

# INTEGRALES.

## 1.- Formas básicas.

- 1)  $\int 0 dx = c$
- 2)  $\int dx = x + c$
- 3)  $\int du = u + c$
- 4)  $\int a dv = a \int dv$
- 5)  $\int a du = au + c$
- 6)  $\int a f(x) dx = a \int f(x) dx$
- 7)  $\int (du + dv - dw) = u + v - w + c$
- 8)  $\int [f(x) + g(x) - h(x) + \dots] dx = \int f(x) dx + \int g(x) dx - \int h(x) dx + \dots$
- 9)  $\int x^n dx = \frac{1}{n+1} x^{n+1} + c \quad (n \neq -1)$
- 10)  $\int u^n du = \frac{u^{n+1}}{n+1} + c \quad (n \neq -1)$
- 11)  $\int \frac{du}{u} = \int u^{-1} du = \ln u + c$
- 12)  $\int u dv = u \int dv - \int v du = uv - \int v du$

## 2.- Formas que contienen seno y coseno.

- 13)  $\int \sin x dx = -\cos x + c$
- 14)  $\int \cos x dx = \sin x + c$
- 15)  $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
- 16)  $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
- 17)  $\int \sin^2(ax) dx = \frac{x}{2} - \frac{1}{4a} \sin(2ax) + c$
- 18)  $\int \sin^2(ax) dx = \frac{1}{2} \left[ x - \frac{1}{a} \sin(ax) \cos(ax) \right] + c$
- 19)  $\int \cos^2(ax) dx = \frac{x}{2} + \frac{1}{4a} \sin(2ax) + c$
- 20)  $\int \cos^2(ax) dx = \frac{1}{2} \left[ x + \frac{1}{a} \sin(ax) \cos(ax) \right] + c$
- 21)  $\int \sin^3(ax) dx = -\frac{1}{3a} \cos(ax) [\sin^2(ax) + 2] + c$
- 22)  $\int \cos^3(ax) dx = \frac{1}{3a} \sin(ax) [\cos^2(ax) + 2] + c$
- 23)  $\int \sin^4(ax) dx = \frac{3x}{8} - \frac{1}{4a} \sin(2ax) + \frac{1}{32a} \sin(4ax) + c$
- 24)  $\int \cos^4(ax) dx = \frac{3x}{8} + \frac{1}{4a} \sin(2ax) + \frac{1}{32a} \sin(4ax) + c$
- 25)  $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
- 26)  $\int \sin^n(ax) \cos(ax) dx = \frac{\sin^{n+1}(ax)}{a(n+1)} + c \quad n \neq -1$

- 27)  $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx \quad n \neq 0$
- 28)  $\int \cos^n(ax) \sin(ax) dx = -\frac{\cos^{n+1}(ax)}{a(n+1)} + c \quad n \neq 0$
- 29)  $\int \sin(ax) \cos(ax) dx = \frac{1}{2a} \sin^2(ax) + c = -\frac{1}{2a} \cos^2(ax) + c$
- 30)  $\int \sin^2(ax) \cos^2(ax) dx = \frac{x}{8} - \frac{1}{32a} \sin(4ax) + c$
- 31)  $\int \sin^m x \cos^n x dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x dx$
- 32)  $\int \sin^m(ax) \cos^n(ax) dx = \frac{\sin^{m+1}(ax) \cos^{n-1}(ax)}{a(m+n)} + \frac{n-1}{a(m+n)} \int \sin^m(ax) \cos^{n-2}(ax) dx$
- 33)  $\int x \sin x dx = \sin x - x \cos x + c$
- 34)  $\int x \cos x dx = \cos x + x \sin x + c$
- 35)  $\int x \sin(ax) dx = \frac{1}{a^2} \sin(ax) - \frac{x}{a} \cos(ax) + c$
- 36)  $\int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax) + c$
- 37)  $\int x^2 \sin(ax) dx = \frac{2x}{a^2} \sin(ax) + \frac{2-a^2 x^2}{a^3} \cos(ax) + c$
- 38)  $\int x^2 \cos(ax) dx = \frac{2x}{a^2} \cos(ax) + \frac{a^2 x^2 - 2}{a^3} \sin(ax) + c$
- 39)  $\int x^3 \sin(ax) dx = \frac{3a^2 x^2 - 6}{a^4} \sin(ax) + \frac{6x - a^2 x^3}{a^3} \cos(ax) + c$
- 40)  $\int x^3 \cos(ax) dx = \frac{3a^2 x^2 - 6}{a^4} \cos(ax) + \frac{a^2 x^3 - 6x}{a^3} \sin(ax) + c$
- 41)  $\int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$
- 42)  $\int x^n \sin(ax) dx = -\frac{1}{a} x^n \cos(ax) + \frac{n}{a} \int x^{n-1} \cos(ax) dx$
- 43)  $\int x^n \cos x dx = x^n \sin x - n \int x^{n-1} \sin x dx$
- 44)  $\int x^n \cos(ax) dx = \frac{1}{a} x^n \sin(ax) - \frac{n}{a} \int x^{n-1} \sin(ax) dx$
- 45)  $\int x \sin^2(ax) dx = \frac{x^2}{4} - \frac{x}{4a} \sin(2ax) - \frac{1}{8a^2} \cos(2ax) + c$
- 46)  $\int x \cos^2(ax) dx = \frac{x^2}{4} + \frac{x}{4a} \sin(2ax) + \frac{1}{8a^2} \cos(2ax) + c$
- 47)  $\int x^2 \sin^2(ax) dx = \frac{x^3}{6} - \left( \frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin(2ax) - \frac{x}{4a^2} \cos(2ax) + c$
- 48)  $\int x^2 \cos^2(ax) dx = \frac{x^3}{6} + \left( \frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin(2ax) + \frac{x}{4a^2} \cos(2ax) + c$
- 49)  $\int x \sin^3(ax) dx = \frac{x}{12a} \cos(3ax) - \frac{1}{36a^2} \sin(3ax) - \frac{3x}{4a} \cos(ax) + \frac{3}{4a^2} \sin(ax)$
- 50)  $\int x \cos^3(ax) dx = \frac{x}{12a} \sin(3ax) + \frac{1}{36a^2} \cos(3ax) + \frac{3x}{4a} \sin(ax) + \frac{3}{4a^2} \cos(ax)$
- 51)  $\int \sin(ax) \sin(bx) dx = -\frac{\sin(a+b)x}{2(a+b)} + \frac{\sin(a-b)x}{2(a-b)} + c \quad a^2 \neq b^2$
- 52)  $\int \sin(ax) \cos(bx) dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + c \quad a^2 \neq b^2$

