		9786600

Then $$\bar{X} = \frac{\Sigma \bar{x}V}{\Sigma V} = \frac{9786600}{209190} \text{ mm}$$ or $\bar{X} = 46.8$ mm ◀

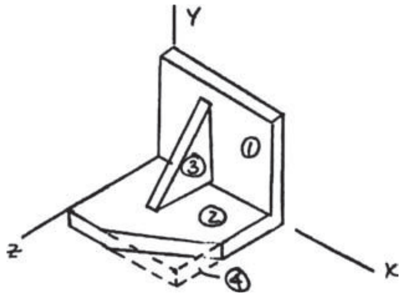


PROBLEM 5.101

For the stop bracket shown, locate the *z* coordinate of the center of gravity.

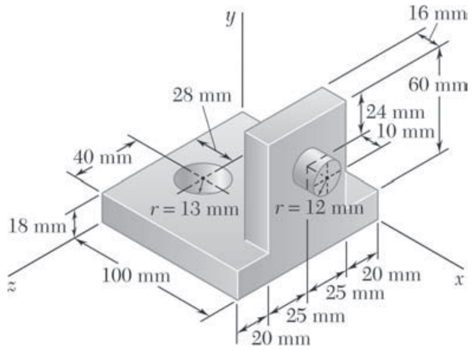
SOLUTION

Assume that the bracket is homogeneous so that it center of gravity coincides with the centroid of the volume.



	V, mm^3	$\bar{z}$, mm	$\bar{z}V$, mm^4
1	$(100)(88)(12) = 105600$	6	633600
2	$(100)(12)(88) = 105600$	$12 + \frac{1}{2}(88) = 56$	5913600
3	$\frac{1}{2}(62)(51)(10) = 15810$	$12 + \frac{1}{3}(51) = 29$	458490
4	$-\frac{1}{2}(66)(45)(12) = -17820$	$55 + \frac{2}{3}(45) = 85$	-1514700
Σ	209190		5491000

Then $$\bar{Z} = \frac{\Sigma \bar{z}V}{\Sigma V} = \frac{5491000}{209190} \text{ mm}$$ or $\bar{Z} = 26.2$ mm ◀

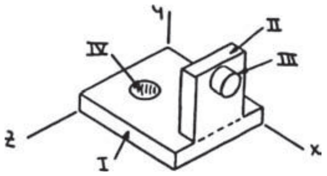


PROBLEM 5.102

For the machine element shown, locate the *y* coordinate of the center of gravity.

SOLUTION

First assume that the machine element is homogeneous so that its center of gravity will coincide with the centroid of the corresponding volume.

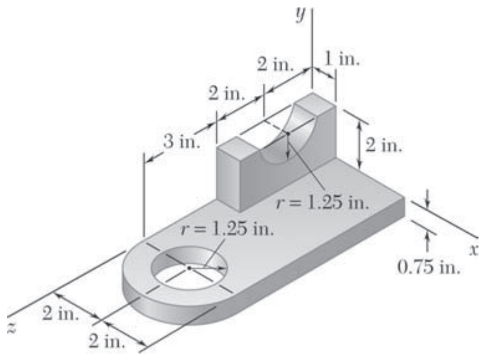


	V, mm^3	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{x}V$, mm^4	$\overline{y}V$, mm^4
I	$(100)(18)(90) = 162000$	50	9	8100000	1458000
II	$(16)(60)(50) = 48000$	92	48	4416000	2304000
III	$\pi(12)^2(10) = 4523.9$	105	54	475010	244290
IV	$-\pi(13)^2(18) = -9556.7$	28	9	–267590	–86010
Σ	204967.2			12723420	3920280

We have $\overline{Y}\Sigma V = \Sigma \overline{y}V$

$$\overline{Y}(204967.2 \text{ mm}^3) = 3920280 \text{ mm}^4$$

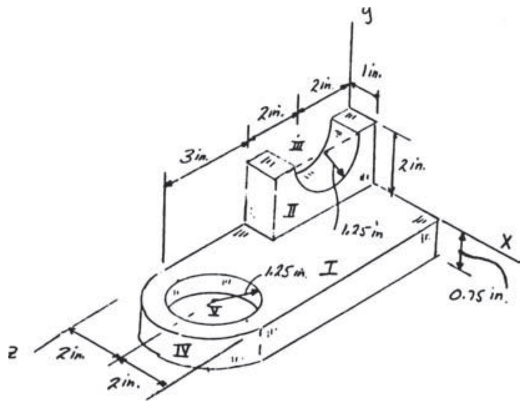
or $\overline{Y} = 19.13$ mm ◀



PROBLEM 5.103

For the machine element shown, locate the y coordinate of the center of gravity.

SOLUTION



For half cylindrical hole:

$$r = 1.25 \text{ in.}$$

$$\bar{y}_{\text{III}} = 2 - \frac{4(1.25)}{3\pi}$$

$$= 1.470 \text{ in.}$$

For half cylindrical plate: $r = 2$ in.

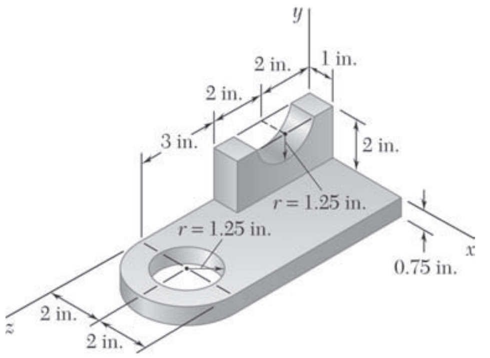
$$\bar{z}_{\text{IV}} = 7 + \frac{4(2)}{3\pi} = 7.85 \text{ in.}$$

		V, in.^{3}	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}V$, in.^{4}	$\bar{z}V$, in.^{4}
I	Rectangular plate	$(7)(4)(0.75) = 21.0$	–0.375	3.5	–7.875	73.50
II	Rectangular plate	$(4)(2)(1) = 8.0$	1.0	2	8.000	16.00
III	–(Half cylinder)	$-\frac{\pi}{2}(1.25)^2(1) = 2.454$	1.470	2	–3.607	–4.908
IV	Half cylinder	$\frac{\pi}{2}(2)^2(0.75) = 4.712$	–0.375	–7.85	–1.767	36.99
V	–(Cylinder)	$-\pi(1.25)^2(0.75) = -3.682$	–0.375	7	1.381	–25.77
	Σ	27.58			–3.868	95.81

$$\bar{Y}\Sigma V = \Sigma \bar{y} V$$

$$\bar{Y}(27.58 \text{ in.}^3) = -3.868 \text{ in.}^4$$

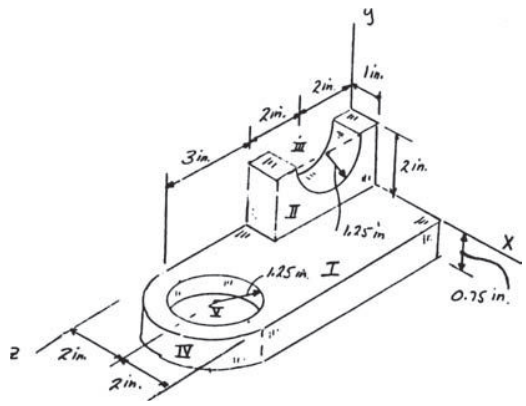
$\bar{Y} = -0.1403$ in. ◀



PROBLEM 5.104

For the machine element shown, locate the z coordinate of the center of gravity.

SOLUTION



For half cylindrical hole:

$$r = 1.25 \text{ in.}$$

$$\bar{y}_{\text{III}} = 2 - \frac{4(1.25)}{3\pi} = 1.470 \text{ in.}$$

For half cylindrical plate: $r = 2$ in.

$$\bar{z}_{\text{IV}} = 7 + \frac{4(2)}{3\pi} = 7.85 \text{ in.}$$

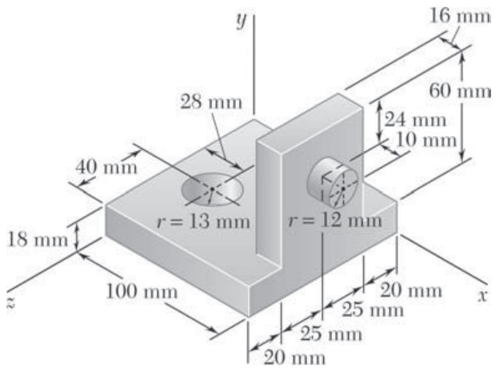
		V, in.^{3}	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}V$, in.^{4}	$\bar{z}V$, in.^{4}
I	Rectangular plate	$(7)(4)(0.75) = 21.0$	–0.375	3.5	–7.875	73.50
II	Rectangular plate	$(4)(2)(1) = 8.0$	1.0	2	8.000	16.00
III	–(Half cylinder)	$-\frac{\pi}{2}(1.25)^2(1) = 2.454$	1.470	2	–3.607	–4.908
IV	Half cylinder	$\frac{\pi}{2}(2)^2(0.75) = 4.712$	–0.375	–7.85	–1.767	36.99
V	–(Cylinder)	$-\pi(1.25)^2(0.75) = -3.682$	–0.375	7	1.381	–25.77
	Σ	27.58			–3.868	95.81

Now

$$\bar{Z}\Sigma V = \bar{z}V$$

$$\bar{Z}(27.58 \text{ in.}^3) = 95.81 \text{ in.}^4$$

$\bar{Z} = 3.47$ in. ◀

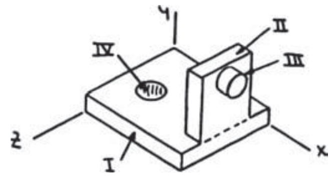


PROBLEM 5.105

For the machine element shown, locate the x coordinate of the center of gravity.

SOLUTION

First assume that the machine element is homogeneous so that its center of gravity will coincide with the centroid of the corresponding volume.

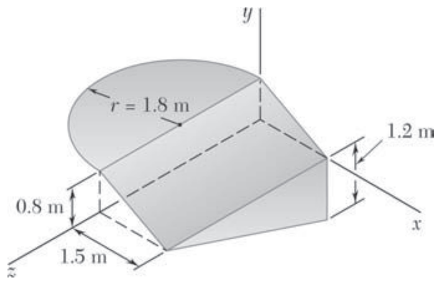


	V, mm^3	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}V$, mm^4	$\bar{y}V$, mm^4
I	$(100)(18)(90) = 162000$	50	9	8100000	1458000
II	$(16)(60)(50) = 48000$	92	48	4416000	2304000
III	$\pi(12)^2(10) = 4523.9$	105	54	475010	244290
IV	$-\pi(13)^2(18) = -9556.7$	28	9	–267590	–86010
Σ	204967.2			12723420	3920280

We have $$\bar{X}\Sigma V = \Sigma \bar{x} V$$

$$\bar{X}(204967.2 \text{ mm}^3) = 12723420 \text{ mm}^4$$ $\bar{X} = 62.1 \text{ mm}$ ◀

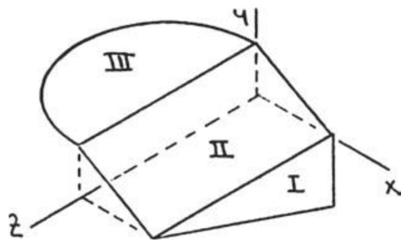
PROBLEM 5.106



Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area.



$$\bar{y}_{\mathrm{I}} = -\frac{1}{3}(1.2) = -0.4 \text{ m}$$

$$\bar{z}_{\mathrm{I}} = \frac{1}{3}(3.6) = 1.2 \text{ m}$$

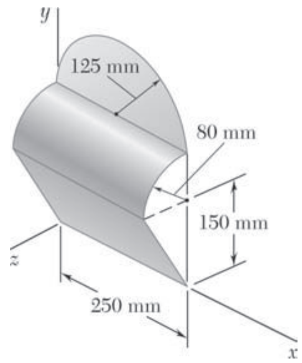
$$\bar{x}_{\mathrm{III}} = -\frac{4(1.8)}{3\pi} = -\frac{2.4}{\pi} \text{ m}$$

	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3	$\bar{z}A$, m^3
I	$\frac{1}{2}(3.6)(1.2) = 2.16$	1.5	–0.4	1.2	3.24	–0.864	2.592
II	$(3.6)(1.7) = 6.12$	0.75	0.4	1.8	4.59	2.448	11.016
III	$\frac{\pi}{2}(1.8)^2 = 5.0894$	$-\frac{2.4}{\pi}$	0.8	1.8	–3.888	4.0715	9.1609
Σ	13.3694				3.942	5.6555	22.769

We have $\bar{X}\Sigma V = \Sigma \bar{x}V$: $\bar{X}(13.3694 \text{ m}^2) = 3.942 \text{ m}^3$ or $\bar{X} = 0.295 \text{ m}$ ◀

$\bar{Y}\Sigma V = \Sigma \bar{y}V$: $\bar{Y}(13.3694 \text{ m}^2) = 5.6555 \text{ m}^3$ or $\bar{Y} = 0.423 \text{ m}$ ◀

$\bar{Z}\Sigma V = \Sigma \bar{z}V$: $\bar{Z}(13.3694 \text{ m}^2) = 22.769 \text{ m}^3$ or $\bar{Z} = 1.703 \text{ m}$ ◀



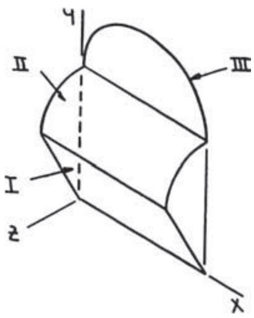
PROBLEM 5.107

Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area. Now note that symmetry implies

$$\bar{X} = 125 \text{ mm} \blacktriangleleft$$



$$\bar{y}_{\text{II}} = 150 + \frac{2 \times 80}{\pi} = 200.93 \text{ mm}$$

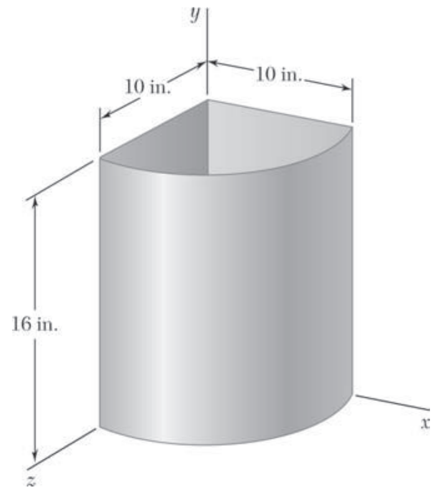
$$\bar{z}_{\text{II}} = \frac{2 \times 80}{\pi} = 50.930 \text{ mm}$$

$$\bar{y}_{\text{III}} = 230 + \frac{4 \times 125}{3\pi} = 283.05 \text{ mm}$$

	A, mm^2	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
I	$(250)(170) = 42500$	75	40	3187500	1700000
II	$\frac{\pi}{2}(80)(250) = 31416$	200.93	50930	6312400	1600000
III	$\frac{\pi}{2}(125)^2 = 24544$	283.05	0	6947200	0
Σ	98460			16447100	3300000

We have $\quad \bar{Y}\Sigma A = \Sigma \bar{y} A: \quad \bar{Y}(98460 \text{ mm}^2) = 16447100 \text{ mm}^3 \qquad$ or $\bar{Y} = 1670 \text{ mm} \blacktriangleleft$

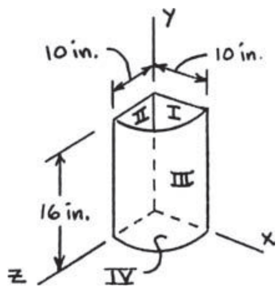
$\bar{Z}\Sigma A = \Sigma \bar{z} A: \quad \bar{Z}(98460 \text{ mm}^2) = 3.300 \times 10^6 \text{ mm}^3 \qquad$ or $\bar{Z} = 33.5 \text{ mm} \blacktriangleleft$



PROBLEM 5.108

A wastebasket, designed to fit in the corner of a room, is 16 in. high and has a base in the shape of a quarter circle of radius 10 in. Locate the center of gravity of the wastebasket, knowing that it is made of sheet metal of uniform thickness.

