

# FUNCIONES HIPERBÓLICAS.

## 1.- Definición de las funciones hiperbólicas.

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \cosh x = \frac{e^x + e^{-x}}{2} \quad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

## 2.- Funciones recíprocas.

$$\operatorname{csch} x = \frac{1}{\sinh x} \quad \operatorname{sech} x = \frac{1}{\cosh x} \quad \operatorname{coth} x = \frac{1}{\tanh x}$$

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}} \quad \operatorname{sech} x = \frac{2}{e^x + e^{-x}} \quad \operatorname{coth} x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 1}{e^{2x} - 1}$$

## 3.- Identidades hiperbólicas fundamentales.

$$\sinh x \operatorname{csch} x = 1 \quad \cosh x \operatorname{sech} x = 1 \quad \tanh x \operatorname{coth} x = 1 \quad \tanh x = \frac{\sinh x}{\cosh x} \quad \operatorname{coth} x = \frac{\cosh x}{\sinh x}$$

## 4.- Identidades pitagóricas.

$$\cosh^2 x - \sinh^2 x = 1 \quad \operatorname{coth}^2 x - 1 = \operatorname{csch}^2 x \quad 1 - \tanh^2 x = \operatorname{sech}^2 x$$

## 5.- Otras identidades.

$$\cosh x + \sinh x = e^x \quad \cosh x - \sinh x = e^{-x}$$

## 6.- Equivalencia entre las funciones hiperbólicas.

	$\sinh x$	$\cosh x$	$\tanh x$	$\operatorname{csch} x$	$\operatorname{sech} x$	$\operatorname{coth} x$
$\sinh x$	1	$\sqrt{\cosh^2 x - 1}$	$\frac{\tanh x}{\sqrt{1 - \tanh^2 x}}$	$\frac{1}{\operatorname{csch} x}$	$\frac{\sqrt{1 - \operatorname{sech}^2 x}}{\operatorname{sech} x}$	$\frac{1}{\sqrt{\operatorname{coth}^2 x - 1}}$
$\cosh x$	$\sqrt{1 + \sinh^2 x}$	1	$\frac{1}{\sqrt{1 - \tanh^2 x}}$	$\frac{\sqrt{1 + \operatorname{csch}^2 x}}{\operatorname{csch} x}$	$\frac{1}{\operatorname{sech} x}$	$\frac{\operatorname{coth} x}{\sqrt{\operatorname{coth}^2 x - 1}}$
$\tanh x$	$\frac{\sinh x}{\sqrt{1 + \sinh^2 x}}$	$\frac{\sqrt{\cosh^2 x - 1}}{\cosh x}$	1	$\frac{1}{\sqrt{1 + \operatorname{csch}^2 x}}$	$\sqrt{1 - \operatorname{sech}^2 x}$	$\frac{1}{\operatorname{coth} x}$
$\operatorname{csch} x$	$\frac{1}{\sinh x}$	$\frac{1}{\sqrt{\cosh^2 x - 1}}$	$\frac{\sqrt{1 - \tanh^2 x}}{\tanh x}$	1	$\frac{\operatorname{sech} x}{\sqrt{1 - \operatorname{sech}^2 x}}$	$\sqrt{\operatorname{coth}^2 x - 1}$
$\operatorname{sech} x$	$\frac{1}{\sqrt{1 + \sinh^2 x}}$	$\frac{1}{\cosh x}$	$\frac{\sqrt{1 - \tanh^2 x}}{\sqrt{1 + \operatorname{csch}^2 x}}$	$\frac{\operatorname{csch} x}{\sqrt{1 + \operatorname{csch}^2 x}}$	1	$\frac{\sqrt{\operatorname{coth}^2 x - 1}}{\operatorname{coth} x}$
$\operatorname{coth} x$	$\frac{\sqrt{1 - \sinh^2 x}}{\sinh x}$	$\frac{\cosh x}{\sqrt{\cosh^2 x - 1}}$	$\frac{1}{\tanh x}$	$\sqrt{1 + \operatorname{csch}^2 x}$	$\frac{1}{\sqrt{1 - \operatorname{sech}^2 x}}$	1

## 7.- Argumento doble.

$$\sinh 2x = 2 \sinh x \cosh x \quad \cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh 2x = 2 \sinh^2 x + 1 \quad \cosh 2x = 2 \cosh^2 x - 1 \quad \tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