$$53) \int \cos(ax) \cos(bx) dx = \frac{\sin(a+b)x}{2(a+b)} + \frac{\sin(a-b)x}{2(a-b)} + c \quad a^2 \neq b^2$$

## 3.- Formas que contienen tangente, secante y cosecante.

- 54)  $\int \tan x dx = -\ln |\cos x| + c$
- 55)  $\int \tan x dx = \ln |\sec x| + c$
- 56)  $\int \tan(ax) dx = -\frac{1}{a} \ln |\cos(ax)| + c$
- 57)  $\int \tan(ax) dx = \frac{1}{a} \ln |\sec(ax)| + c$
- 58)  $\int \cot x dx = -\ln |\csc x| + c$
- 59)  $\int \cot x dx = \ln |\sin x| + c$
- 60)  $\int \cot(ax) dx = -\frac{1}{a} \ln |\csc(ax)| + c$
- 61)  $\int \cot(ax) dx = \frac{1}{a} \ln |\sin(ax)| + c$
- 62)  $\int \sec x dx = \ln |\sec x + \tan x| + c$
- 63)  $\int \sec x dx = \ln \left[ \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \right] + c$
- 64)  $\int \sec(ax) dx = \frac{1}{a} \ln |\sec(ax) + \tan(ax)| + c$
- 65)  $\int \sec(ax) dx = \frac{1}{a} \ln \left[ \tan \left( \frac{\pi}{4} + \frac{ax}{2} \right) \right] + c$
- 66)  $\int \csc x dx = \ln |\csc x - \cot x| + c$
- 67)  $\int \csc x dx = -\ln |\csc x + \cot x| + c$
- 68)  $\int \csc x dx = \ln \left[ \tan \left( \frac{x}{2} \right) \right] + c$
- 69)  $\int \csc(ax) dx = \frac{1}{a} \ln |\csc(ax) - \cot(ax)| + c$
- 70)  $\int \csc(ax) dx = \frac{1}{a} \ln \left[ \tan \left( \frac{ax}{2} \right) \right] + c$
- 71)  $\int \sec x \tan x dx = \sec x + c$
- 72)  $\int \sec(ax) \tan(ax) dx = \frac{1}{a} \sec(ax) + c$
- 73)  $\int \csc x \cot x dx = -\csc x + c$
- 74)  $\int \csc(ax) \cot(ax) dx = -\frac{1}{a} \csc(ax) + c$
- 75)  $\int \sec^2 x dx = \tan x + c$
- 76)  $\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
- 77)  $\int \csc^2 x dx = -\cot x + c$
- 78)  $\int \csc^2(ax) dx = -\frac{1}{a} \cot(ax) + c$
- 79)  $\int \sec(ax) \csc(ax) dx = \ln |\csc(2ax) - \cot(2ax)| + c$
- 80)  $\int x \sec(ax) \tan(ax) dx = \frac{1}{a} x \sec(ax) - \frac{1}{a} \ln |\sec(ax) + \tan(ax)| + c$

$$81) \int x \csc(ax) \cot(ax) dx = -\frac{1}{a} x \csc(ax) + \frac{1}{a} \ln[\csc(ax) - \cot(ax)] + c$$

$$82) \int \tan^2(ax) dx = \frac{1}{a} \tan(ax) - x + c$$

$$83) \int \cot^2(ax) dx = -\frac{1}{a} \cot(ax) - x + c$$

$$84) \int \tan^3(ax) dx = \frac{1}{2a} \tan^2(ax) + \frac{1}{a} \ln|\cos(ax)| + c$$

$$85) \int \cot^3(ax) dx = -\frac{1}{2a} \cot^2(ax) - \frac{1}{a} \ln|\sin(ax)| + c$$

$$86) \int \tan^4(ax) dx = \frac{1}{3a} \tan^3(ax) - \frac{1}{a} \tan(ax) + x + c$$

$$87) \int \cot^4(ax) dx = -\frac{1}{3a} \cot^3(ax) + \frac{1}{a} \cot(ax) + x + c$$

$$88) \int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx \quad n \neq 1$$

$$89) \int \cot^n x dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx \quad n \neq 1$$

$$90) \int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + c$$

$$91) \int \sec^3(ax) dx = \frac{1}{2a} \sec(ax) \tan(ax) + \frac{1}{2a} \ln|\sec(ax) + \tan(ax)| + c$$