SOLUTION



By symmetry: $\overline{X} = \overline{Z}$

For III (Cylindrical surface)

$$\overline{x} = \frac{2r}{\pi} = \frac{2(10)}{\pi} = 6.3662 \text{ in.}$$

$$A = \frac{\pi}{2}rh = \frac{\pi}{2}(10)(16) = 251.33 \text{ in.}^2$$

For IV (Quarter-circle bottom)

$$\overline{x} = \frac{4r}{3\pi} = \frac{4(10)}{3\pi} = 4.2441 \text{ in.}$$

$$A = \frac{\pi}{4}r^2 = \frac{\pi}{4}(10)^2 = 78.540 \text{ in.}^2$$

	A, in.^{2}	$\overline{x}$, in.	$\overline{x}$, in.	$\overline{x}A$, in.^{3}	$\overline{y}A$, in.^{3}
I	$(10)(16) = 160$	5	8	800	1280
II	$(10)(16) = 160$	0	8	0	1280
III	251.33	6.3662	8	1600.0	2010.6
IV	78.540	4.2441	0	333.33	0
Σ	649.87			2733.3	4570.6

$\overline{X}\Sigma A = \Sigma\overline{x}A$: $\overline{X}(649.87 \text{ in.}^2) = 2733.3 \text{ in.}^3$

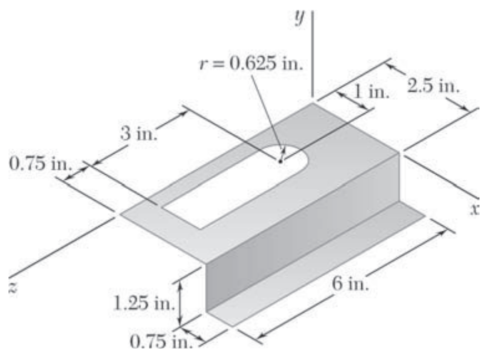
$$\overline{X} = 4.2059 \text{ in.}$$

$\overline{X} = \overline{Z} = 4.21 \text{ in.}$ ◀

$\overline{Y}\Sigma A = \Sigma\overline{y}A$: $\overline{Y}(649.87 \text{ in.}^2) = 4570.6 \text{ in.}^3$

$$\overline{Y} = 7.0331 \text{ in.}$$

$\overline{Y} = 7.03 \text{ in.}$ ◀

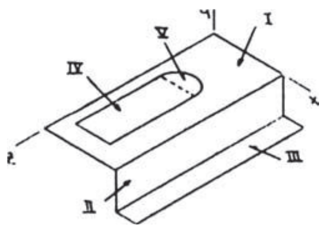


PROBLEM 5.109

A mounting bracket for electronic components is formed from sheet metal of uniform thickness. Locate the center of gravity of the bracket.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the bracket coincides with the centroid of the corresponding area. Then (see diagram)



$$\bar{z}_{\mathrm{V}} = 2.25 - \frac{4(0.625)}{3\pi}$$
$$= 1.98474 \text{ in.}$$
$$A_{\mathrm{V}} = -\frac{\pi}{2}(0.625)^2$$
$$= -0.61359 \text{ in.}^2$$

	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}	$\bar{z}A$, in.^{3}
I	$(2.5)(6) = 15$	1.25	0	3	18.75	0	45
II	$(1.25)(6) = 7.5$	2.5	–0.625	3	18.75	–4.6875	22.5
III	$(0.75)(6) = 4.5$	2.875	–1.25	3	12.9375	–5.625	13.5
IV	$-\left(\frac{5}{4}\right)(3) = -3.75$	1.0	0	3.75	3.75	0	–14.0625
V	-0.61359	1.0	0	1.98474	0.61359	0	–1.21782
Σ	22.6364				46.0739	10.3125	65.7197

We have $\bar{X}\Sigma A = \Sigma \bar{x} A$

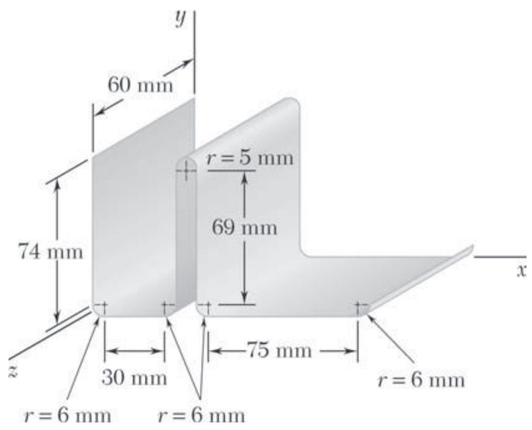
$$\bar{X}(22.6364 \text{ in.}^2) = 46.0739 \text{ in.}^3 \qquad \text{or} \qquad \bar{X} = 2.04 \text{ in.} \blacktriangleleft$$

$$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}(22.6364 \text{ in.}^2) = -10.3125 \text{ in.}^3 \qquad \text{or} \qquad \bar{Y} = -0.456 \text{ in.} \blacktriangleleft$$

$$\bar{Z}\Sigma A = \Sigma \bar{z} A$$

$$\bar{Z}(22.6364 \text{ in.}^2) = 65.7197 \text{ in.}^3 \qquad \text{or} \qquad \bar{Z} = 2.90 \text{ in.} \blacktriangleleft$$



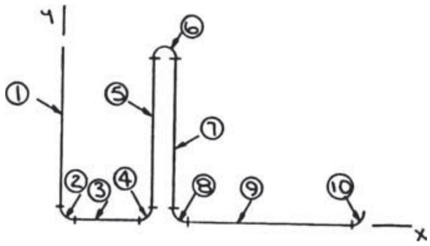
PROBLEM 5.110

A thin sheet of plastic of uniform thickness is bent to form a desk organizer. Locate the center of gravity of the organizer.

SOLUTION

First assume that the plastic is homogeneous so that the center of gravity of the organizer will coincide with the centroid of the corresponding area. Now note that symmetry implies

$$\overline{Z} = 30.0 \text{ mm} \blacktriangleleft$$



$$\overline{x}_2 = 6 - \frac{2 \times 6}{\pi} = 2.1803 \text{ mm}$$

$$\overline{x}_4 = 36 + \frac{2 \times 6}{\pi} = 39.820 \text{ mm}$$

$$\overline{x}_8 = 58 - \frac{2 \times 6}{\pi} = 54.180 \text{ mm}$$

$$\overline{x}_{10} = 133 + \frac{2 \times 6}{\pi} = 136.820 \text{ mm}$$

$$\overline{y}_2 = \overline{y}_4 = \overline{y}_8 = \overline{y}_{10} = 6 - \frac{2 \times 6}{\pi} = 2.1803 \text{ mm}$$

$$\overline{y}_6 = 75 + \frac{2 \times 5}{\pi} = 78.183 \text{ mm}$$

$$A_2 = A_4 = A_8 = A_{10} = \frac{\pi}{2} \times 6 \times 60 = 565.49 \text{ mm}^2$$

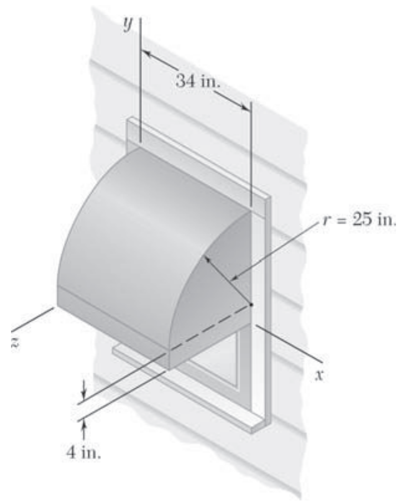
$$A_6 = \pi \times 5 \times 60 = 942.48 \text{ mm}^2$$

PROBLEM 5.110 (Continued)

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$(74)(60) = 4440$	0	43	0	190920
2	565.49	2.1803	2.1803	1233	1233
3	$(30)(60) = 1800$	21	0	37800	0
4	565.49	39.820	2.1803	22518	1233
5	$(69)(60) = 4140$	42	40.5	173880	167670
6	942.48	47	78.183	44297	73686
7	$(69)(60) = 4140$	52	40.5	215280	167670
8	565.49	54.180	2.1803	30638	1233
9	$(75)(60) = 4500$	95.5	0	429750	0
10	565.49	136.820	2.1803	77370	1233
Σ	22224.44			1032766	604878

We have $\bar{X}\Sigma A = \Sigma \bar{x} A$: $\bar{X}(22224.44 \text{ mm}^2) = 1032766 \text{ mm}^3$ or $\bar{X} = 46.5 \text{ mm}$ ◀

$\bar{Y}\Sigma A = \Sigma \bar{y} A$: $\bar{Y}(22224.44 \text{ mm}^2) = 604878 \text{ mm}^3$ or $\bar{Y} = 27.2 \text{ mm}$ ◀

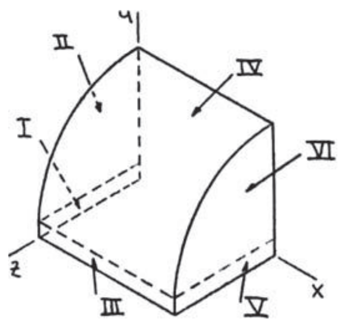


PROBLEM 5.111

A window awning is fabricated from sheet metal of uniform thickness. Locate the center of gravity of the awning.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the awning coincides with the centroid of the corresponding area.



$$\overline{y}_{\text{II}} = \overline{y}_{\text{VI}} = 4 + \frac{(4)(25)}{3\pi} = 14.6103 \text{ in.}$$

$$\overline{z}_{\text{II}} = \overline{z}_{\text{VI}} = \frac{(4)(25)}{3\pi} = \frac{100}{3\pi} \text{ in.}$$

$$\overline{y}_{\text{IV}} = 4 + \frac{(2)(25)}{\pi} = 19.9155 \text{ in.}$$

$$\overline{z}_{\text{IV}} = \frac{(2)(25)}{\pi} = \frac{50}{\pi} \text{ in.}$$

$$A_{\text{II}} = A_{\text{VI}} = \frac{\pi}{4}(25)^2 = 490.87 \text{ in.}^2$$

$$A_{\text{IV}} = \frac{\pi}{2}(25)(34) = 1335.18 \text{ in.}^2$$

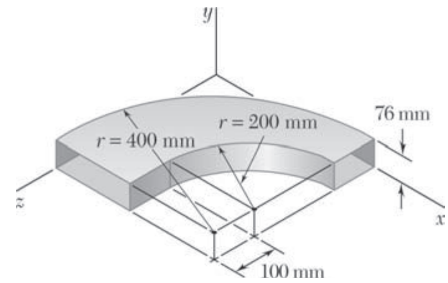
	A, in.^{2}	$\overline{y}$, in.	$\overline{z}$, in.	$\overline{y}A$, in.^{3}	$\overline{z}A$, in.^{3}
I	$(4)(25) = 100$	2	12.5	200	1250
II	490.87	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
III	$(4)(34) = 136$	2	25	272	3400
IV	1335.18	19.9155	$\frac{50}{\pi}$	26591	21250
V	$(4)(25) = 100$	2	12.5	200	1250
VI	490.87	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
Σ	2652.9			41607	37567

PROBLEM 5.111 (Continued)

Now, symmetry implies $\bar{X} = 17.00 \text{ in.}$ ◀

and $\bar{Y}\Sigma A = \Sigma \bar{y} A: \quad \bar{Y}(2652.9 \text{ in.}^2) = 41607 \text{ in.}^3$ or $\bar{Y} = 15.68 \text{ in.}$ ◀

$\bar{Z}\Sigma A = \Sigma \bar{z} A: \quad \bar{Z}(2652.9 \text{ in.}^2) = 37567$ or $\bar{Z} = 14.16 \text{ in.}$ ◀



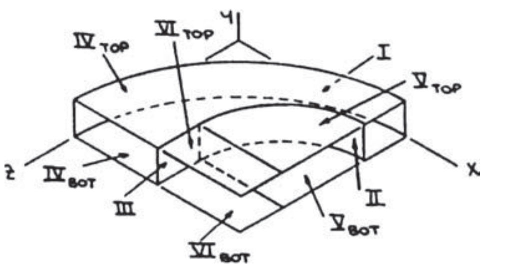
PROBLEM 5.112

An elbow for the duct of a ventilating system is made of sheet metal of uniform thickness. Locate the center of gravity of the elbow.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the duct coincides with the centroid of the corresponding area. Also, note that the shape of the duct implies

$$\overline{Y} = 38.0 \text{ mm} \blacktriangleleft$$



Note that $\quad \overline{x}_{\text{I}} = \overline{z}_{\text{I}} = 400 - \dfrac{2}{\pi}(400) = 145.352 \text{ mm}$

$$\overline{x}_{\text{II}} = 400 - \frac{2}{\pi}(200) = 272.68 \text{ mm}$$

$$\overline{z}_{\text{II}} = 300 - \frac{2}{\pi}(200) = 172.676 \text{ mm}$$

$$\overline{x}_{\text{IV}} = \overline{z}_{\text{IV}} = 400 - \frac{4}{3\pi}(400) = 230.23 \text{ mm}$$

$$\overline{x}_{\text{V}} = 400 - \frac{4}{3\pi}(200) = 315.12 \text{ mm}$$

$$\overline{z}_{\text{V}} = 300 - \frac{4}{3\pi}(200) = 215.12 \text{ mm}$$

Also note that the corresponding top and bottom areas will contribute equally when determining $\overline{x}$ and $\overline{z}$.

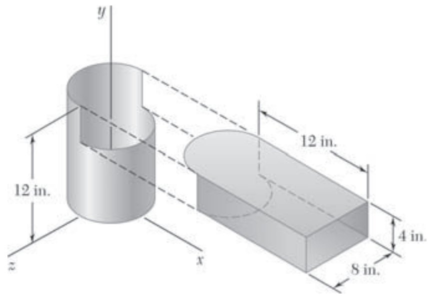
Thus

	A, mm^2	$\overline{x}$, mm	$\overline{z}$, mm	$\overline{x}A$, mm^3	$\overline{z}A$, mm^3
I	$\frac{\pi}{2}(400)(76) = 47752$	145.352	145.352	6940850	6940850
II	$\frac{\pi}{2}(200)(76) = 23876$	272.68	172.676	6510510	4122810
III	$100(76) = 7600$	200	350	1520000	2660000
IV	$2\left(\frac{\pi}{4}\right)(400)^2 = 251327$	230.23	230.23	57863020	57863020
V	$-2\left(\frac{\pi}{4}\right)(200)^2 = -62832$	315.12	215.12	–19799620	–13516420
VI	$-2(100)(200) = -40000$	300	350	–12000000	–14000000
Σ	227723			41034760	44070260

PROBLEM 5.112 (Continued)

We have $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\bar{X}(227723 \text{ mm}^2) = 41034760 \text{ mm}^3$ or $\bar{X} = 180.2 \text{ mm}$ ◀

$\bar{Z}\Sigma A = \Sigma \bar{z}A$: $\bar{Z}(227723 \text{ mm}^2) = 44070260 \text{ mm}^3$ or $\bar{Z} = 193.5 \text{ mm}$ ◀

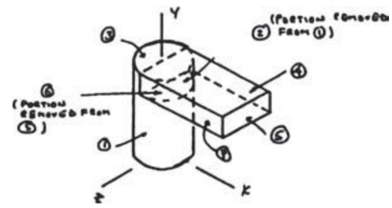


PROBLEM 5.113

An 8-in.-diameter cylindrical duct and a 4 × 8-in. rectangular duct are to be joined as indicated. Knowing that the ducts were fabricated from the same sheet metal, which is of uniform thickness, locate the center of gravity of the assembly.

SOLUTION

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

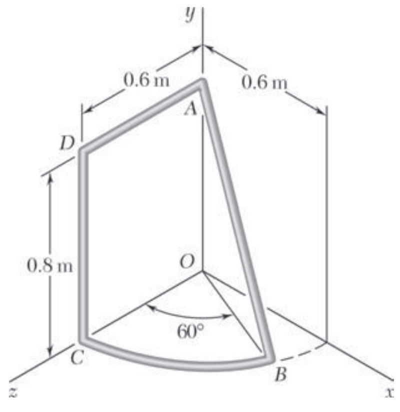


	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$\pi(8)(12) = 96\pi$	0	6	0	576π
2	$-\frac{\pi}{2}(8)(4) = -16\pi$	$\frac{2(4)}{\pi} = \frac{8}{\pi}$	10	-128	-160π
3	$\frac{\pi}{2}(4)^2 = 8\pi$	$-\frac{4(4)}{3\pi} = -\frac{16}{3\pi}$	12	-42.667	96π
4	$(8)(12) = 96$	6	12	576	1152
5	$(8)(12) = 96$	6	8	576	768
6	$-\frac{\pi}{2}(4)^2 = -8\pi$	$\frac{4(4)}{3\pi} = \frac{16}{3\pi}$	8	-42.667	-64π
7	$(4)(12) = 48$	6	10	288	480
8	$(4)(12) = 48$	6	10	288	480
Σ	539.33			1514.6	4287.4

Then
$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{1514.67}{539.33}\text{ in.}$$
or $\bar{X} = 2.81$ in. ◀

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{4287.4}{539.33}\text{ in.}$$
or $\bar{Y} = 7.95$ in. ◀

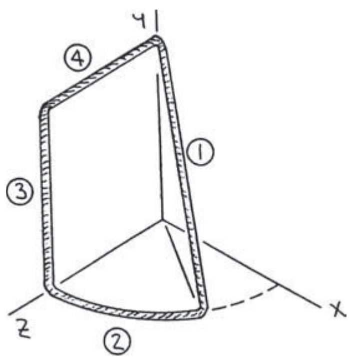
PROBLEM 5.114



A thin steel wire of uniform cross section is bent into the shape shown. Locate its center of gravity.