$$92) \int \csc^3 x dx = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln|\csc x - \cot x| + c$$

$$93) \int \csc^3(ax) dx = -\frac{1}{2a} \csc(ax) \cot(ax) + \frac{1}{2a} \ln|\csc(ax) - \cot(ax)| + c$$

$$94) \int \sec^5 x dx = \frac{1}{4} \sec^3 x \tan x + \frac{3}{8} \sec x \tan x + \frac{3}{8} \ln|\sec x + \tan x| + c$$

$$95) \int \csc^5 x dx = -\frac{1}{4} \csc^3 x \cot x - \frac{3}{8} \csc x \cot x + \frac{3}{8} \ln|\csc x - \cot x| + c$$

$$96) \int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx \quad n \neq 1$$

$$97) \int \csc^n x dx = -\frac{\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x dx \quad n \neq 1$$

$$98) \int x \sec^2 x dx = x \tan x - \ln|\sec x| + c$$

$$99) \int x \csc^2 x dx = -x \cot x - \ln|\csc x| + c$$

#### 4.- Formas que contienen funciones hiperbólicas.

$$100) \int \sinh(ax) dx = \frac{1}{a} \cosh(ax) + c$$

$$101) \int \cosh(ax) dx = \frac{1}{a} \sinh(ax) + c$$

$$102) \int \tanh(ax) dx = \frac{1}{a} \ln|\cosh(ax)| + c$$

$$103) \int \tanh(ax) dx = -\frac{1}{a} \ln|\operatorname{sech}(ax)| + c$$

$$104) \int \coth(ax) dx = \frac{1}{a} \ln|\sinh(ax)| + c$$

$$105) \int \coth(ax) dx = -\frac{1}{a} \ln|\operatorname{csch}(ax)| + c$$

$$106) \int \operatorname{sech}(ax) dx = \frac{1}{a} \tan^{-1}[\sinh(ax)] + c$$

$$107) \int \operatorname{sech}(ax) dx = \frac{2}{a} \tan^{-1}(e^{ax}) + c$$

$$108) \int \operatorname{csch}(ax) dx = \frac{1}{2a} \ln \left| \frac{\cosh(ax) - 1}{\cosh(ax) + 1} \right| + c$$

$$109) \int \operatorname{csch}(ax) dx = \frac{1}{a} \ln|\tanh(\frac{1}{2}ax)| + c$$

$$110) \int \operatorname{sech}(ax) \tanh(ax) dx = -\frac{1}{a} \operatorname{sech}(ax) + c$$

$$111) \int \operatorname{csch}(ax) \coth(ax) dx = -\frac{1}{a} \operatorname{csch}(ax) + c$$

$$112) \int \operatorname{sech}^2(ax) dx = \frac{1}{a} \tanh(ax) + c$$

$$113) \int \operatorname{csch}^2(ax) dx = -\frac{1}{a} \coth(ax) + c$$

$$114) \int \operatorname{senh}^2(ax) dx = \frac{1}{4a} \operatorname{senh}(2ax) - \frac{x}{2} + c$$

$$115) \int \operatorname{senh}^2(ax) dx = \frac{1}{2} \left[ \frac{1}{a} \operatorname{senh}(ax) \cosh(ax) - x \right] + c$$

$$116) \int \cosh^2(ax) dx = \frac{1}{4a} \operatorname{senh}(2ax) + \frac{x}{2} + c$$

$$117) \int \cosh^2(ax) dx = \frac{1}{2} \left[ \frac{1}{a} \operatorname{senh}(ax) \cosh(ax) + x \right] + c$$

$$118) \int \tanh^2(ax) dx = x - \frac{1}{a} \tanh(ax)$$

$$119) \int \coth^2(ax) dx = x - \frac{1}{a} \coth(ax)$$

$$120) \int \operatorname{senh}^n x dx = \frac{\operatorname{senh}^{n-1} x \cosh x}{n} - \frac{n-1}{n} \int \operatorname{senh}^{n-2} x dx \quad n \neq 0$$

$$121) \int \cosh^n x dx = \frac{\cosh^{n-1} x \operatorname{senh} x}{n} + \frac{n-1}{n} \int \cosh^{n-2} x dx \quad n \neq 0$$

$$122) \int \tanh^n x dx = -\frac{\tanh^{n-1} x}{n-1} + \int \tanh^{n-2} x dx \quad n \neq 1$$

$$123) \int \operatorname{csch}^n x dx = -\frac{\operatorname{csch}^{n-2} x \coth x}{n-1} - \frac{n-2}{n-1} \int \operatorname{csch}^{n-2} x dx \quad n \neq 1$$

$$124) \int \operatorname{sech}^n x dx = \frac{\operatorname{sech}^{n-2} x \tanh x}{n-1} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} x dx \quad n \neq 1$$

$$125) \int \coth^n x dx = -\frac{\coth^{n-1} x}{n-1} + \int \coth^{n-2} x dx \quad n \neq 1$$

$$126) \int x \operatorname{senh}(ax) dx = \frac{x}{a} \cosh(ax) - \frac{1}{a^2} \operatorname{senh}(ax) + c$$

$$127) \int x \cosh(ax) dx = \frac{x}{a} \operatorname{senh}(ax) - \frac{1}{a^2} \cosh(ax) + c$$

$$128) \int e^{ax} \operatorname{senh}(bx) dx = \frac{e^{ax}}{a^2 - b^2} [a \operatorname{senh}(bx) - b \cosh(bx)] + c$$

$$129) \int e^{ax} \cosh(bx) dx = \frac{e^{ax}}{a^2 - b^2} [a \cosh(bx) + b \operatorname{senh}(bx)] + c$$

#### 5.- Formas que contienen logaritmos.

$$130) \int \ln x dx = x \ln x - x + c$$

$$131) \int \ln x dx = x(\ln x - 1) + c$$

$$132) \int x \ln x dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$$

$$133) \int x \ln x dx = \frac{x^2}{4} (2 \ln x - 1) + c$$

$$134) \int x^2 \ln x dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + c$$

$$135) \int x^2 \ln x dx = \frac{x^3}{9} (3 \ln x - 1) + c$$

$$136) \int x^n \ln(ax) dx = \frac{x^{n+1}}{n+1} \ln(ax) - \frac{x^{n+1}}{(n+1)^2} + c$$

$$137) \int x^n \ln(ax) dx = \frac{x^{n+1}}{(n+1)^2} [(n+1) \ln(ax) - 1] + c$$

$$138) \int \frac{1}{x \ln x} dx = \ln(\ln x) + c$$

$$139) \int (\ln x)^2 dx = x(\ln x)^2 - 2x \ln x + 2x + c$$

$$140) \int (\ln x)^2 dx = x[(\ln x)^2 - 2 \ln x + 2] + c$$

$$141) \int x(\ln x)^2 dx = \frac{1}{2} x^2 (\ln x)^2 - \frac{1}{2} x^2 \ln x + \frac{1}{4} x^2 + c$$

$$142) \int x(\ln x)^2 dx = \frac{1}{4} x^2 [2(\ln x)^2 - 2 \ln x + 1] + c$$

$$143) \int x^2 (\ln x)^2 dx = \frac{1}{3} x^3 (\ln x)^2 - \frac{2}{9} x^3 \ln x + \frac{2}{27} x^3 + c$$

$$144) \int x^2 (\ln x)^2 dx = \frac{2}{27} x^3 [\frac{9}{2} (\ln x)^2 - 3 \ln x + 1] + c$$

$$145) \int (\ln x)^m dx = x(\ln x)^m - m \int (\ln x)^{m-1} dx$$

$$146) \int x^n (\ln x)^m dx = \frac{1}{n+1} x^{n+1} (\ln x)^m - \frac{m}{n+1} \int x^n (\ln x)^{m-1} dx \quad n \neq -1$$

#### 6.- Formas que contienen exponenciales.

$$147) \int a^x dx = \frac{1}{\ln a} a^x + c \quad a > 0$$

$$148) \int a^x \ln a dx = a^x + c \quad a > 0$$

$$149) \int b^{ax} dx = \frac{b^{ax}}{a \ln b} + c \quad b > 0$$

$$150) \int e^x dx = e^x + c$$

$$151) \int e^{-x} dx = -e^{-x} + c$$

$$152) \int x e^x dx = e^x (x - 1) + c$$

$$153) \int x e^{-x^2} dx = -\frac{1}{2} e^{-x^2} + c$$

$$154) \int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$155) \int x e^{ax} dx = \frac{ax - 1}{a^2} e^{ax} + c$$

$$156) \int x^2 e^{ax} dx = \frac{a^2 x^2 - 2ax + 2}{a^3} e^{ax} + c$$

$$157) \int x^n e^{ax} dx = \frac{x^n}{a} e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$158) \int x^n b^{ax} dx = \frac{x^n}{a \ln b} b^{ax} - \frac{n}{a \ln b} \int x^{n-1} b^{ax} dx$$

$$159) \int \frac{e^{ax}}{x^n} dx = -\frac{1}{(n-1)x^{n-1}} e^{ax} + \frac{a}{n-1} \int \frac{e^{ax}}{x^{n-1}} dx$$

$$160) \int \frac{b^{ax}}{x^n} dx = -\frac{1}{(n-1)x^{n-1}} b^{ax} + \frac{a \ln b}{n-1} \int \frac{b^{ax}}{x^{n-1}} dx$$

$$161) \int e^{ax} \ln x dx = \frac{e^{ax} \ln x}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx$$

$$162) \int e^{ax} \operatorname{sen}(bx) dx = \frac{e^{ax}}{a^2 + b^2} [a \operatorname{sen}(bx) - b \cos(bx)] + c$$

$$163) \int e^{ax} \cos(bx) dx = \frac{e^{ax}}{a^2 + b^2} [a \cos(bx) + b \operatorname{sen}(bx)] + c$$

$$164) \int e^{ax} \operatorname{senh}(bx) dx = \frac{e^{ax}}{a^2 - b^2} [a \operatorname{senh}(bx) - b \cosh(bx)] + c$$

$$165) \int e^{ax} \cosh(bx) dx = \frac{e^{ax}}{a^2 - b^2} [a \cosh(bx) - b \operatorname{senh}(bx)] + c$$