SOLUTION

First assume that the wire is homogeneous so that its center of gravity will coincide with the centroid of the corresponding line.



$$\bar{x}_1 = 0.3\sin 60° = 0.15\sqrt{3}\text{ m}$$

$$\bar{z}_1 = 0.3\cos 60° = 0.15\text{ m}$$

$$\bar{x}_2 = \left(\frac{0.6\sin 30°}{\frac{\pi}{6}}\right)\sin 30° = \frac{0.9}{\pi}\text{ m}$$

$$\bar{z}_2 = \left(\frac{0.6\sin 30°}{\frac{\pi}{6}}\right)\cos 30° = \frac{0.9}{\pi}\sqrt{3}\text{ m}$$

$$L_2 = \left(\frac{\pi}{3}\right)(0.6) = (0.2\pi)\text{ m}$$

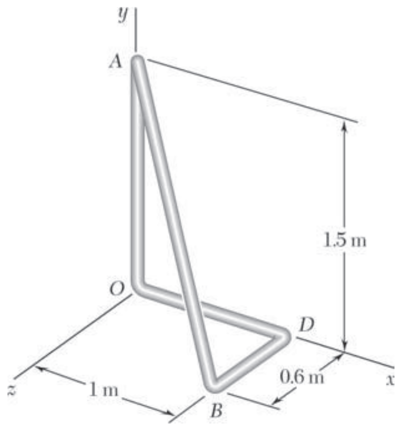
	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	$\bar{z}L$, m^2
1	1.0	$0.15\sqrt{3}$	0.4	0.15	0.25981	0.4	0.15
2	0.2π	$\frac{0.9}{\pi}$	0	$\frac{0.9\sqrt{3}}{\pi}$	0.18	0	0.31177
3	0.8	0	0.4	0.6	0	0.32	0.48
4	0.6	0	0.8	0.3	0	0.48	0.18
Σ	3.0283				0.43981	1.20	1.12177

We have $\quad \bar{X}\Sigma L = \Sigma \bar{x}L: \quad \bar{X}(3.0283\text{ m}) = 0.43981\text{ m}^2 \quad$ or $\bar{X} = 0.1452\text{ m}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y}L: \quad \bar{Y}(3.0283\text{ m}) = 1.20\text{ m}^2 \quad$ or $\bar{Y} = 0.396\text{ m}$ ◀

$\bar{Z}\Sigma L = \Sigma \bar{z}L: \quad \bar{Z}(3.0283\text{ m}) = 1.12177\text{ m}^2 \quad$ or $\bar{Z} = 0.370\text{ m}$ ◀

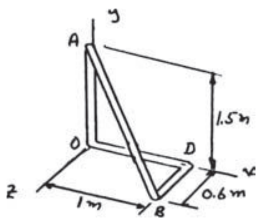
PROBLEM 5.115



Locate the center of gravity of the figure shown, knowing that it is made of thin brass rods of uniform diameter.

SOLUTION

Uniform rod



$$AB^2 = (1\text{ m})^2 + (0.6\text{ m})^2 + (1.5\text{ m})^2$$

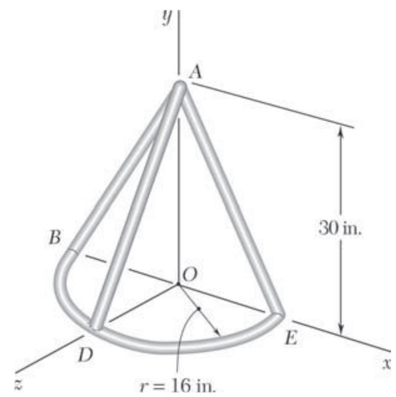
$$AB = 1.9\text{ m}$$

	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	ΣL, m^2
AB	1.9	0.5	0.75	0.3	0.95	1.425	0.57
BD	0.6	1.0	0	0.3	0.60	0	0.18
DO	1.0	0.5	0	0	0.50	0	0
OA	1.5	0	0.75	0	0	1.125	0
Σ	5.0				2.05	2.550	0.75

$\bar{X}\Sigma L = \Sigma\bar{x}L$: $\quad \bar{X}(5.0\text{ m}) = 2.05\text{ m}^2$ $\qquad \bar{X} = 0.410\text{ m}$ ◀

$\bar{Y}\Sigma L = \Sigma\bar{y}L$: $\quad \bar{Y}(5.0\text{ m}) = 2.55\text{ m}^2$ $\qquad \bar{Y} = 0.510\text{ m}$ ◀

$\bar{Z}\Sigma L = \Sigma\bar{z}L$: $\quad \bar{Z}(5.0\text{ m}) = 0.75\text{ m}^2$ $\qquad \bar{Z} = 0.1500\text{ m}$ ◀

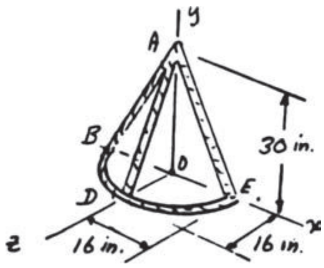


PROBLEM 5.116

Locate the center of gravity of the figure shown, knowing that it is made of thin brass rods of uniform diameter.

SOLUTION

By symmetry: $\bar{X} = 0$ ◀



	L, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}L$, in.^{2}	$\bar{z}L$, in.^{2}
AB	$\sqrt{30^2+16^2} = 34$	15	0	510	0
AD	$\sqrt{30^2+16^2} = 34$	15	8	510	272
AE	$\sqrt{30^2+16^2} = 34$	15	0	510	0
BDE	$\pi(16) = 50.265$	0	$\dfrac{2(16)}{\pi} = 10.186$	0	512
Σ	152.265			1530	784

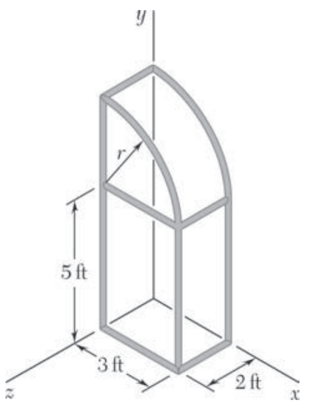
$$\bar{Y}\Sigma L = \Sigma \bar{y} L: \quad \bar{Y}(152.265 \text{ in.}) = 1530 \text{ in.}^2$$

$$\bar{Y} = 10.048 \text{ in.} \qquad \bar{Y} = 10.05 \text{ in.} ◀$$

$$\bar{Z}\Sigma L = \Sigma \bar{z} L: \quad \bar{Z}(152.265 \text{ in.}) = 784 \text{ in.}^2$$

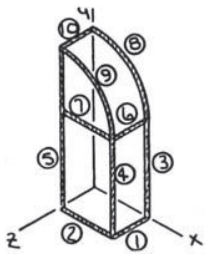
$$\bar{Z} = 5.149 \text{ in.} \qquad \bar{Z} = 5.15 \text{ in.} ◀$$

PROBLEM 5.117



The frame of a greenhouse is constructed from uniform aluminum channels. Locate the center of gravity of the portion of the frame shown.

SOLUTION



First assume that the channels are homogeneous so that the center of gravity of the frame will coincide with the centroid of the corresponding line.

$$\bar{x}_8 = \bar{x}_9 = \frac{2\times 3}{\pi} = \frac{6}{\pi}\text{ ft}$$

$$\bar{y}_8 = \bar{y}_9 = 5 + \frac{2\times 3}{\pi} = 6.9099\text{ ft}$$

	L, ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$\bar{x}L$, ft^2	$\bar{y}L$, ft^2	$\bar{z}L$, ft^2
1	2	3	0	1	6	0	2
2	3	1.5	0	2	4.5	0	6
3	5	3	2.5	0	15	12.5	0
4	5	3	2.5	2	15	12.5	10
5	8	0	4	2	0	32	16
6	2	3	5	1	6	10	2
7	3	1.5	5	2	4.5	15	6
8	$\frac{\pi}{2}\times 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	0	9	32.562	0
9	$\frac{\pi}{2}\times 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	2	9	32.562	9.4248
10	2	0	8	1	0	16	2
Σ	39.4248				69	163.124	53.4248

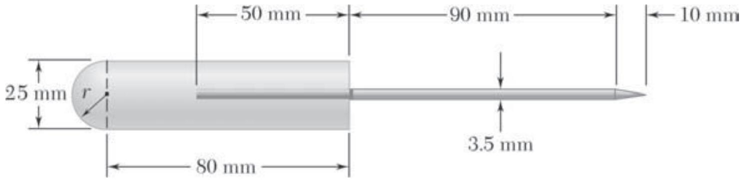
We have $\bar{X}\Sigma L = \Sigma \bar{x} L$: $\bar{X}(39.4248\text{ ft}) = 69\text{ ft}^2$ or $\bar{X} = 1.750\text{ ft}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y} L$: $\bar{Y}(39.4248\text{ ft}) = 163.124\text{ ft}^2$ or $\bar{Y} = 4.14\text{ ft}$ ◀

$\bar{Z}\Sigma L = \Sigma \bar{z} L$: $\bar{Z}(39.4248\text{ ft}) = 53.4248\text{ ft}^2$ or $\bar{Z} = 1.355\text{ ft}$ ◀

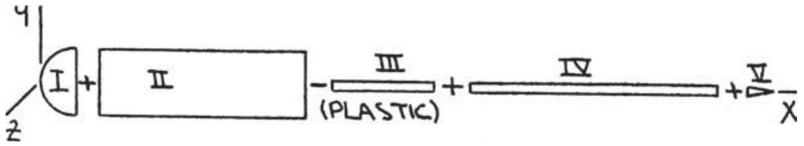
PROBLEM 5.118

A scratch awl has a plastic handle and a steel blade and shank. Knowing that the density of plastic is 1030 kg/m^3 and of steel is 7860 kg/m^3, locate the center of gravity of the awl.



SOLUTION

First, note that symmetry implies $\bar{Y} = \bar{Z} = 0$ ◀



$$\bar{x}_{\mathrm{I}} = \frac{5}{8}(12.5 \text{ mm}) = 7.8125 \text{ mm}$$

$$W_{\mathrm{I}} = (1030 \text{ kg/m}^3)\left(\frac{2\pi}{3}\right)(0.0125 \text{ m})^3$$

$$= 4.2133 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{\mathrm{II}} = 52.5 \text{ mm}$$

$$W_{\mathrm{II}} = (1030 \text{ kg/m}^3)\left(\frac{\pi}{4}\right)(0.025 \text{ m})^2(0.08 \text{ m})$$

$$= 40.448 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{\mathrm{III}} = 92.5 \text{ mm} - 25 \text{ mm} = 67.5 \text{ mm}$$

$$W_{\mathrm{III}} = -(1030 \text{ kg/m}^3)\left(\frac{\pi}{4}\right)(0.0035 \text{ m})^2(.05 \text{ m})$$

$$= -0.49549 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{\mathrm{IV}} = 182.5 \text{ mm} - 70 \text{ mm} = 112.5 \text{ mm}$$

$$W_{\mathrm{IV}} = (7860 \text{ kg/m}^3)\left(\frac{\pi}{4}\right)(0.0035 \text{ m})^2(0.14 \text{ m})^2 = 10.5871 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{\mathrm{V}} = 182.5 \text{ mm} + \frac{1}{4}(10 \text{ mm}) = 185 \text{ mm}$$

$$W_{\mathrm{V}} = (7860 \text{ kg/m}^3)\left(\frac{\pi}{3}\right)(0.00175 \text{ m})^2(0.01 \text{ m}) = 0.25207 \times 10^{-3} \text{ kg}$$

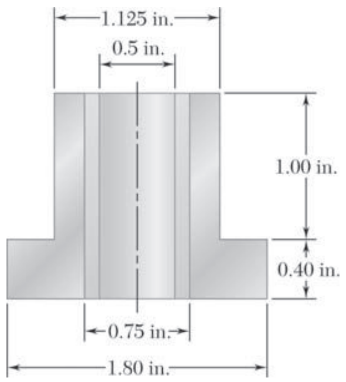
PROBLEM 5.118 (Continued)

	W, kg	$\bar{x}$, mm	$\bar{x}W$, kg·mm
I	4.123×10^{-3}	7.8125	32.916×10^{-3}
II	40.948×10^{-3}	52.5	2123.5×10^{-3}
III	-0.49549×10^{-3}	67.5	-33.447×10^{-3}
IV	10.5871×10^{-3}	112.5	1191.05×10^{-3}
V	0.25207×10^{-3}	185	46.633×10^{-3}
Σ	55.005×10^{-3}		3360.7×10^{-3}

We have $\quad \bar{X}\Sigma W = \Sigma\bar{x}W: \quad \bar{X}(55.005\times10^{-3}\text{ kg}) = 3360.7\times10^{-3}\text{ kg}\cdot\text{mm}$

or $\quad \bar{X} = 61.1\text{ mm}$ ◀

(From the end of the handle)

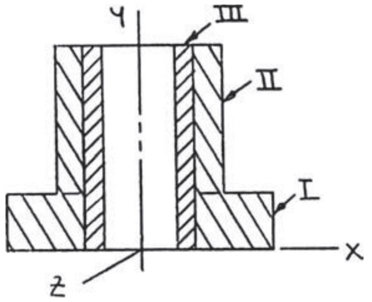


PROBLEM 5.119

A bronze bushing is mounted inside a steel sleeve. Knowing that the specific weight of bronze is 0.318 lb/in.^3 and of steel is 0.284 lb/in.^3, determine the location of the center of gravity of the assembly.

SOLUTION

First, note that symmetry implies $\qquad \overline{X} = \overline{Z} = 0$ ◀



Now $\qquad W = (pg)V$

$$\overline{y}_{\text{I}} = 0.20 \text{ in.} \quad W_{\text{I}} = (0.284 \text{ lb/in.}^3)\left\{\left(\frac{\pi}{4}\right)\left[\left(1.8^2 - 0.75^2\right)\text{in.}^2\right](0.4 \text{ in.})\right\} = 0.23889 \text{ lb}$$

$$\overline{y}_{\text{II}} = 0.90 \text{ in.} \quad W_{\text{II}} = (0.284 \text{ lb/in.}^3)\left\{\left(\frac{\pi}{4}\right)\left[\left(1.125^2 - 0.75^2\right)\text{in.}^2\right](1 \text{ in.})\right\} = 0.156834 \text{ lb}$$

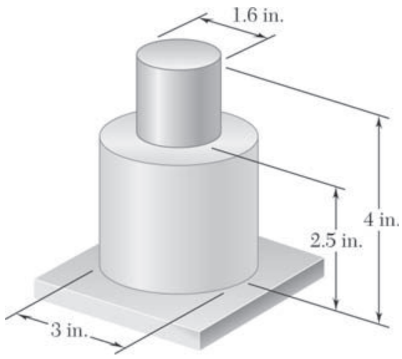
$$\overline{y}_{\text{III}} = 0.70 \text{ in.} \quad W_{\text{III}} = (0.318 \text{ lb/in.}^3)\left\{\left(\frac{\pi}{4}\right)\left[\left(0.75^2 - 0.5^2\right)\text{in.}^2\right](1.4 \text{ in.})\right\} = 0.109269 \text{ lb}$$

We have $\qquad \overline{Y}\Sigma W = \Sigma \overline{y} W$

$$\overline{Y} = \frac{(0.20 \text{ in.})(0.23889 \text{ lb}) + (0.90 \text{ in.})(0.156834 \text{ lb}) + (0.70 \text{ in.})(0.109269 \text{ lb})}{0.23889 \text{ lb} + 0.156834 \text{ lb} + 0.109269 \text{ lb}}$$

or $\qquad \overline{Y} = 0.526 \text{ in.}$ ◀

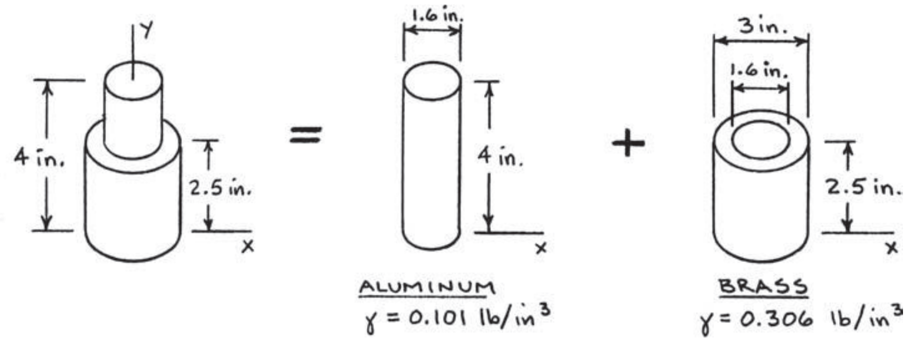
(above base)