### 7.- Formas que contienen funciones trigonométricas inversas.

$$166) \int \operatorname{sen}^{-1} x dx = x \operatorname{sen}^{-1} x + \sqrt{1-x^2}$$

$$167) \int \cos^{-1} x dx = x \cos^{-1} x - \sqrt{1-x^2}$$

$$168) \int \tan^{-1} x dx = x \tan^{-1} x - \ln \sqrt{1+x^2} + c$$

$$169) \int \cot^{-1} x dx = x \cot^{-1} x + \ln \sqrt{1+x^2} + c$$

$$170) \int \sec^{-1} x dx = x \sec^{-1} x - \ln(x + \sqrt{x^2-1}) + c$$

$$171) \int \operatorname{csc}^{-1} x dx = x \operatorname{csc}^{-1} x - \cosh^{-1} x + c$$

$$172) \int \csc^{-1} x dx = x \csc^{-1} x + \ln(x + \sqrt{x^2-1}) + c$$

$$173) \int \csc^{-1} x dx = x \csc^{-1} x + \cosh^{-1} x + c$$

$$174) \int x \operatorname{sen}^{-1} x dx = \frac{1}{4}(2x^2-1) \operatorname{sen}^{-1} x + \frac{1}{4} x \sqrt{1-x^2} + c$$

$$175) \int x \cos^{-1} x dx = \frac{1}{4}(2x^2-1) \cos^{-1} x - \frac{1}{4} x \sqrt{1-x^2} + c$$

$$176) \int x \tan^{-1} x dx = \frac{1}{2}(x^2+1) \tan^{-1} x - \frac{1}{2} x + c$$

$$177) \int x \sec^{-1} x dx = \frac{1}{2} x^2 \sec^{-1} x - \frac{1}{2} \sqrt{x^2-1} + c$$

$$178) \int x \operatorname{csc}^{-1} x dx = \frac{1}{2} x^2 \operatorname{csc}^{-1} x + \frac{1}{2} \sqrt{x^2-1} + c$$

$$179) \int x \cot^{-1} x dx = \frac{1}{2}(x^2-1) \cot^{-1} x + \frac{1}{2} x + c$$

$$180) \int x^n \operatorname{sen}^{-1} x dx = \frac{1}{n+1} \left( x^{n+1} \operatorname{sen}^{-1} x - \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx \right)$$

$$181) \int x^n \cos^{-1} x dx = \frac{1}{n+1} \left( x^{n+1} \cos^{-1} x + \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx \right)$$

$$182) \int x^n \tan^{-1} x dx = \frac{1}{n+1} \left( x^{n+1} \tan^{-1} x - \int \frac{x^{n+1}}{1+x^2} dx \right)$$

### 8.- Formas que contienen funciones hiperbólicas inversas.

$$183) \int \operatorname{senh}^{-1} x dx = x \operatorname{senh}^{-1} x - \sqrt{1+x^2} + c$$

$$184) \int \cosh^{-1} x dx = x \cosh^{-1} x - \sqrt{x^2-1} + c$$

$$185) \int x \operatorname{senh}^{-1} x dx = \frac{1}{4}(2x^2+1) \operatorname{senh}^{-1} x - \frac{1}{4} x \sqrt{1+x^2} + c$$

$$186) \int x \cosh^{-1} x dx = \frac{1}{4}(2x^2-1) \cosh^{-1} x - \frac{1}{4} x \sqrt{x^2-1} + c$$

### 9.- Formas que contienen funciones racionales.

$$187) \int \frac{x+a}{x+b} dx = x + (a-b) \ln(x+b) + c$$

$$188) \int \frac{mx+a}{nx+b} dx = \frac{m}{n} x - \frac{bm-an}{n^2} \ln(nx+b) + c$$

$$189) \int \frac{mx+a}{(nx+b)^2} dx = \frac{m}{n^2} \ln(nx+b) + \frac{bm-an}{n^2(nx+b)} + c$$

$$190) \int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \left( \frac{x+a}{x+b} \right) + c \quad a \neq b$$

$$191) \int \frac{x}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \left[ \frac{(x+b)^b}{(x+a)^a} \right] + c \quad a \neq b$$

$$192) \int \frac{u du}{a+bu} = \frac{1}{b^2} [a+bu - a \ln(a+bu)] + c$$

$$193) \int \frac{u^2 du}{a+bu} = \frac{1}{b^3} \left[ \frac{1}{2}(a+bu)^2 - 2a(a+bu) + a^2 \ln(a+bu) \right] + c$$

$$194) \int \frac{u du}{(a+bu)^2} = \frac{1}{b^2} \left[ \frac{a}{a+bu} + \ln(a+bu) \right] + c$$

$$195) \int \frac{u^2 du}{(a+bu)^2} = \frac{1}{b^2} \left[ a+bu - \frac{a^2}{a+bu} - 2a \ln(a+bu) \right] + c$$

$$196) \int \frac{u du}{(a+bu)^3} = \frac{1}{b^2} \left[ \frac{a}{2(a+bu)^2} - \frac{1}{a+bu} \right] + c$$

$$197) \int \frac{u du}{(a+bu)^n} = \frac{(a+bu)^{1-n}}{b^2} \left[ \frac{a+bu}{2-n} - \frac{a}{1-n} \right] + c$$

$$198) \int \frac{1}{u(a+bu)} du = \frac{1}{a} \ln \left( \frac{u}{a+bu} \right) + c$$

$$199) \int \frac{1}{u(a+bu)^2} du = \frac{1}{a(a+bu)} + \frac{1}{a^2} \ln \left( \frac{u}{a+bu} \right) + c$$

$$200) \int \frac{1}{u^2(a+bu)} du = -\frac{1}{au} + \frac{b}{a^2} \ln \left( \frac{bu+a}{u} \right) + c$$

$$201) \int \frac{1}{u^2(a+bu)^2} du = -\frac{1}{a^2 u} - \frac{b}{a^2(a+bu)} + \frac{2b}{a^3} \ln \left( \frac{a+bu}{u} \right) + c$$

### 10.- Formas que contienen suma y diferencia de cuadrados.

$$202) \int \frac{1}{a^2+b^2u^2} du = \frac{1}{ab} \tan^{-1} \left( \frac{b}{a} u \right) + c$$

$$203) \int \frac{u}{a^2+b^2u^2} du = \frac{1}{2b^2} \ln(a^2+b^2u^2) + c$$

$$204) \int \frac{1}{a^2-b^2u^2} du = \frac{1}{2ab} \ln \left( \frac{a+bu}{a-bu} \right) + c$$

$$205) \int \frac{u}{a^2-b^2u^2} du = -\frac{1}{2b^2} \ln(a^2-b^2u^2) + c$$

$$206) \int \frac{1}{u(a^2 \pm b^2u^2)} du = \frac{1}{2a^2} \ln \left( \frac{u^2}{a^2 \pm b^2u^2} \right) + c$$

$$207) \int \frac{1}{u^2(a^2+b^2u^2)} du = -\frac{1}{a^2u} - \frac{b}{a^3} \tan^{-1} \left( \frac{bu}{a} \right) + c$$

$$208) \int \frac{1}{u^2(a^2-b^2u^2)} du = -\frac{1}{a^2u} - \frac{b}{2a^3} \ln \left( \frac{a-bx}{a+bx} \right) + c$$

$$209) \int \frac{du}{(a^2 \pm u^2)^n} = \frac{1}{2a^2(n-1)} \left[ \frac{u}{(a^2 \pm u^2)^{n-1}} + (2n-3) \int \frac{du}{(a^2 \pm u^2)^{n-1}} \right]$$

$$n \neq 1$$

$$210) \int \frac{du}{(u^2-a^2)^n} = -\frac{1}{2a^2(n-1)} \left[ \frac{u}{(u^2-a^2)^{n-1}} + (2n-3) \int \frac{du}{(u^2-a^2)^{n-1}} \right] \quad n \neq 1$$

$$211) \int (a^2 \pm u^2)^n du = \frac{u(a^2 \pm u^2)^n}{2n+1} + \frac{2ma^2}{2n+1} \int (a^2 \pm u^2)^{n-1} du \quad n \neq -\frac{1}{2}$$

$$212) \int (u^2-a^2)^n du = \frac{u(u^2-a^2)^n}{2n+1} - \frac{2ma^2}{2n+1} \int (u^2-a^2)^{n-1} du \quad n \neq -\frac{1}{2}$$

### 11.- Formas que contienen radicales.

$$213) \int \sqrt{a+bu} du = \frac{2}{3b} (a+bu)^{\frac{3}{2}} + c$$

$$214) \int u \sqrt{a+bu} du = \frac{2}{15b^2} (3bu-2a)(a+bu)^{\frac{3}{2}} + c$$

$$215) \int u^2 \sqrt{a+bu} du = \frac{2}{105b^3} (15b^2u^2-12abu+8a^2)(a+bu)^{\frac{3}{2}} + c$$

$$216) \int u^n \sqrt{a+bu} du = \frac{2u^n(a+bu)^{\frac{3}{2}}}{b(2n+3)} - \frac{2an}{b(2n+3)} \int u^{n-1} \sqrt{a+bu} du$$

$$217) \int \frac{du}{\sqrt{a+bu}} = \frac{2}{b} (a+bu)^{\frac{1}{2}} + c$$

$$218) \int \frac{u du}{\sqrt{a+bu}} = \frac{2}{3b^2} (bu-2a) \sqrt{a+bu} + c$$

$$219) \int \frac{u^2 du}{\sqrt{a+bu}} = \frac{2}{15b^3} (3b^2u^2-4abu+8a^2) \sqrt{a+bu} + c$$

$$220) \int \frac{u^n du}{\sqrt{a+bu}} = \frac{2u^n \sqrt{a+bu}}{b(2n+1)} - \frac{2an}{b(2n+1)} \int \frac{u^{n-1} du}{\sqrt{a+bu}}$$

$$221) \int \frac{du}{u \sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}} \right| + c \quad a > 0$$

$$222) \int \frac{du}{u \sqrt{a+bu}} = \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bu}{-a}} + c \quad a < 0$$

$$223) \int \frac{du}{u^n \sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a+bu}} + c$$

$$224) \int \frac{\sqrt{a+u}}{\sqrt{b-u}} du = -\sqrt{a+u} \sqrt{b-u} + (a+b) \operatorname{sen}^{-1} \sqrt{\frac{a+u}{a+b}} + c$$

$$225) \int \frac{\sqrt{a+bu} du}{u} = 2\sqrt{a+bu} + a \int \frac{du}{u \sqrt{a+bu}}$$

$$226) \int \frac{\sqrt{a+bu} du}{u^n} = -\frac{(a+bu)^{\frac{3}{2}}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)} \int \frac{\sqrt{a+bu} du}{u^n}$$