PROBLEM 5.120

A brass collar, of length 2.5 in., is mounted on an aluminum rod of length 4 in. Locate the center of gravity of the composite body. (Specific weights: brass = 0.306 lb/in.^{3}, aluminum = 0.101 lb/in.^{3})

SOLUTION



Aluminum rod: $W = \gamma V$

$$= (0.101 \text{ lb/in.}^3)\left[\frac{\pi}{4}(1.6 \text{ in.})^2(4 \text{ in.})\right]$$

$$= 0.81229 \text{ lb}$$

Brass collar: $W = \gamma V$

$$= (0.306 \text{ lb/in.}^3)\frac{\pi}{4}[(3 \text{ in.})^2 - (1.6 \text{ in.})^2](2.5 \text{ in.})$$

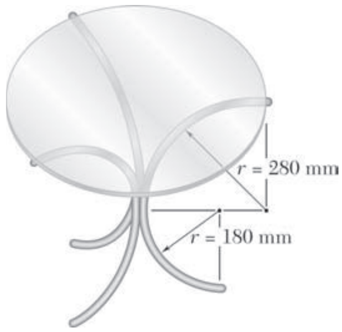
$$= 3.8693 \text{ lb}$$

Component	W(lb)	$\bar{y}$ (in.)	$\bar{y}W$ (lb · in.)
Rod	0.81229	2	1.62458
Collar	3.8693	1.25	4.8366
Σ	4.6816		6.4612

$$\bar{Y}\Sigma W = \Sigma \bar{y} W: \quad \bar{Y}(4.6816 \text{ lb}) = 6.4612 \text{ lb} \cdot \text{in.}$$

$$\bar{Y} = 1.38013 \text{ in.} \qquad \bar{Y} = 1.380 \text{ in.} \blacktriangleleft$$

PROBLEM 5.121

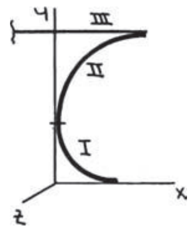


The three legs of a small glass-topped table are equally spaced and are made of steel tubing, which has an outside diameter of 24 mm and a cross-sectional area of 150 mm^2. The diameter and the thickness of the table top are 600 mm and 10 mm, respectively. Knowing that the density of steel is 7860 kg/m^3 and of glass is 2190 kg/m^3, locate the center of gravity of the table.

SOLUTION

First note that symmetry implies $\qquad \overline{X} = \overline{Z} = 0$ ◀

Also, to account for the three legs, the masses of components I and II will each bex multiplied by three



$$\overline{y}_{\text{I}} = 12 + 180 - \frac{2\times 180}{\pi} = 77.408\text{ mm} \qquad m_{\text{I}} = \rho_{ST}V_{\text{I}} = 7860\text{ kg/m}^3 \times (150\times 10^{-6}\text{ m}^2)\times \frac{\pi}{2}(0.180\text{ m}) = 0.33335\text{ kg}$$

$$\overline{y}_{\text{II}} = 12 + 180 + \frac{2\times 280}{\pi} = 370.25\text{ mm} \qquad m_{\text{II}} = \rho_{ST}V_{\text{II}} = 7860\text{ kg/m}^3 \times (150\times 10^{-6}\text{ m}^2)\times \frac{\pi}{2}(0.280\text{ m}) = 0.51855\text{ kg}$$

$$\overline{y}_{\text{III}} = 24 + 180 + 280 + 5 = 489\text{ mm} \qquad m_{\text{III}} = \rho_{GL}V_{\text{III}} = 2190\text{ kg/m}^3 \times \frac{\pi}{4}(0.6\text{ m})^2 \times (0.010\text{ m}) = 6.1921\text{ kg}$$

	m, kg	$\overline{y}$, mm	$\overline{y}m$, kg·mm
I	3(0.33335)	77.408	77.412
II	3(0.51855)	370.25	515.98
III	6.1921	489	3027.9
Σ	8.7478		3681.3

We have $\qquad Y\Sigma m = \Sigma \overline{y}m: \quad \overline{Y}(8.7478\text{ kg}) = 3681.3\text{ kg}\cdot\text{mm}$

or $\qquad \overline{Y} = 420.8\text{ mm}$

The center of gravity is 421 mm ◀

(above the floor)

PROBLEM 5.122

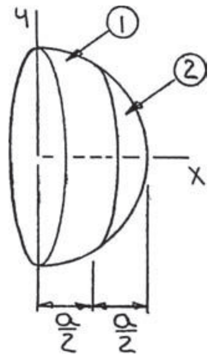
Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A hemisphere

SOLUTION

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$



The equation of the generating curve is $x^2 + y^2 = a^2$ so that $r^2 = a^2 - x^2$ and then

$$dV = \pi(a^2 - x^2)dx$$

Component 1

$$V_1 = \int_0^{a/2} \pi(a^2 - x^2)dx = \pi\left[a^2 x - \frac{x^3}{3}\right]_0^{a/2}$$

$$= \frac{11}{24}\pi a^3$$

and

$$\int_1 \bar{x}_{EL} dV = \int_0^{a/2} x\left[\pi(a^2 - x^2)dx\right]$$

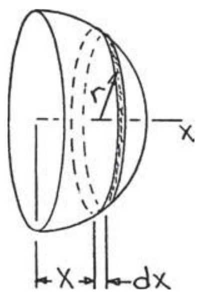
$$= \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{a/2}$$

$$= \frac{7}{64}\pi a^4$$

Now

$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1\left(\frac{11}{24}\pi a^3\right) = \frac{7}{64}\pi a^4 \qquad \text{or} \quad \bar{x}_1 = \frac{21}{88}a \blacktriangleleft$$

Component 2



$$V_2 = \int_{a/2}^{a} \pi(a^2 - x^2)dx = \pi\left[a^2 x - \frac{x^3}{3}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2(a) - \frac{a^3}{3}\right] - \left[a^2\left(\frac{a}{2}\right) - \frac{\left(\frac{a}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{5}{24}\pi a^3$$

PROBLEM 5.122 (Continued)

and $$\int_2 \bar{x}_{EL}\,dV = \int_{a/2}^{a} x\left[\pi(a^2 - x^2)dx\right] = \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2\frac{(a)^2}{2} - \frac{(a)^4}{4}\right] - \left[a^2\frac{\left(\frac{a}{2}\right)^2}{2} - \frac{\left(\frac{a}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^4$$

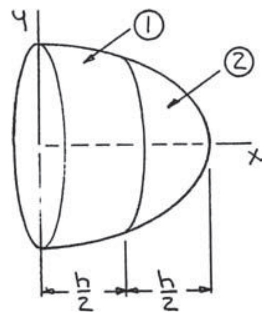
Now $$\bar{x}_2 V_2 = \int_2 \bar{x}_{EL}\,dV: \quad \bar{x}_2\left(\frac{5}{24}\pi a^3\right) = \frac{9}{64}\pi a^4$$ or $$\bar{x}_2 = \frac{27}{40}a \blacktriangleleft$$

PROBLEM 5.123

Determine by direct integration the values of $\overline{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A semiellipsoid of revolution

SOLUTION



Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \overline{x}_{EL} = x$$

The equation of the generating curve is $\dfrac{x^2}{h^2} + \dfrac{y^2}{a^2} = 1$ so that

$$r^2 = \frac{a^2}{h^2}(h^2 - x^2)$$

and then

$$dV = \pi \frac{a^2}{h^2}(h^2 - x^2)dx$$

Component 1

$$V_1 = \int_0^{h/2} \pi \frac{a^2}{h^2}(h^2 - x^2)dx = \pi \frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_0^{h/2}$$

$$= \frac{11}{24}\pi a^2 h$$

and

$$\int_1 \overline{x}_{EL} dV = \int_0^{h/2} x\left[\pi \frac{a^2}{h^2}(h^2 - x^2)dx\right]$$

$$= \pi \frac{a^2}{h^2}\left[h^2 \frac{x^2}{2} - \frac{x^4}{4}\right]_0^{h/2}$$

$$= \frac{7}{64}\pi a^2 h^2$$

Now

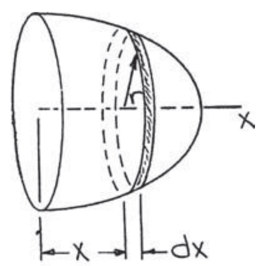
$$\overline{x}_1 V_1 = \int_1 \overline{x}_{EL} dV: \quad \overline{x}_1\left(\frac{11}{24}\pi a^2 h\right) = \frac{7}{64}\pi a^2 h^2 \qquad \text{or} \quad \overline{x}_1 = \frac{21}{88}h \blacktriangleleft$$

Component 2

$$V_2 = \int_{h/2}^{h} \pi \frac{a^2}{h^2}(h^2 - x^2)dx = \pi \frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_{h/2}^{h}$$

$$= \pi \frac{a^2}{h^2}\left\{\left[h^2(h) - \frac{(h)^3}{3}\right] - \left[h^2\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{5}{24}\pi a^2 h$$



PROBLEM 5.123 (Continued)

and $$\int_2 \bar{x}_{EL}\,dV = \int_{h/2}^{h} x\left[\pi\frac{a^2}{h^2}(h^2-x^2)dx\right]$$

$$= \pi\frac{a^2}{h^2}\left[h^2\frac{x^2}{2}-\frac{x^4}{4}\right]_{h/2}^{h}$$

$$= \pi\frac{a^2}{h^2}\left\{\left[h^2\frac{(h)^2}{2}-\frac{(h)^4}{4}\right]-\left[h^2\frac{\left(\frac{h}{2}\right)^2}{2}-\frac{\left(\frac{h}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^2h^2$$

Now $$\bar{x}_2V_2 = \int_2 \bar{x}_{EL}\,dV: \quad \bar{x}_2\left(\frac{5}{24}\pi a^2h\right) = \frac{9}{64}\pi a^2h^2$$

or $\bar{x}_2 = \dfrac{27}{40}h$ ◀

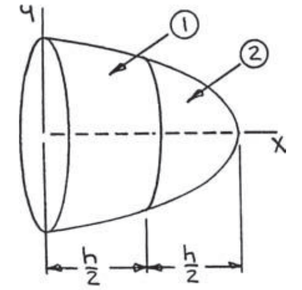
PROBLEM 5.124

Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A paraboloid of revolution

SOLUTION

Choose as the element of volume a disk of radius r and thickness dx. Then



$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $x = h - \dfrac{h}{a^2}y^2$ so that $r^2 = \dfrac{a^2}{h}(h - x)$

and then $$dV = \pi \frac{a^2}{h}(h - x)dx$$

Component 1 $$V_1 = \int_0^{h/2} \pi \frac{a^2}{h}(h - x)dx$$

$$= \pi \frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_0^{h/2}$$

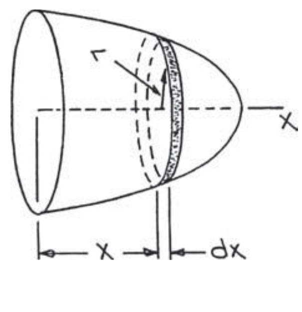
$$= \frac{3}{8}\pi a^2 h$$

and $$\int_1 \bar{x}_{EL}\, dV = \int_0^{h/2} x\left[\pi \frac{a^2}{h}(h - x)dx\right]$$

$$= \pi \frac{a^2}{h}\left[h\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{h/2} = \frac{1}{12}\pi a^2 h^2$$

Now $$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL}\, dV: \quad \bar{x}_1\left(\frac{3}{8}\pi a^2 h\right) = \frac{1}{12}\pi a^2 h^2$$ or $\bar{x}_1 = \dfrac{2}{9}h$ ◀

Component 2



$$V_2 = \int_{h/2}^{h} \pi \frac{a^2}{h}(h - x)dx = \pi \frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_{h/2}^{h}$$

$$= \pi \frac{a^2}{h}\left\{\left[h(h) - \frac{(h)^2}{2}\right] - \left[h\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^2}{2}\right]\right\}$$

$$= \frac{1}{8}\pi a^2 h$$

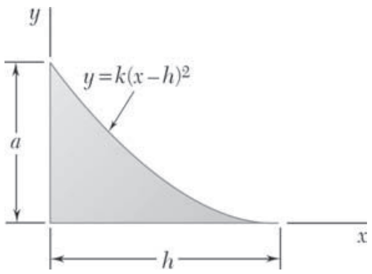
PROBLEM 5.124 (Continued)

and $$\int_2 \overline{x}_{EL}\,dV = \int_{h/2}^{h} x\left[\pi\frac{a^2}{h}(h-x)dx\right] = \pi\frac{a^2}{h}\left[h\frac{x^2}{2}-\frac{x^3}{3}\right]_{h/2}^{h}$$

$$= \pi\frac{a^2}{h}\left\{\left[h\frac{(h)^2}{2}-\frac{(h)^3}{3}\right]-\left[h\frac{\left(\frac{h}{2}\right)^2}{2}-\frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{1}{12}\pi a^2 h^2$$

Now $$\overline{x}_2 V_2 = \int_2 \overline{x}_{EL}\,dV:\quad \overline{x}_2\left(\frac{1}{8}\pi a^2 h\right) = \frac{1}{12}\pi a^2 h^2$$ or $\overline{x}_2 = \dfrac{2}{3}h$ ◀



PROBLEM 5.125

Locate the centroid of the volume obtained by rotating the shaded area about the x axis.

SOLUTION

First note that symmetry implies $\qquad \bar{y} = 0$ ◀

and $\qquad \bar{z} = 0$ ◀

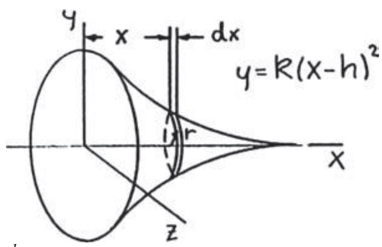
We have $\qquad y = k(X - h)^2$

at $\qquad x = 0, \quad y = a: \quad a = k(-h)^2$

or $\qquad k = \dfrac{a}{h^2}$

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \ \bar{X}_{EL} = x$$



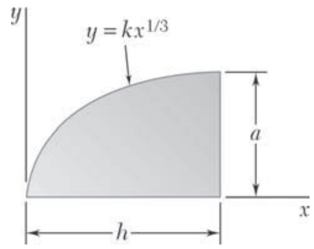
Now $\qquad r = \dfrac{a}{h^2}(x-h)^2$

so that $\qquad dV = \pi \dfrac{a^2}{h^4}(x-h)^4 dx$

Then
$$V = \int_0^h \pi \frac{a^2}{h^4}(x-h)^4 dx = \frac{\pi}{5}\frac{a^2}{h^4}\left[(x-h)^5\right]_0^h$$
$$= \frac{1}{5}\pi a^2 h$$

and
$$\int \bar{x}_{EL} dV = \int_0^h x\left[\pi \frac{a^2}{h^4}(x-h)^4 dx\right]$$
$$= \pi \frac{a^2}{h^4}\int_0^h (x^5 - 4hx^4 + 6h^2x^3 - 4h^3x^2 + h^4x)dx$$
$$= \pi \frac{a^2}{h^4}\left[\frac{1}{6}x^6 - \frac{4}{5}hx^5 + \frac{3}{2}h^2x^4 - \frac{4}{3}h^3x^3 + \frac{1}{2}h^4x^2\right]_0^h$$
$$= \frac{1}{30}\pi a^2 h^2$$

Now $\qquad \bar{x}V = \int \bar{x}_{EL} dV: \quad \bar{x}\left(\dfrac{\pi}{5}a^2 h\right) = \dfrac{\pi}{30}a^2h^2 \qquad$ or $\quad \bar{x} = \dfrac{1}{6}h$ ◀



PROBLEM 5.126

Locate the centroid of the volume obtained by rotating the shaded area about the x axis.

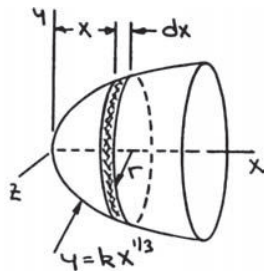
SOLUTION

First note that symmetry implies $\overline{y} = 0$ ◀

$\overline{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad x_{EL} = x$$



Now $\quad r = kx^{1/3}$

so that $\quad dV = \pi k^2 x^{2/3} dx$

at $x = h, \ y = a$: $\quad a = kh^{1/3}$

or $\quad k = \dfrac{a}{h^{1/3}}$

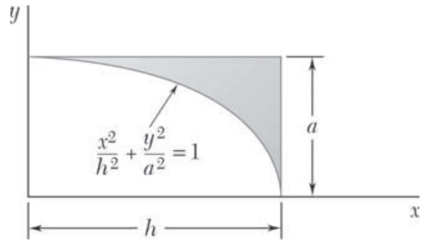
Then $\quad dV = \pi \dfrac{a^2}{h^{2/3}} x^{2/3} dx$

and
$$V = \int_0^h \pi \frac{a^2}{h^{2/3}} x^{2/3} dx$$
$$= \pi \frac{a^2}{h^{2/3}} \left[\frac{3}{5} x^{5/3} \right]_0^h$$
$$= \frac{3}{5} \pi a^2 h$$

Also
$$\int \overline{x}_{EL} dV = \int_0^h x \left(\pi \frac{a^2}{h^{2/3}} x^{2/3} dx \right) = \pi \frac{a^2}{h^{2/3}} \left[\frac{3}{8} x^{8/3} \right]_0^h$$
$$= \frac{3}{8} \pi a^2 h^2$$

Now $\quad \overline{x}V = \int \overline{x} dV: \quad \overline{x} \left(\dfrac{3}{5} \pi a^2 h \right) = \dfrac{3}{8} \pi a^2 h^2 \qquad$ or $\overline{x} = \dfrac{5}{8} h$ ◀

PROBLEM 5.127



Locate the centroid of the volume obtained by rotating the shaded area about the line $x = h$.

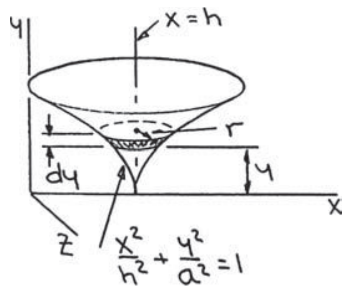
SOLUTION

First, note that symmetry implies $\overline{x} = h$ ◀

$\overline{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dy, \quad \overline{y}_{EL} = y$$



Now $x^2 = \frac{h^2}{a^2}(a^2 - y^2)$ so that $r = h - \frac{h}{a}\sqrt{a^2 - y^2}$

Then $$dV = \pi \frac{h^2}{a^2}\left(a - \sqrt{a^2 - y^2}\right)^2 dy$$

and $$V = \int_0^a \pi \frac{h^2}{a^2}\left(a - \sqrt{a^2 - y^2}\right)^2 dy$$

Let $$y = a\sin\theta \Rightarrow dy = a\cos\theta\, d\theta$$

Then
$$\begin{aligned}
V &= \pi \frac{h^2}{a^2}\int_0^{\pi/2}\left(a - \sqrt{a^2 - a^2\sin^2\theta}\right)^2 a\cos\theta\, d\theta \\
&= \pi \frac{h^2}{a^2}\int_0^{\pi/2}\left[a^2 - 2a(a\cos\theta) + (a^2 - a^2\sin^2\theta)\right] a\cos\theta\, d\theta \\
&= \pi a h^2 \int_0^{\pi/2}(2\cos\theta - 2\cos^2\theta - \sin^2\theta\cos\theta)\, d\theta \\
&= \pi a h^2\left[2\sin\theta - 2\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - \frac{1}{3}\sin^3\theta\right]_0^{\pi/2} \\
&= \pi a h^2\left[2 - 2\left(\frac{\frac{\pi}{2}}{2}\right) - \frac{1}{3}\right] \\
&= 0.095870\pi a h^2
\end{aligned}$$

PROBLEM 5.127 (Continued)

and $$\int \overline{y}_{EL}\,dV = \int_0^a y\left[\pi\frac{h^2}{a^2}\left(a-\sqrt{a^2-y^2}\right)^2 dy\right]$$

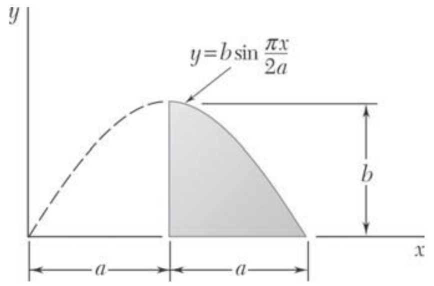
$$= \pi\frac{h^2}{a^2}\int_0^a\left(2a^2y - 2ay\sqrt{a^2-y^2} - y^3\right)dy$$

$$= \pi\frac{h^2}{a^2}\left[a^2y^2 + \frac{2}{3}a(a^2-y^2)^{3/2} - \frac{1}{4}y^4\right]_0^a$$

$$= \pi\frac{h^2}{a^2}\left\{\left[a^2(a)^2 - \frac{1}{4}a^4\right] - \left[\frac{2}{3}a(a^2)^{3/2}\right]\right\}$$

$$= \frac{1}{12}\pi a^2h^2$$

Now $$\overline{y}V = \int \overline{y}_{EL}\,dV: \quad \overline{y}(0.095870\pi a h^2) = \frac{1}{12}\pi a^2 h^2$$ or $\overline{y} = 0.869a$ ◀



PROBLEM 5.128*

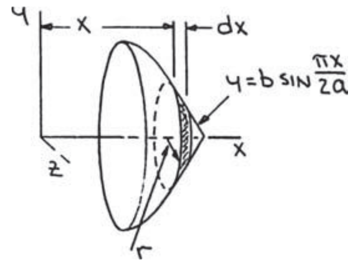
Locate the centroid of the volume generated by revolving the portion of the sine curve shown about the x axis.

SOLUTION

First, note that symmetry implies $\quad \bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx.



Then $\quad dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$

Now $\quad r = b\sin\dfrac{\pi x}{2a}$

so that $\quad dV = \pi b^2 \sin^2 \dfrac{\pi x}{2a} dx$

Then
$$V = \int_a^{2a} \pi b^2 \sin^2 \frac{\pi x}{2a} dx$$
$$= \pi b^2 \left[\frac{x}{2} - \frac{\sin\frac{\pi x}{2a}}{2\frac{\pi}{a}} \right]_a^{2a}$$
$$= \pi b^2 \left[\left(\tfrac{2a}{2}\right) - \left(\tfrac{a}{2}\right) \right]$$
$$= \frac{1}{2}\pi a b^2$$

and
$$\int \bar{x}_{EL} dV = \int_a^{2a} x \left(\pi b^2 \sin^2 \frac{\pi x}{2a} dx \right)$$

Use integration by parts with

$$u = x \qquad dV = \sin^2 \frac{\pi x}{2a}$$
$$du = dx \qquad V = \frac{x}{2} - \frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}$$

PROBLEM 5.128* (Continued)

Then $$\int \bar{x}_{EL}\,dV = \pi b^2\left\{\left[x\left(\frac{x}{2}-\frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}\right)\right]_a^{2a} - \int_a^{2a}\left(\frac{x}{2}-\frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}\right)dx\right\}$$

$$= \pi b^2\left\{\left[2a\left(\frac{2a}{2}\right)-a\left(\frac{a}{2}\right)\right]-\left[\frac{1}{4}x^2+\frac{a^2}{2\pi^2}\cos\frac{\pi x}{a}\right]_a^{2a}\right\}$$

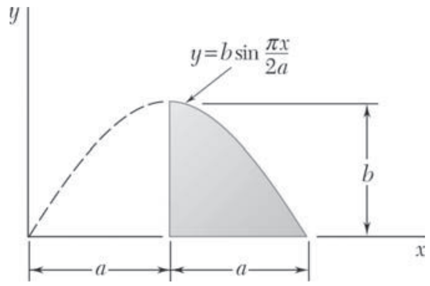
$$= \pi b^2\left\{\left(\frac{3}{2}a^2\right)-\left[\frac{1}{4}(2a)^2+\frac{a^2}{2\pi^2}-\frac{1}{4}(a)^2+\frac{a^2}{2\pi^2}\right]\right\}$$

$$= \pi a^2 b^2\left(\frac{3}{4}-\frac{1}{\pi^2}\right)$$

$$= 0.64868\pi a^2 b^2$$

Now $$\bar{x}V = \int \bar{x}_{EL}\,dV: \quad \bar{x}\left(\frac{1}{2}\pi ab^2\right) = 0.64868\pi a^2 b^2$$

or $\bar{x} = 1.297a$ ◀



PROBLEM 5.129*

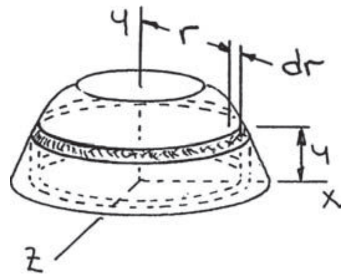
Locate the centroid of the volume generated by revolving the portion of the sine curve shown about the y axis. (*Hint:* Use a thin cylindrical shell of radius r and thickness dr as the element of volume.)

SOLUTION

First note that symmetry implies $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a cylindrical shell of radius r and thickness dr.



Then $$dV = (2\pi r)(y)(dr), \quad \bar{y}_{EL} = \frac{1}{2}y$$

Now $$y = b\sin\frac{\pi r}{2a}$$

so that $$dV = 2\pi br\sin\frac{\pi r}{2a}dr$$

Then $$V = \int_a^{2a} 2\pi br\sin\frac{\pi r}{2a}dr$$

Use integration by parts with

$$u = rd \qquad dv = \sin\frac{\pi r}{2a}dr$$

$$du = dr \qquad v = -\frac{2a}{\pi}\cos\frac{\pi r}{2a}$$

Then $$V = 2\pi b\left\{\left[(r)\left(-\frac{2a}{\pi}\cos\frac{\pi r}{2a}\right)\right]_a^{2a} - \int_a^{2a}\left(\frac{2a}{\pi}\cos\frac{\pi r}{2a}\right)dr\right\}$$

$$= 2\pi b\left\{-\frac{2a}{\pi}[(2a)(-1)] + \left[\frac{4a^2}{\pi^2}\sin\frac{\pi r}{2a}\right]_a^{2a}\right\}$$

$$V = 2\pi b\left(\frac{4a^2}{\pi} - \frac{4a^2}{\pi^2}\right)$$

$$= 8a^2b\left(1 - \frac{1}{\pi}\right)$$

$$= 5.4535a^2b$$

PROBLEM 5.129* (Continued)

Also
$$\int \bar{y}_{EL}\,dV = \int_a^{2a}\left(\tfrac{1}{2}b\sin\frac{\pi r}{2a}\right)\left(2\pi br\sin\frac{\pi r}{2a}\,dr\right)$$
$$= \pi b^2\int_a^{2a} r\sin^2\frac{\pi r}{2a}\,dr$$

Use integration by parts with

$$u = r \qquad dv = \sin^2\frac{\pi r}{2a}\,dr$$
$$du = dr \qquad v = \frac{r}{2} - \frac{\sin\frac{\pi r}{a}}{\frac{2\pi}{a}}$$

Then
$$\int \bar{y}_{EL}\,dV = \pi b^2\left\{\left[(r)\left(\frac{r}{2}-\frac{\sin\frac{\pi r}{a}}{\frac{2\pi}{a}}\right)\right]_a^{2a} - \int_a^{2a}\left(\frac{r}{2}-\frac{\sin\frac{\pi r}{a}}{\frac{2\pi}{a}}\right)dr\right\}$$
$$= \pi b^2\left\{\left[(2a)\left(\frac{2a}{2}\right)-(a)\left(\frac{a}{2}\right)\right]-\left[\frac{r^2}{4}+\frac{a^2}{2\pi^2}\cos\frac{\pi r}{a}\right]_a^{2a}\right\}$$
$$= \pi b^2\left\{\frac{3}{2}a^2 - \left[\frac{(2a)^2}{4}+\frac{a^2}{2\pi^2}-\frac{(a)^2}{4}+\frac{a^2}{2\pi^2}\right]\right\}$$
$$= \pi a^2 b^2\left(\frac{3}{4}-\frac{1}{\pi^2}\right)$$
$$= 2.0379a^2b^2$$

Now $\quad \bar{y}V = \int \bar{y}_{EL}\,dV: \quad \bar{y}(5.4535a^2b) = 2.0379a^2b^2 \qquad$ or $\bar{y} = 0.374b$ ◀

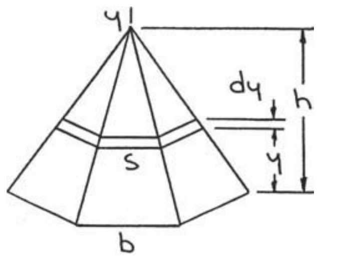
PROBLEM 5.130*

Show that for a regular pyramid of height h and n sides $(n = 3, 4, \ldots)$ the centroid of the volume of the pyramid is located at a distance $h/4$ above the base.

SOLUTION

Choose as the element of a horizontal slice of thickness dy. For any number N of sides, the area of the base of the pyramid is given by

$$A_{\text{base}} = kb^2$$



where $k = k(N)$; see note below. Using similar triangles, have

$$\frac{s}{b} = \frac{h-y}{h}$$

or $$s = \frac{b}{h}(h-y)$$

Then $$dV = A_{\text{slice}}dy = ks^2 dy = k\frac{b^2}{h^2}(h-y)^2 dy$$

and $$V = \int_0^h k\frac{b^2}{h^2}(h-y)^2 dy = k\frac{b^2}{h^2}\left[-\frac{1}{3}(h-y)^3\right]_0^h$$

$$= \frac{1}{3}kb^2 h$$

Also $$\overline{y}_{EL} = y$$

So then $$\int \overline{y}_{EL} dV = \int_0^h y\left[k\frac{b^2}{h^2}(h-y)^2 dy\right] = k\frac{b^2}{h^2}\int_0^h (h^2 y - 2hy^2 + y^3)\,dy$$

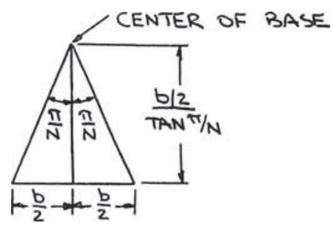
$$= k\frac{b^2}{h^2}\left[\frac{1}{2}h^2 y^2 - \frac{2}{3}hy^3 + \frac{1}{4}y^4\right]_0^h = \frac{1}{12}kb^2h^2$$

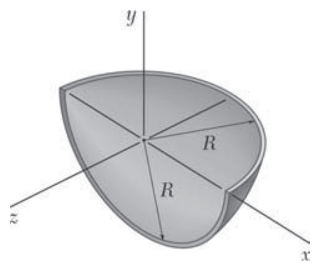
Now $$\overline{y}V = \int \overline{y}_{EL} dV: \quad \overline{y}\left(\frac{1}{3}kb^2 h\right) = \frac{1}{12}kb^2h^2$$ or $y = \dfrac{1}{4}h$ Q.E.D. ◀

Note $$A_{\text{base}} = N\left(\frac{1}{2}\times b\times\frac{\frac{b}{2}}{\tan\frac{\pi}{N}}\right)$$

$$= \frac{N}{4\tan\frac{\pi}{N}}b^2$$

$$= k(N)b^2$$





PROBLEM 5.131

Determine by direct integration the location of the centroid of one-half of a thin, uniform hemispherical shell of radius R.