### 12.- Formas que contienen suma y diferencia de cuadrados con radicales.

$$227) \int \sqrt{u^2 \pm a^2} du = \frac{1}{2} u \sqrt{u^2 \pm a^2} \pm \frac{1}{2} a^2 \ln(u + \sqrt{u^2 \pm a^2}) + c$$

$$228) \int u \sqrt{u^2 \pm a^2} du = \frac{1}{3} (x^2 \pm u^2)^{\frac{3}{2}} + c$$

$$229) \int u^2 \sqrt{u^2 \pm a^2} du = \frac{1}{8} u (2u^2 \mp a^2) \sqrt{u^2 \pm a^2} - \frac{1}{8} a^4 \ln(u + \sqrt{u^2 \pm a^2}) + c$$

$$230) \int u^3 \sqrt{u^2 \pm a^2} du = \frac{1}{5} (u^2 \pm a^2)^{\frac{3}{2}} (u^2 \pm a^2) + c$$

$$231) \int \sqrt{a^2-u^2} du = \frac{1}{2} u \sqrt{a^2-u^2} + \frac{1}{2} a^2 \operatorname{sen}^{-1} \frac{u}{a} + c$$

$$232) \int u \sqrt{a^2 - u^2} du = -\frac{1}{3}(a^2 - u^2)^{\frac{3}{2}} + c$$

$$233) \int u^2 \sqrt{a^2 - u^2} du = \frac{1}{8}u(2u^2 - a^2)\sqrt{a^2 - u^2} + \frac{1}{8}a^4 \operatorname{sen}^{-1} \frac{u}{a} + c$$

$$234) \int u^3 \sqrt{a^2 - u^2} du = -\frac{1}{5}(u^2 + \frac{2}{3}a^2)(a^2 - u^2)^{\frac{3}{2}} + c$$

$$235) \int (u^2 \pm a^2)^{\frac{3}{2}} du = \frac{1}{8}u(2u^2 + 5a^2)\sqrt{u^2 \pm a^2} + \frac{3}{8}a^4 \ln(u + \sqrt{u^2 \pm a^2}) + c$$

$$236) \int (a^2 - u^2)^{\frac{3}{2}} du = -\frac{1}{8}u(2u^2 - 5a^2)\sqrt{a^2 - u^2} + \frac{3}{8}a^4 \operatorname{sen}^{-1} \frac{u}{a} + c$$

$$237) \int \frac{u du}{\sqrt{a^2 \pm u^2}} = \pm \sqrt{a^2 \pm u^2} + c$$

$$238) \int \frac{u du}{\sqrt{u^2 \pm a^2}} = \sqrt{u^2 \pm a^2} + c$$

$$239) \int \frac{u du}{\sqrt{(u^2 \pm a^2)^3}} = -\frac{1}{\sqrt{u^2 \pm a^2}} + c$$

$$240) \int \frac{u^2 du}{\sqrt{u^2 \pm a^2}} = \frac{1}{2}u\sqrt{u^2 \pm a^2} \mp \frac{1}{2}a^2 \ln(u + \sqrt{u^2 \pm a^2}) + c$$

$$241) \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{1}{2}u\sqrt{a^2 - u^2} + \frac{1}{2}a^2 \operatorname{sen}^{-1} \frac{u}{a} + c$$

$$242) \int \frac{u^2 du}{\sqrt{(u^2 + a^2)^3}} = \ln(u + \sqrt{u^2 + a^2}) - \frac{u}{\sqrt{u^2 + a^2}} + c$$

$$243) \int \frac{du}{\sqrt{(u^2 \pm a^2)^3}} = \pm \frac{u}{a^2 \sqrt{u^2 \pm a^2}} + c$$

$$244) \int \frac{du}{\sqrt{(a^2 - u^2)^3}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + c$$

$$245) \int \frac{du}{u^2 \sqrt{u^2 \pm a^2}} = \mp \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + c$$

$$246) \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + c$$

$$247) \int \frac{\sqrt{a^2 \pm u^2}}{u} du = \sqrt{a^2 \pm u^2} - a \ln \left( \frac{a + \sqrt{a^2 \pm u^2}}{u} \right) + c$$

$$248) \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \operatorname{sec}^{-1} \frac{u}{a} + c$$

$$249) \int \frac{\sqrt{u^2 \pm a^2}}{u^2} du = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln(u + \sqrt{u^2 \pm a^2}) + c$$

$$250) \int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{\sqrt{a^2 - u^2}}{u} - \operatorname{sen}^{-1} \frac{u}{a} + c$$

### 13.- Formas que conducen a funciones trigonométricas inversas.

$$251) \int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1} \frac{u}{a} + c$$

$$252) \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{sec}^{-1} \frac{u}{a} + c$$

$$253) \int \frac{1}{u^2 + a^2} du = \frac{1}{a} \operatorname{tan}^{-1} \frac{u}{a} + c$$

### 14.- Formas que conducen a funciones hiperbólicas inversas.

$$254) \int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2}) + c = \operatorname{senh}^{-1} \frac{u}{a} + c$$

$$255) \int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2}) + c = \operatorname{cosh}^{-1} \frac{u}{a} + c$$

$$256) \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 - u^2}}{u} \right) + c = -\frac{1}{a} \operatorname{sech}^{-1} \frac{u}{a} + c$$

$$257) \int \frac{du}{u \sqrt{u^2 + a^2}} = -\frac{1}{a} \ln \left( \frac{a + \sqrt{u^2 + a^2}}{u} \right) + c = -\frac{1}{a} \operatorname{csch}^{-1} \frac{u}{a} + c$$

$$258) \int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + c$$

$$259) \int \frac{1}{a^2 - u^2} du = \frac{1}{a} \operatorname{tanh}^{-1} \frac{u}{a} + c \quad |u| < a$$

$$260) \int \frac{1}{a^2 - u^2} du = \frac{1}{a} \operatorname{coth}^{-1} \frac{u}{a} + c \quad |u| > a$$