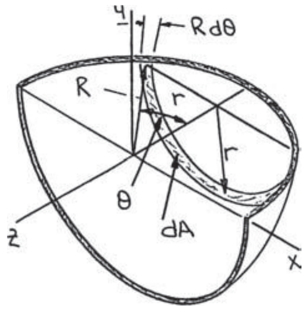
SOLUTION

First note that symmetry implies $\bar{x} = 0$ ◀

The element of area dA of the shell shown is obtained by cutting the shell with two planes parallel to the xy plane. Now

$$dA = (\pi r)(R d\theta)$$

$$\bar{y}_{EL} = -\frac{2r}{\pi}$$



Where $r = R\sin\theta$

so that $dA = \pi R^2 \sin\theta d\theta$

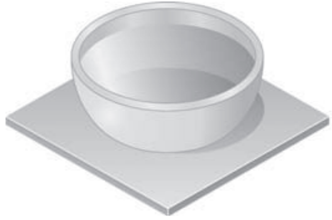
$$\bar{y}_{EL} = -\frac{2R}{\pi}\sin\theta$$

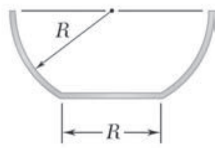
Then
$$A = \int_0^{\pi/2} \pi R^2 \sin\theta d\theta = \pi R^2 [-\cos\theta]_0^{\pi/2}$$
$$= \pi R^2$$

and
$$\int \bar{y}_{EL} dA = \int_0^{\pi/2} \left(-\frac{2R}{\pi}\sin\theta\right)(\pi R^2 \sin\theta d\theta)$$
$$= -2R^3\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$
$$= -\frac{\pi}{2}R^3$$

Now $\bar{y}A = \int \bar{y}_{EL} dA$: $\quad \bar{y}(\pi R^2) = -\frac{\pi}{2}R^3$ or $\bar{y} = -\frac{1}{2}R$ ◀

Symmetry implies $\bar{z} = \bar{y}$ $\quad \bar{z} = -\frac{1}{2}R$ ◀





PROBLEM 5.132

The sides and the base of a punch bowl are of uniform thickness t. If $t << R$ and $R = 250$ mm, determine the location of the center of gravity of (*a*) the bowl, (*b*) the punch.

SOLUTION

(*a*) Bowl

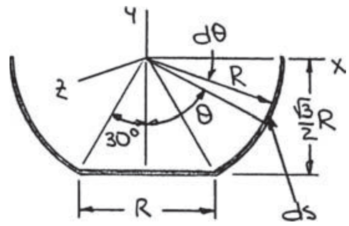
First note that symmetry implies $\qquad \bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

for the coordinate axes shown below. Now assume that the bowl may be treated as a shell; the center of gravity of the bowl will coincide with the centroid of the shell. For the walls of the bowl, an element of area is obtained by rotating the arc ds about the y axis. Then

$$dA_{\text{wall}} = (2\pi R\sin\theta)(Rd\theta)$$

and $\qquad (\bar{y}_{EL})_{\text{wall}} = -R\cos\theta$



Then
$$A_{\text{wall}} = \int_{\pi/6}^{\pi/2} 2\pi R^2 \sin\theta d\theta$$
$$= 2\pi R^2[-\cos\theta]_{\pi/6}^{\pi/2}$$
$$= \pi\sqrt{3}R^2$$

and
$$\bar{y}_{\text{wall}} A_{\text{wall}} = \int (\bar{y}_{EL})_{\text{wall}}\, dA$$
$$= \int_{\pi/6}^{\pi/2} (-R\cos\theta)(2\pi R^2 \sin\theta d\theta)$$
$$= \pi R^3[\cos^2\theta]_{\pi/6}^{\pi/2}$$
$$= -\frac{3}{4}\pi R^3$$

By observation $\qquad A_{\text{base}} = \frac{\pi}{4}R^2, \quad \bar{y}_{\text{base}} = -\frac{\sqrt{3}}{2}R$

Now $\qquad \bar{y}\Sigma A = \Sigma \bar{y} A$

or
$$\bar{y}\left(\pi\sqrt{3}R^2 + \frac{\pi}{4}R^2\right) = -\frac{3}{4}\pi R^3 + \frac{\pi}{4}R^2\left(-\frac{\sqrt{3}}{2}R\right)$$

or $\qquad \bar{y} = -0.48763R \quad R = 250 \text{ mm} \qquad \bar{y} = -121.9 \text{ mm}$ ◀

PROBLEM 5.132 (Continued)

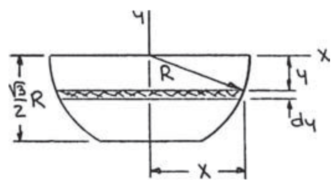
(*b*) Punch

First note that symmetry implies $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

and that because the punch is homogeneous, its center of gravity will coincide with the centroid of the corresponding volume. Choose as the element of volume a disk of radius *x* and thickness *dy*. Then

$$dV = \pi x^2 dy, \quad \bar{y}_{EL} = y$$



Now $x^2 + y^2 = R^2$

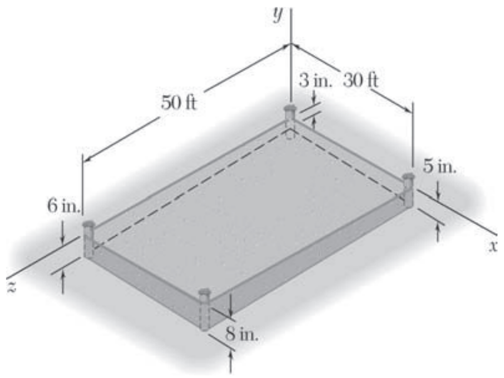
so that $dV = \pi(R^2 - y^2)dy$

Then
$$V = \int_{-\sqrt{3}/2R}^{0} \pi(R^2 - y^2)\,dy$$
$$= \pi\left[R^2 y - \frac{1}{3}y^3\right]_{-\sqrt{3}/2R}^{0}$$
$$= -\pi\left[R^2\left(-\frac{\sqrt{3}}{2}R\right) - \frac{1}{3}\left(-\frac{\sqrt{3}}{2}R\right)^3\right] = \frac{3}{8}\pi\sqrt{3}R^3$$

and
$$\int \bar{y}_{EL}\,dV = \int_{-\sqrt{3}/2R}^{0} (y)\left[\pi\left(R^2 - y^2\right)dy\right]$$
$$= \pi\left[\frac{1}{2}R^2y^2 - \frac{1}{4}y^4\right]_{-\sqrt{3}/2R}^{0}$$
$$= -\pi\left[\frac{1}{2}R^2\left(-\frac{\sqrt{3}}{2}R\right)^2 - \frac{1}{4}\left(-\frac{\sqrt{3}}{2}R\right)^4\right] = -\frac{15}{64}\pi R^4$$

Now
$$\bar{y}V = \int \bar{y}_{EL}\,dV: \quad \bar{y}\left(\frac{3}{8}\pi\sqrt{3}R^3\right) = -\frac{15}{64}\pi R^4$$

or
$$\bar{y} = -\frac{5}{8\sqrt{3}}R \quad R = 250 \text{ mm} \qquad \bar{y} = -90.2 \text{ mm} \blacktriangleleft$$



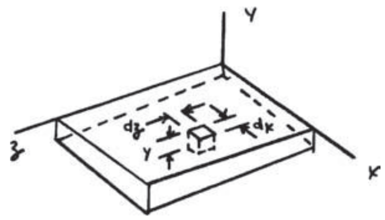
PROBLEM 5.133

After grading a lot, a builder places four stakes to designate the corners of the slab for a house. To provide a firm, level base for the slab, the builder places a minimum of 3 in. of gravel beneath the slab. Determine the volume of gravel needed and the x coordinate of the centroid of the volume of the gravel. (*Hint:* The bottom surface of the gravel is an oblique plane, which can be represented by the equation $y = a + bx + cz$.)

SOLUTION

The centroid can be found by integration. The equation for the bottom of the gravel is:

$y = a + bx + cz$, where the constants a, b, and c can be determined as follows:



For $x = 0$, and $z = 0$: $\quad y = -3$ in., and therefore

$$-\frac{3}{12}\text{ ft} = a, \quad \text{or} \quad a = -\frac{1}{4}\text{ ft}$$

For $x = 30$ ft, and $z = 0$: $\quad y = -5$ in., and therefore

$$-\frac{5}{12}\text{ ft} = -\frac{1}{4}\text{ ft} + b(30\text{ ft}), \quad \text{or} \quad b = -\frac{1}{180}$$

For $x = 0$, and $z = 50$ ft: $\quad y = -6$ in., and therefore

$$-\frac{6}{12}\text{ ft} = -\frac{1}{4}\text{ ft} + c(50\text{ ft}), \quad \text{or} \quad c = -\frac{1}{200}$$

Therefore:
$$y = -\frac{1}{4}\text{ft} - \frac{1}{180}x - \frac{1}{200}z$$

Now
$$\overline{x} = \frac{\int x_{EL}\, dV}{V}$$

A volume element can be chosen as:

$$dV = |y|\, dxdz$$

or
$$dV = \frac{1}{4}\left(1 + \frac{1}{45}x + \frac{1}{50}z\right)dx\, dz$$

and
$$\overline{x}_{EL} = x$$

PROBLEM 5.133 (Continued)

Then
$$\int x_{EL}\,dV = \int_0^{50}\int_0^{30}\frac{x}{4}\left(1+\frac{1}{45}x+\frac{1}{50}z\right)dx\,dz$$

$$=\frac{1}{4}\int_0^{50}\left[\frac{x^2}{2}+\frac{1}{135}x^3+\frac{z}{100}x^2\right]_0^{30}dz$$

$$=\frac{1}{4}\int_0^{50}(650+9z)dz$$

$$=\frac{1}{4}\left[650\,z+\frac{9}{2}z^2\right]_0^{50}$$

$$=10937.5\text{ ft}^4$$

The volume is:
$$V\int dV = \int_0^{50}\int_0^{30}\frac{1}{4}\left(1+\frac{1}{45}x+\frac{1}{50}z\right)dx\,dz$$

$$=\frac{1}{4}\int_0^{50}\left[x+\frac{1}{90}x^2+\frac{z}{50}x\right]_0^{30}dz$$

$$=\frac{1}{4}\int_0^{50}\left(40+\frac{3}{5}z\right)dz$$

$$=\frac{1}{4}\left[40z+\frac{3}{10}z^2\right]_0^{50}$$

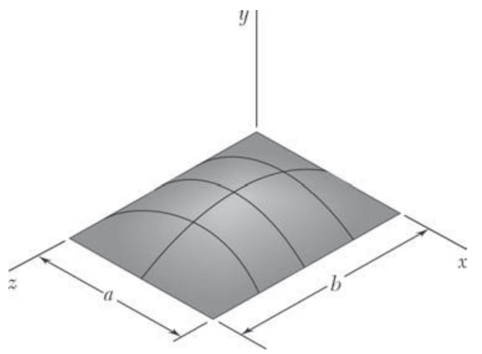
$$=687.50\text{ ft}^3$$

Then
$$\bar{x}=\frac{\int \bar{x}_{EL}\,dV}{V}=\frac{10937.5\text{ ft}^4}{687.5\text{ ft}^3}=15.9091\text{ ft}$$

Therefore:

$$V = 688\text{ ft}^3 \blacktriangleleft$$

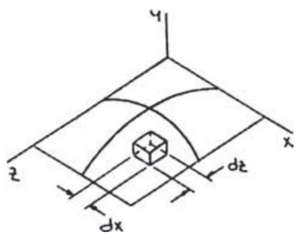
$$\bar{x}=15.91\text{ ft} \blacktriangleleft$$



PROBLEM 5.134

Determine by direct integration the location of the centroid of the volume between the *xz* plane and the portion shown of the surface $y = 16h(ax - x^2)(bz - z^2)/a^2b^2$.

SOLUTION



First note that symmetry implies $\qquad \bar{x} = \dfrac{a}{2}$ ◀

$\bar{z} = \dfrac{b}{2}$ ◀

Choose as the element of volume a filament of base $dx \times dz$ and height y. Then

$$dV = ydxdz, \quad \bar{y}_{EL} = \frac{1}{2}y$$

or

$$dV = \frac{16h}{a^2b^2}(ax - x^2)(bz - z^2)dx\,dz$$

Then

$$V = \int_0^b \int_0^a \frac{16h}{a^2b^2}(ax - x^2)(bz - z^2)dx\,dz$$

$$V = \frac{16h}{a^2b^2}\int_0^b (bz - z^2)\left[\frac{a}{z}x^2 - \frac{1}{3}x^3\right]_0^a dz$$

$$= \frac{16h}{a^2b^2}\left[\frac{a}{2}(a^2) - \frac{1}{3}(a)^3\right]\left[\frac{b}{2}z^2 - \frac{1}{3}z^3\right]_0^b$$

$$= \frac{8ah}{3b^2}\left[\frac{b}{2}(b)^2 - \frac{1}{3}(b)^3\right]$$

$$= \frac{4}{9}abh$$

PROBLEM 5.134 (Continued)

and $$\int \bar{y}_{EL}\,dV = \int_0^b \int_0^a \frac{1}{2}\left[\frac{16h}{a^2b^2}(ax-x^2)(bz-z^2)\right]\left[\frac{16h}{a^2b^2}(ax-x^2)(bz-z^2)dx\,dz\right]$$

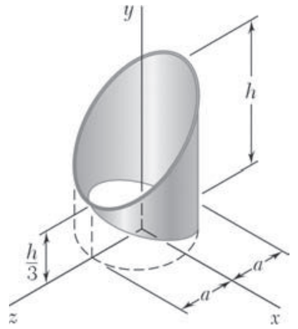
$$= \frac{128h^2}{a^4b^4}\int_0^b \int_0^a (a^2x^2-2ax^3+x^4)(b^2z^2-2bz^3+z^4)dx\,dz$$

$$= \frac{128h^2}{a^2b^4}\int_0^b (b^2z^2-2bz^3+z^4)\left[\frac{a^2}{3}x^3-\frac{a}{2}x^4+\frac{1}{5}x^5\right]_0^a dz$$

$$= \frac{128h^2}{a^4b^4}\left[\frac{a^2}{3}(a)^3-\frac{a}{2}(a)^4+\frac{1}{5}(a)^5\right]\left[\frac{b^2}{3}Z^3-\frac{b}{Z}Z^4+\frac{1}{5}Z^5\right]_0^b$$

$$= \frac{64ah^2}{15b^4}\left[\frac{b^3}{3}(b)^3-\frac{b}{2}(b)^4+\frac{1}{5}(b)^5\right] = \frac{32}{225}abh^2$$

Now $$\bar{y}V = \int \bar{y}_{EL}\,dV: \quad \bar{y}\left(\frac{4}{9}abh\right) = \frac{32}{225}abh^2$$ or $\bar{y} = \dfrac{8}{25}h$ ◀



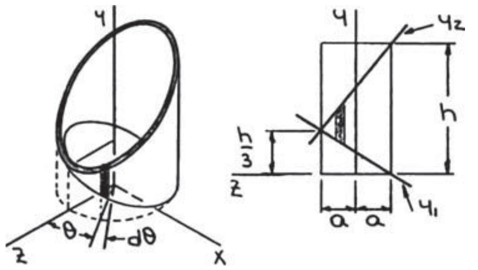
PROBLEM 5.135

Locate the centroid of the section shown, which was cut from a thin circular pipe by two oblique planes.

SOLUTION

First note that symmetry implies $\overline{x} = 0$ ◀

Assume that the pipe has a uniform wall thickness t and choose as the element of volume A vertical strip of width $ad\theta$ and height $(y_2 - y_1)$. Then



$$dV = (y_2 - y_1)tad\theta, \quad \overline{y}_{EL} = \frac{1}{2}(y_1 + \overline{y}_2)\overline{z}_{EL} = z$$

Now

$$y_1 = \frac{\frac{h}{3}}{2a}z + \frac{h}{6} \qquad y_2 = -\frac{\frac{2h}{3}}{2a}z + \frac{2}{3}h$$

$$= \frac{h}{6a}(z + a) \qquad = \frac{h}{3a}(-z + 2a)$$

and

$$z = a\cos\theta$$

Then

$$(y_2 - y_1) = \frac{h}{3a}(-a\cos\theta + 2a) - \frac{h}{6a}(a\cos\theta + a)$$

$$= \frac{h}{2}(1 - \cos\theta)$$

and

$$(y_1 + y_2) = \frac{h}{6a}(a\cos\theta + a) + \frac{h}{3a}(-a\cos\theta + 2a)$$

$$= \frac{h}{6}(5 - \cos\theta)$$

$$dV = \frac{aht}{2}(1 - \cos\theta)d\theta \quad \overline{y}_{EL} = \frac{h}{12}(5 - \cos\theta), \quad \overline{z}_{EL} = a\cos\theta$$

PROBLEM 5.135 (Continued)

Then $$V = 2\int_0^{\pi} \frac{aht}{2}(1-\cos\theta)d\theta = aht[\theta - \sin\theta]_0^{\pi}$$
$$= \pi aht$$

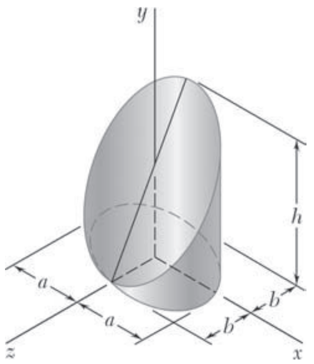
and $$\int \overline{y}_{EL} dV = 2\int_0^{\pi} \frac{h}{12}(5-\cos\theta)\left[\frac{aht}{2}(1-\cos\theta)d\theta\right]$$
$$= \frac{ah^2 t}{12}\int_0^{\pi}(5 - 6\cos\theta + \cos^2\theta)d\theta$$
$$= \frac{ah^2 t}{12}\left[5\theta - 6\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi}$$
$$= \frac{11}{24}\pi ah^2 t$$

$$\int \overline{z}_{EL} dV = 2\int_0^{\pi} a\cos\theta\left[\frac{aht}{2}(1-\cos\theta)d\theta\right]$$
$$= a^2 ht\left[\sin\theta - \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}$$
$$= -\frac{1}{2}\pi a^2 ht$$

Now $$\overline{y}V = \int \overline{y}_{EL} dV: \quad \overline{y}(\pi aht) = \frac{11}{24}\pi ah^2 t$$ or $\overline{y} = \frac{11}{24}h$ ◀

and $$\overline{z}V = \int \overline{z}_{EL} dV: \quad \overline{z}(\pi aht) = -\frac{1}{2}\pi a^2 ht$$ or $\overline{z} = -\frac{1}{2}a$ ◀

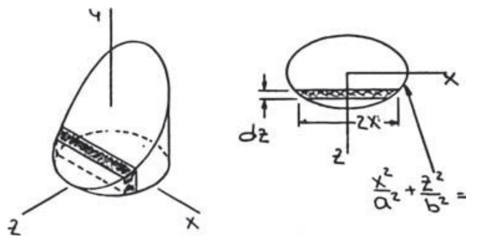
PROBLEM 5.136*



Locate the centroid of the section shown, which was cut from an elliptical cylinder by an oblique plane.

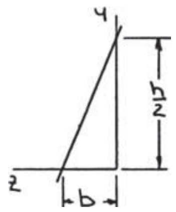
SOLUTION

First note that symmetry implies $x = 0$ ◀



Choose as the element of volume a vertical slice of width zx, thickness dz, and height y. Then

$$dV = 2xydz, \quad \overline{y}_{EL} = \frac{1}{24}, \quad \overline{z}_{EL} = z$$



Now $$x = \frac{a}{b}\sqrt{b^2 - z^2}$$

and $$y = -\frac{h/2}{b}z + \frac{h}{2} = \frac{h}{2b}(b - z)$$

Then $$V = \int_{-b}^{b}\left(2\frac{a}{b}\sqrt{b^2 - z^2}\right)\left[\frac{h}{2b}(b - z)\right]dz$$

Let $$z = b\sin\theta \quad dz = b\cos\theta d\theta$$

Then $$V = \frac{ah}{b^2}\int_{\pi/2}^{\pi/2}(b\cos\theta)[b(1 - \sin\theta)]b\cos\theta\, d\theta$$

$$= abh\int_{-\pi/2}^{\pi/2}(\cos^2\theta - \sin\theta\cos^2\theta)\, d\theta$$

$$= abh\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} + \frac{1}{3}\cos^3\theta\right]_{-\pi/2}^{\pi/2}$$

$$V = \frac{1}{2}\pi abh$$

PROBLEM 5.136* (Continued)

and $$\int \bar{y}_{EL}\,dV = \int_{-b}^{b}\left[\frac{1}{2}\times\frac{h}{2b}(b-z)\right]\left\{\left(2\frac{a}{b}\sqrt{b^2-z^2}\right)\left[\frac{h}{2b}(b-z)\right]dz\right\}$$

$$=\frac{1}{4}\frac{ah^2}{b^3}\int_{-b}^{b}(b-z)^2\sqrt{b^2-z^2}\,dz$$

Let $$z = b\sin\theta \quad dz = b\cos\theta d\theta$$

Then $$\int \bar{y}_{EL}\,dV = \frac{1}{4}\frac{ah^2}{b^3}\int_{-\pi/2}^{\pi/2}[b(1-\sin\theta)]^2(b\cos\theta)\times(b\cos\theta\,d\theta)$$

$$=\frac{1}{4}abh^2\int_{-\pi/2}^{\pi/2}(\cos^2\theta - 2\sin\theta\cos^2\theta + \sin^2\theta\cos^2\theta)\,d\theta$$

Now $$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

So that $$\sin^2\theta\cos^2\theta = \frac{1}{4}(1-\cos^2 2\theta)$$

Then $$\int \bar{y}_{EL}\,dV = \frac{1}{4}abh^2\int_{-\pi/2}^{\pi/2}\left[\cos^2\theta - 2\sin\theta\cos^2\theta + \frac{1}{4}(1-\cos^2 2\theta)\right]d\theta$$

$$=\frac{1}{4}abh^2\left[\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)+\frac{1}{3}\cos^3\theta + \frac{1}{4}\theta - \frac{1}{4}\left(\frac{\theta}{2}+\frac{\sin 4\theta}{8}\right)\right]_{-\pi/2}^{\pi/2}$$

$$=\frac{5}{32}\pi abh^2$$

Also $$\int \bar{z}_{EL}\,dV = \int_{-b}^{b} z\left\{2\frac{a}{b}\sqrt{a^2-z^2}\left[\frac{h}{2b}(b-z)\right]dz\right\}$$

$$=\frac{ah}{b^2}\int_{-b}^{b} z(b-z)\sqrt{b^2-z^2}\,dz$$

Let $$z = b\sin\theta \quad dz = b\cos\theta\,d\theta$$

Then $$\int \bar{z}_{EL}\,dV = \frac{ah}{b^2}\int_{-\pi/2}^{\pi/2}(b\sin\theta)[b(1-\sin\theta)](b\cos\theta)\times(b\cos\theta\,d\theta)$$

$$= ab^2h\int_{-\pi/2}^{\pi/2}(\sin\theta\cos^2\theta - \sin^2\theta\cos^2\theta)d\theta$$

Using $$\sin^2\theta\cos^2\theta = \frac{1}{4}(1-\cos^2 2\theta) \text{ from above}$$

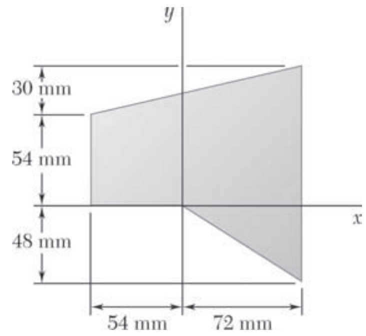
$$\int z_{EL}\,dV = ab^2h\int_{-\pi/2}^{\pi/2}\left[\sin\theta\cos^2\theta - \frac{1}{4}(1-\cos^2 2\theta)\right]d\theta$$

$$= ab^2h\left[-\frac{1}{3}\cos^3\theta - \frac{1}{4}\theta + \frac{1}{4}\left(\frac{\theta}{2}+\frac{\sin 4\theta}{8}\right)\right]_{-\pi/2}^{\pi/2} = -\frac{1}{8}\pi ab^2h$$

PROBLEM 5.136* (Continued)

Now $\quad \bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y}\left(\frac{1}{2}\pi abh\right) = \frac{5}{32}\pi abh^2 \qquad$ or $\bar{y} = \frac{5}{16}h$ ◀

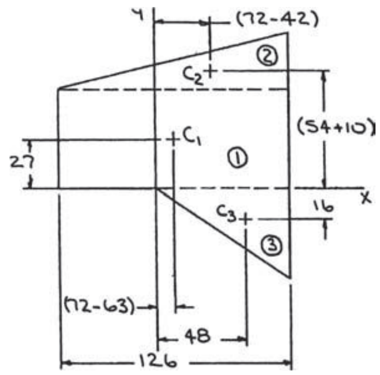
and $\quad \bar{z}V = \int \bar{z}_{EL} dV: \quad \bar{z}\left(\frac{1}{2}\pi abh\right) = -\frac{1}{8}\pi ab^2 h \qquad$ or $\bar{z} = -\frac{1}{4}b$ ◀



PROBLEM 5.137

Locate the centroid of the plane area shown.

SOLUTION



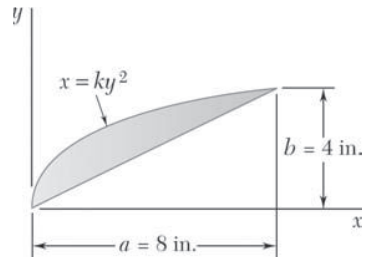
	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$126 \times 54 = 6804$	9	27	61236	183708
2	$\frac{1}{2} \times 126 \times 30 = 1890$	30	64	56700	120960
3	$\frac{1}{2} \times 72 \times 48 = 1728$	48	−16	82944	−27648
Σ	10422			200880	277020

Then $$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(10422 \text{ m}^2) = 200880 \text{ mm}^2 \qquad \text{or } \bar{X} = 19.27 \text{ mm} \blacktriangleleft$$

and $$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

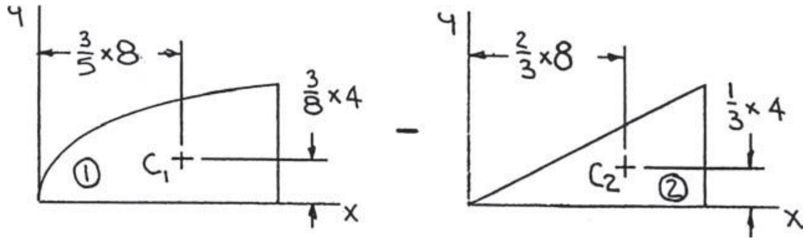
$$\bar{Y}(10422 \text{ m}^2) = 270020 \text{ mm}^3 \qquad \text{or } \bar{Y} = 26.6 \text{ mm} \blacktriangleleft$$



PROBLEM 5.138

Locate the centroid of the plane area shown.

SOLUTION



Dimensions in in.

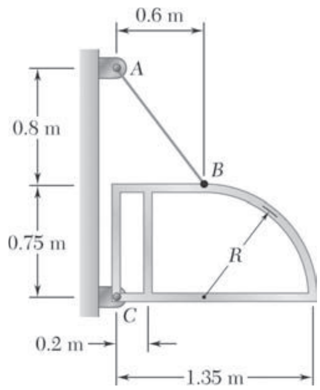
	A, in.^{2}	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in.^{3}	$\bar{y}A$, in.^{3}
1	$\frac{2}{3}(4)(8) = 21.333$	4.8	1.5	102.398	32.000
2	$-\frac{1}{2}(4)(8) = -16.0000$	5.3333	1.33333	85.333	−21.333
Σ	5.3333			17.0650	10.6670

Then $$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(5.3333 \text{ in.}^2) = 17.0650 \text{ in.}^3$$ or $\bar{X} = 3.20$ in. ◀

and $$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}(5.3333 \text{ in.}^2) = 10.6670 \text{ in.}^3$$ or $\bar{Y} = 2.00$ in. ◀

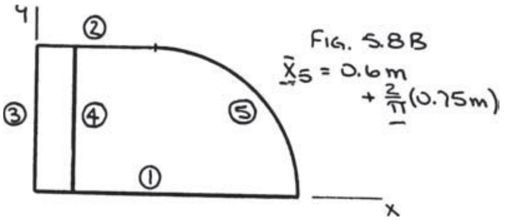


PROBLEM 5.139

The frame for a sign is fabricated from thin, flat steel bar stock of mass per unit length 4.73 kg/m. The frame is supported by a pin at *C* and by a cable *AB*. Determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION

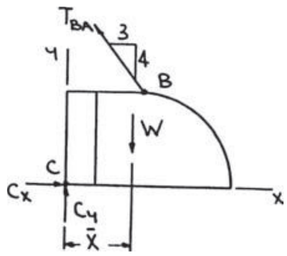
First note that because the frame is fabricates from uniform bar stock, its center of gravity will coincide with the centroid of the corresponding line.



	L, m	$\bar{x}$, m	$\bar{x}L$, m^2
1	1.35	0.675	0.91125
2	0.6	0.3	0.18
3	0.75	0	0
4	0.75	0.2	0.15
5	$\frac{\pi}{2}(0.75) = 1.17810$	1.07746	1.26936
Σ	4.62810		2.5106

Then
$$\bar{X}\Sigma L = \Sigma\bar{x}L$$
$$\bar{X}(4.62810) = 2.5106$$

or
$$\bar{X} = 0.54247 \text{ m}$$



The free-body diagram of the frame is then

Where
$$W = (m'\Sigma L)g$$
$$= 4.73 \text{ kg/m} \times 4.62810 \text{ m} \times 9.81 \text{ m/s}^2$$
$$= 214.75 \text{ N}$$

PROBLEM 5.139 (Continued)

Equilibrium then requires

(*a*) $$\Sigma M_C = 0: \quad (1.55 \text{ m})\left(\frac{3}{5}T_{BA}\right) - (0.54247 \text{ m})(214.75 \text{ N}) = 0$$

or $$T_{BA} = 125.264 \text{ N} \qquad \text{or } T_{BA} = 125.3 \text{ N} \blacktriangleleft$$

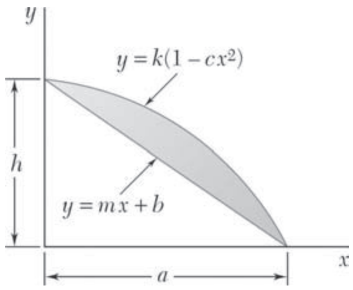
(*b*) $$\Sigma F_x = 0: \quad C_x - \frac{3}{5}(125.264 \text{ N}) = 0$$

or $$\mathbf{C}_x = 75.158 \text{ N} \longrightarrow$$

$$\Sigma F_y = 0: \quad C_y + \frac{4}{5}(125.264 \text{ N}) - (214.75 \text{ N}) = 0$$

or $$\mathbf{C}_y = 114.539 \text{ N} \uparrow$$

Then $$\mathbf{C} = 137.0 \text{ N} \measuredangle 56.7° \blacktriangleleft$$

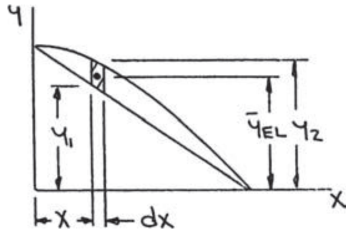


PROBLEM 5.140

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION

By observation $$y_1 = -\frac{h}{a}x + h = h\left(1 - \frac{x}{a}\right)$$



For y_2: at $x = 0$, $y = h$: $h = k(1-0)$ or $k = h$

at $x = a$, $y = 0$: $0 = h(1 - ca^2)$ or $C = \dfrac{1}{a^2}$

Then $$y_2 = h\left(1 - \frac{x^2}{a^2}\right)$$

Now $$dA = (y_2 - y_1)dx$$

$$= h\left[\left(1 - \frac{x^2}{a^2}\right) - \left(1 - \frac{x}{a}\right)\right]dx$$

$$= h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx$$

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}(y_1 - y_2)$$

$$= \frac{h}{2}\left[\left(1 - \frac{x}{a}\right) + \left(1 - \frac{x^2}{a^2}\right)\right]$$

$$= \frac{h}{2}\left(2 - \frac{x}{a} - \frac{x^2}{a^2}\right)$$

Then $$A = \int dA = \int_0^a h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx = h\left[\frac{x^2}{2a} - \frac{x^3}{3a^2}\right]_0^a$$

$$= \frac{1}{6}ah$$

PROBLEM 5.140 (Continued)

and
$$\int \overline{x}_{EL}\,dA = \int_0^a x\left[h\left(\frac{x}{a}-\frac{x^2}{a^2}\right)dx\right] = h\left[\left(\frac{x^3}{3a}-\frac{x^4}{4a^2}\right)\right]_0^a$$

$$= \frac{1}{12}a^2 h$$

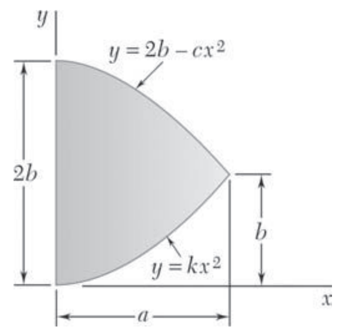
$$\int \overline{y}_{EL}\,dA = \int_0^a \frac{h}{2}\left(2-\frac{x}{a}-\frac{x^2}{a^2}\right)\left[h\left(\frac{x}{a}-\frac{x^2}{a^2}\right)dx\right]$$

$$= \frac{h^2}{2}\int_0^a\left(2\frac{x}{a}-3\frac{x^2}{a^2}+\frac{x^4}{a^4}\right)dx$$

$$= \frac{h^2}{2}\left[\frac{x^2}{a}-\frac{x^3}{a^2}+\frac{x^5}{5a^4}\right]_0^a = \frac{1}{10}ah^2$$

$$\overline{x}A = \int \overline{x}_{EL}\,dA: \quad x\left(\frac{1}{6}ah\right) = \frac{1}{12}a^2 h \qquad \overline{x} = \frac{1}{2}a \blacktriangleleft$$

$$\overline{y}A = \int \overline{y}_{EL}\,dA: \quad y\left(\frac{1}{6}ah\right) = \frac{1}{10}a^2 h \qquad \overline{y} = \frac{3}{5}h \blacktriangleleft$$



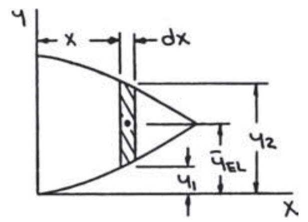
PROBLEM 5.141

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b.