### 15.- Integrales que no tienen primitivas en términos de funciones elementales.

Para  $f = \operatorname{sen}, \operatorname{cos}, \operatorname{tan}, \operatorname{exp}, \operatorname{senh}, \operatorname{cosh}, \operatorname{tanh}, \dots$ :

$$261) \int f(ax^n) dx \quad n > 1$$

$$262) \int \frac{f(ax)}{x^n} dx \quad n \geq 1$$

$$263) \int \frac{e^{ax}}{x} dx = \ln(x) + \frac{ax}{1.1!} + \frac{a^2 x^2}{2.2!} + \frac{a^3 x^3}{3.3!} + \frac{a^4 x^4}{4.4!} + \dots + c$$

$$264) \int \frac{\operatorname{sen}(ax)}{x} dx = \frac{ax}{1.1!} - \frac{a^3 x^3}{3.3!} + \frac{a^5 x^5}{5.5!} - \frac{a^7 x^7}{7.7!} + \frac{a^9 x^9}{9.9!} - \dots + c$$

$$265) \int \frac{\operatorname{cos}(ax)}{x} dx = \ln(x) - \frac{a^2 x^2}{2.2!} + \frac{a^4 x^4}{4.4!} - \frac{a^6 x^6}{6.6!} + \frac{a^8 x^8}{8.8!} - \dots + c$$

### 16.- Teorema Fundamental del Cálculo.

$$266) \int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

### 17.- Fórmulas de Wallis.

$$267) \int_0^{\pi/2} \operatorname{sen}^n(x) dx = \begin{cases} \frac{1.3.5 \dots (n-1) \pi}{2.4.6 \dots n} \frac{\pi}{2}, & n \text{ es par positivo } \wedge \neq 1 \\ \frac{2.4.6 \dots (n-1) \pi}{1.3.5 \dots n} \frac{\pi}{2}, & n \text{ es impar } \wedge > 1 \end{cases}$$

$$268) \int_0^{\pi/2} \operatorname{cos}^n(x) dx = \begin{cases} \frac{1.3.5 \dots (n-1) \pi}{2.4.6 \dots n} \frac{\pi}{2}, & n \text{ es par positivo } \wedge \neq 1 \\ \frac{2.4.6 \dots (n-1) \pi}{1.3.5 \dots n} \frac{\pi}{2}, & n \text{ es impar } \wedge > 1 \end{cases}$$

$$269) \int_0^{\infty} \frac{\operatorname{sen}(mx)}{x} dx = \begin{cases} \frac{1}{2} \pi, & \text{si } m > 0 \\ 0, & \text{si } m = 0 \\ -\frac{1}{2} \pi, & \text{si } m < 0 \end{cases}$$

$$270) \int_0^{\infty} \frac{\operatorname{cos}(mx)}{x} dx = 0$$

### 18.- Integrales definidas inmediatas.

$$271) \int_0^{\pi} \operatorname{sen}(ax) \operatorname{sen}(bx) dx = \int_0^{\pi} \operatorname{cos}(ax) \operatorname{cos}(bx) dx = 0; \quad a \vee b \text{ son enteros.}$$

$$272) \int_0^{\pi/2} \operatorname{sen}(ax) \operatorname{cos}(ax) dx = 0$$

$$273) \int_0^{\pi} \operatorname{sen}(ax) \operatorname{cos}(ax) dx = 0$$

$$274) \int_0^{\pi} \operatorname{sen}(ax) \operatorname{cos}(bx) dx = \begin{cases} \frac{2a}{a^2 - b^2}, & \text{si } a - b \text{ es par;} \\ 0, & \text{si } a - b \text{ es impar} \end{cases} \quad a \vee b \text{ son enteros}$$

$$275) \int_0^{\infty} \frac{\operatorname{sen}(ax) \operatorname{sen}(bx) dx}{x^2} = \frac{\pi a}{2} \quad a \leq b$$

$$276) \int_0^{\pi} \operatorname{sen}^2(mx) dx = \frac{\pi}{2}$$

$$277) \int_0^{\pi} \operatorname{cos}^2(mx) dx = \frac{\pi}{2}$$

$$278) \int_0^{\infty} \operatorname{sen}(x^2) dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}$$

$$279) \int_0^{\infty} \operatorname{cos}(x^2) dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}$$

$$280) \int_0^{\infty} e^{-ax} dx = \frac{1}{a}; \quad a > 0$$

$$281) \int_0^{\infty} x^n e^{-ax} dx = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}}, & \text{si } n > -1, a > 0 \\ \frac{n!}{a^{n+1}}, & a > 0, n \in \mathbb{N}. \end{cases}$$

$$282) \int_0^{\infty} e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{2a}$$

$$283) \int_0^{\infty} x e^{-x^2} dx = \frac{1}{2}$$

$$284) \int_0^{\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

$$285) \int_0^{\infty} x^{2n} e^{-a x^2} dx = \frac{1.3.5 \dots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$

$$286) \int_0^{\infty} x^{2n+1} e^{-a x^2} dx = \frac{n!}{2 a^{n+1}}, \quad a > 0$$

$$287) \int_0^1 x^m e^{-ax} dx = \frac{m!}{a^{m+1}} \left[ 1 - e^{-a} \sum_{r=0}^m \frac{a^r}{r!} \right]$$

$$288) \int_0^{\infty} \sqrt{x} e^{-nx} dx = \frac{1}{2n} \sqrt{\frac{\pi}{n}}$$

$$289) \int_0^{\infty} \frac{e^{-nx}}{\sqrt{x}} dx = \sqrt{\frac{\pi}{n}}$$

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