SOLUTION

First note that symmetry implies $\qquad \bar{y} = b$ ◀

at $\qquad x = a, \quad y = b$



$$y_1: \quad b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

Then
$$y_1 = \frac{b}{a^2}x^2$$

$$y_2: \quad b = 2b - ca^2$$

or
$$c = \frac{b}{a^2}$$

Then
$$y_2 = b\left(2 - \frac{x^2}{a^2}\right)$$

Now
$$dA = (y_2 - y_1)dx_2 = \left[b\left(2 - \frac{x^2}{a^2}\right) - \frac{b}{a^2}x^2\right]dx$$

$$= 2b\left(1 - \frac{x^2}{a^2}\right)dx$$

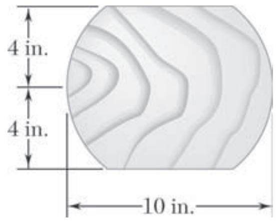
and
$$\bar{x}_{EL} = x$$

Then
$$A = \int dA \int_0^a 2b\left(1 - \frac{x^2}{a^2}\right)dx = 2b\left[x - \frac{x^3}{3a^2}\right]_0^a = \frac{4}{3}ab$$

and
$$\int \bar{x}_{EL}\,dA = \int_0^a x\left[2b\left(1 - \frac{x^2}{a^2}\right)dx\right] = 2b\left[\frac{x^2}{2} - \frac{x^4}{4a^2}\right]_0^a = \frac{1}{2}a^2b$$

$$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{4}{3}ab\right) = \frac{1}{2}a^2b$$

$\bar{x} = \frac{3}{8}a$ ◀



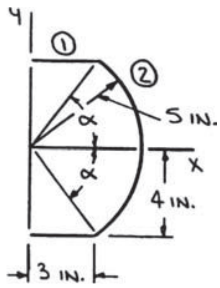
PROBLEM 5.142

Knowing that two equal caps have been removed from a 10-in.-diameter wooden sphere, determine the total surface area of the remaining portion.

SOLUTION

The surface area can be generated by rotating the line shown about the *y* axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = 2\pi \overline{X} L = 2\pi \Sigma \overline{x} L$$
$$= 2\pi(2\overline{x}_1 L_1 + \overline{x}_2 L_2)$$



Now $\tan\alpha = \dfrac{4}{3}$

or $\alpha = 53.130°$

Then $\overline{x}_2 = \dfrac{5 \text{ in.} \times \sin 53.130°}{53.130° \times \frac{\pi}{180°}}$

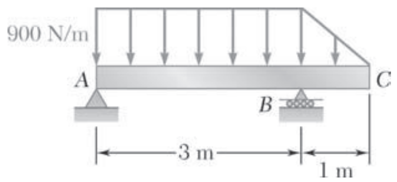
$= 4.3136$ in.

and $L_2 = 2\left(53.130° \times \dfrac{\pi}{180°}\right)(5 \text{ in.})$

$= 9.2729$ in.

$$A = 2\pi\left[2\left(\frac{3}{2}\text{in.}\right)(3 \text{ in.}) + (4.3136 \text{ in.})(9.2729 \text{ in.})\right]$$

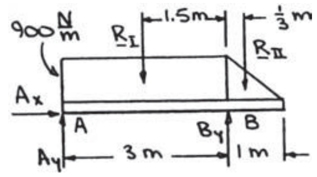
or $A = 308 \text{ in.}^2$ ◀



PROBLEM 5.143

Determine the reactions at the beam supports for the given loading.

SOLUTION



$$R_{\text{I}} = (3\text{ m})(900\text{ N/m}) = 2700\text{ N}$$

$$R_{\text{II}} = \frac{1}{2}(1\text{ m})(900\text{ N/m}) = 450\text{ N}$$

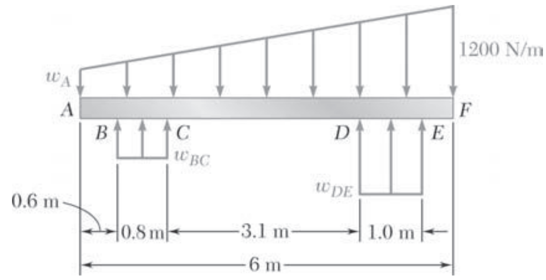
Now $\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$$+\circlearrowleft \Sigma M_B = 0: \quad -(3\text{ m})A_y + (1.5\text{ m})(2700\text{ N}) - \left(\frac{1}{3}\text{m}\right)(450\text{ N}) = 0$$

or $A_y = 1300\text{ N}$ $\quad$ $\mathbf{A} = 1300\text{ N} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 1300\text{ N} - 2700\text{ N} + B_y - 450\text{ N} = 0$$

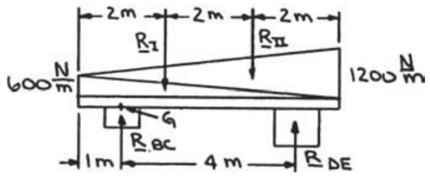
or $B_y = 1850\text{ N}$ $\quad$ $\mathbf{B} = 1850\text{ N} \uparrow$ ◀



PROBLEM 5.144

A beam is subjected to a linearly distributed downward load and rests on two wide supports *BC* and *DE*, which exert uniformly distributed upward loads as shown. Determine the values of w_{BC} and w_{DE} corresponding to equilibrium when $w_A = 600$ N/m.

SOLUTION



We have $$R_{\mathrm{I}} = \frac{1}{2}(6 \text{ m})(600 \text{ N/m}) = 1800 \text{ N}$$

$$R_{\mathrm{II}} = \frac{1}{2}(6 \text{ m})(1200 \text{ N/m}) = 3600 \text{ N}$$

$$R_{BC} = (0.8 \text{ m})(W_{BC} \text{ N/m}) = (0.8\,W_{BC})\text{N}$$

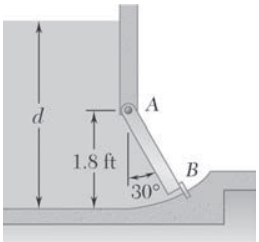
$$R_{DE} = (1.0 \text{ m})(W_{DE} \text{ N/m}) = (W_{DE}) \text{ N}$$

Then $$+\circlearrowleft \Sigma M_G = 0: \quad -(1 \text{ m})(1800 \text{ N}) - (3 \text{ m})(3600 \text{ N}) + (4 \text{ m})(W_{DE} \text{ N}) = 0$$

or $$W_{DE} = 3150 \text{ N/m} \blacktriangleleft$$

and $$+\uparrow \Sigma F_y = 0: \quad (0.8W_{BC}) \text{ N} - 1800 \text{ N} - 3600 \text{ N} + 3150 \text{ N} = 0$$

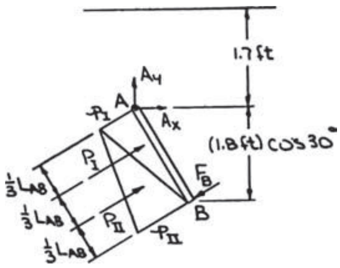
or $$W_{BC} = 2812.5 \text{ N/m} \qquad W_{BC} = 2810 \text{ N/m} \blacktriangleleft$$



PROBLEM 5.145

The square gate AB is held in the position shown by hinges along its top edge A and by a shear pin at B. For a depth of water $d = 3.5$ ft, determine the force exerted on the gate by the shear pin.

SOLUTION



First consider the force of the water on the gate. We have

$$P = \frac{1}{2}Ap$$

$$= \frac{1}{2}A(\gamma h)$$

Then $$P_I = \frac{1}{2}(1.8\text{ ft})^2(62.4\text{ lb/ft}^3)(1.7\text{ ft})$$

$$= 171.850\text{ lb}$$

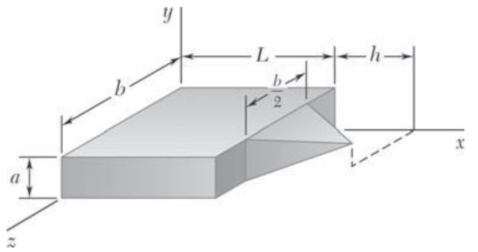
$$P_{II} = \frac{1}{2}(1.8\text{ ft})^2(62.4\text{ lb/ft}^3)\times(1.7 + 1.8\cos 30°)\text{ ft}$$

$$= 329.43\text{ lb}$$

Now $$\Sigma M_A = 0: \quad \left(\frac{1}{3}L_{AB}\right)P_I + \left(\frac{2}{3}L_{AB}\right)P_{II} - L_{AB}F_B = 0$$

or $$\frac{1}{3}(171.850\text{ lb}) + \frac{2}{3}(329.43\text{ lb}) - F_B = 0$$

or $F_B = 276.90$ lb $\qquad \mathbf{F}_B = 277$ lb $\measuredangle$ 30.0° ◀



PROBLEM 5.146

Consider the composite body shown. Determine (*a*) the value of $\bar{x}$ when $h = L/2$, (*b*) the ratio h/L for which $\bar{x} = L$.

SOLUTION

	V	$\bar{x}$	$\bar{x}V$
Rectangular prism	Lab	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
Pyramid	$\frac{1}{3}a\left(\frac{b}{2}\right)h$	$L+\frac{1}{4}h$	$\frac{1}{6}abh\left(L+\frac{1}{4}h\right)$

Then
$$\Sigma V = ab\left(L+\frac{1}{6}h\right)$$

$$\Sigma\bar{x}V = \frac{1}{6}ab\left[3L^2 + h\left(L+\frac{1}{4}h\right)\right]$$

Now
$$\bar{X}\Sigma V = \Sigma\bar{x}V$$

so that
$$\bar{X}\left[ab\left(L+\frac{1}{6}h\right)\right] = \frac{1}{6}ab\left(3L^2 + hL + \frac{1}{4}h^2\right)$$

or
$$\bar{X}\left(1+\frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right) \tag{1}$$

(*a*) $\bar{X} = ?$ when $h = \frac{1}{2}L$

Substituting $\frac{h}{L} = \frac{1}{2}$ into Eq. (1)

$$\bar{X}\left[1+\frac{1}{6}\left(\frac{1}{2}\right)\right] = \frac{1}{6}L\left[3+\left(\frac{1}{2}\right)+\frac{1}{4}\left(\frac{1}{2}\right)^2\right]$$

or
$$\bar{X} = \frac{57}{104}L$$

$\bar{X} = 0.548L$ ◀

PROBLEM 5.146 (Continued)

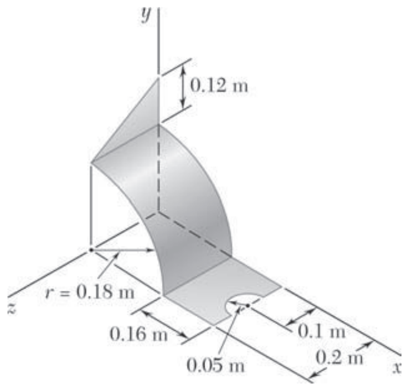
(*b*) $\dfrac{h}{L} = ?$ when $\overline{X} = L$

Substituting into Eq. (1)
$$L\left(1+\frac{1}{6}\frac{h}{L}\right)=\frac{1}{6}L\left(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right)$$

or
$$1+\frac{1}{6}\frac{h}{L}=\frac{1}{2}+\frac{1}{6}\frac{h}{L}+\frac{1}{24}\frac{h^2}{L^2}$$

or
$$\frac{h^2}{L^2}=12$$

$\dfrac{h}{L} = 2\sqrt{3}$ ◀



PROBLEM 5.147

Locate the center of gravity of the sheet-metal form shown.

SOLUTION

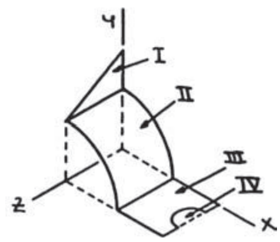
First assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area.

$$\bar{y}_{\mathrm{I}} = 0.18 + \frac{1}{3}(0.12) = 0.22 \text{ m}$$

$$\bar{z}_{\mathrm{I}} = \frac{1}{3}(0.2 \text{ m})$$

$$\bar{x}_{\mathrm{II}} = \bar{y}_{\mathrm{II}} = \frac{2 \times 0.18}{\pi} = \frac{0.36}{\pi} \text{ m}$$

$$\bar{x}_{\mathrm{IV}} = 0.34 - \frac{4 \times 0.05}{3\pi}$$
$$= 0.31878 \text{ m}$$



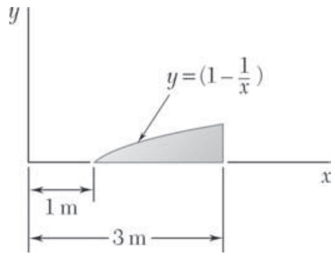
	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3	$\bar{z}A$, m^3
I	$\frac{1}{2}(0.2)(0.12) = 0.012$	0	0.22	$\frac{0.2}{3}$	0	0.00264	0.0008
II	$\frac{\pi}{2}(0.18)(0.2) = 0.018\pi$	$\frac{0.36}{\pi}$	$\frac{0.36}{\pi}$	0.1	0.00648	0.00648	0.005655
III	$(0.16)(0.2) = 0.032$	0.26	0	0.1	0.00832	0	0.0032
IV	$-\frac{\pi}{2}(0.05)^2 = -0.00125\pi$	0.31878	0	0.1	–0.001258	0	–0.000393
Σ	0.096622				0.013542	0.00912	0.009262

PROBLEM 5.147 (Continued)

We have $\bar{X}\Sigma V = \Sigma \bar{x}V: \quad \bar{X}(0.096622 \text{ m}^2) = 0.013542 \text{ m}^3$ or $\bar{X} = 0.1402 \text{ m}$ ◀

$\bar{Y}\Sigma V = \Sigma \bar{y}V: \quad \bar{Y}(0.096622 \text{ m}^2) = 0.00912 \text{ m}^3$ or $\bar{Y} = 0.0944 \text{ m}$ ◀

$\bar{Z}\Sigma V = \Sigma \bar{z}V: \quad \bar{Z}(0.096622 \text{ m}^2) = 0.009262 \text{ m}^3$ or $\bar{Z} = 0.0959 \text{ m}$ ◀



PROBLEM 5.148

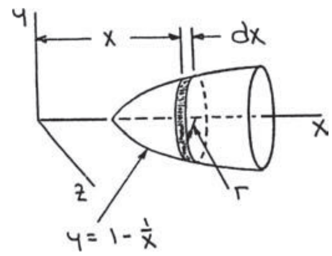
Locate the centroid of the volume obtained by rotating the shaded area about the x axis.

SOLUTION

First, note that symmetry implies $\overline{y} = 0$ ◀

$\overline{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx.



Then $$dV = \pi r^2 dx, \quad \overline{x}_{EL} = x$$

Now $r = 1 - \dfrac{1}{x}$ so that $$dV = \pi\left(1 - \frac{1}{x}\right)^2 dx$$
$$= \pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx$$

Then $$V = \int_1^3 \pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx = \pi\left[x - 2\ln x - \frac{1}{x}\right]_1^3$$
$$= \pi\left[\left(3 - 2\ln 3 - \frac{1}{3}\right) - \left(1 - 2\ln 1 - \frac{1}{1}\right)\right]$$
$$= (0.46944\pi)\ \text{m}^3$$

and $$\int \overline{x}_{EL} dV = \int_1^3 x\left[\pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx\right] = \pi\left[\frac{x^2}{2} - 2x + \ln x\right]_1^3$$
$$= \pi\left\{\left[\frac{3^2}{2} - 2(3) + \ln 3\right] - \left[\frac{1^3}{2} - 2(1) + \ln 1\right]\right\}$$
$$= (1.09861\pi)\ \text{m}$$

Now $$\overline{x}V = \int \overline{x}_{EL} dV: \quad \overline{x}(0.46944\pi\ \text{m}^3) = 1.09861\pi\ \text{m}^4$$ or $\overline{x} = 2.34$ m ◀