## 8.- Argumento triple.

$$\sinh 3x = 4 \sinh^3 x + 3 \sinh x \quad \cosh 3x = 4 \cosh^3 x - 3 \cosh x$$

$$\tanh 3x = \frac{3 \tanh x + \tanh^3 x}{1 + 3 \tanh^2 x}$$

## 9.- Argumento mitad.

$$\sinh\left(\frac{x}{2}\right) = \sqrt{\frac{\cosh x - 1}{2}} \quad \cosh\left(\frac{x}{2}\right) = \sqrt{\frac{\cosh x + 1}{2}} \quad \tanh\left(\frac{x}{2}\right) = \sqrt{\frac{\cosh x - 1}{\cosh x + 1}}$$

$$\tanh\left(\frac{x}{2}\right) = \frac{\sinh x}{\cosh x + 1} \quad \tanh\left(\frac{x}{2}\right) = \frac{\cosh x - 1}{\sinh x} \quad \tanh\left(\frac{x}{2}\right) = \operatorname{coth} x - \operatorname{csch} x$$

$$\operatorname{coth}\left(\frac{x}{2}\right) = \sqrt{\frac{\cosh x + 1}{\cosh x - 1}} \quad \operatorname{coth}\left(\frac{x}{2}\right) = \frac{\sinh x}{\cosh x - 1} \quad \operatorname{coth}\left(\frac{x}{2}\right) = \frac{\cosh x + 1}{\sinh x}$$

$$\operatorname{coth}\left(\frac{x}{2}\right) = \operatorname{coth} x + \operatorname{csch} x \quad \sinh^2 x = \frac{1}{2}(\cosh 2x - 1) \quad \cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$$

## 10.- Suma y diferencia de dos argumentos.

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y \quad \cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

## 11.- Fórmulas de reducción.

$$\sinh(-x) = -\sinh x \quad \operatorname{csch}(-x) = -\operatorname{csch} x \quad \cosh(-x) = \cosh x$$

$$\operatorname{sech}(-x) = \operatorname{sech} x \quad \tanh(-x) = -\tanh x \quad \operatorname{coth}(-x) = -\operatorname{coth} x$$

## 12.- Suma y diferencia de funciones hiperbólicas.

$$\sinh x + \sinh y = 2 \sinh\left(\frac{x+y}{2}\right) \cosh\left(\frac{x-y}{2}\right) \quad \sinh x - \sinh y = 2 \sinh\left(\frac{x-y}{2}\right) \cosh\left(\frac{x+y}{2}\right)$$

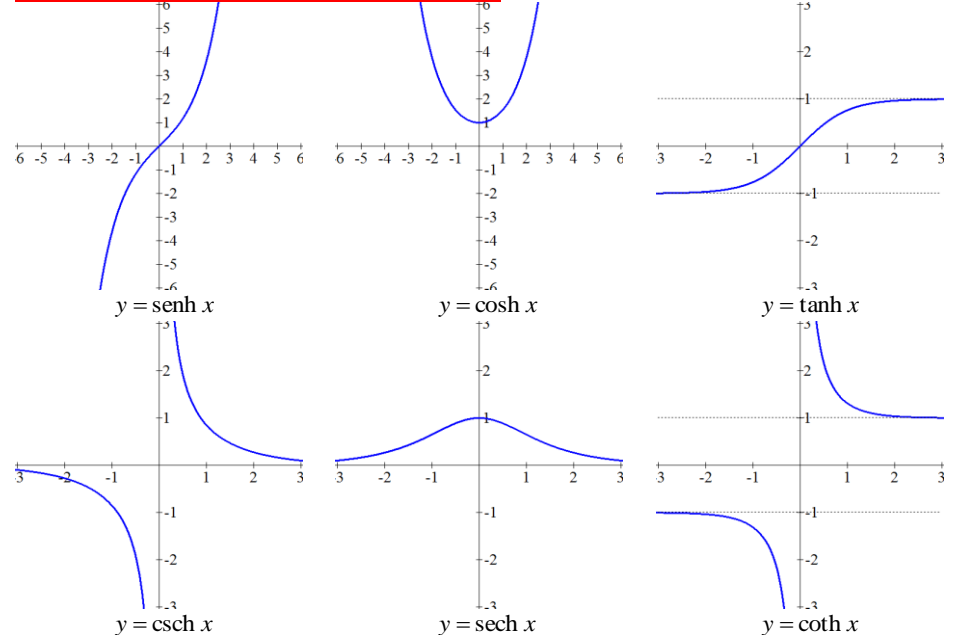
$$\cosh x + \cosh y = 2 \cosh\left(\frac{x+y}{2}\right) \cosh\left(\frac{x-y}{2}\right) \quad \cosh x - \cosh y = 2 \sinh\left(\frac{x+y}{2}\right) \sinh\left(\frac{x-y}{2}\right)$$

## 13.- Producto.

$$\sinh x \sinh y = \frac{1}{2}[\cosh(x+y) - \cosh(x-y)] \quad \cosh x \cosh y = \frac{1}{2}[\cosh(x+y) + \cosh(x-y)]$$

$$\sinh x \cosh y = \frac{1}{2}[\sinh(x+y) - \sinh(x-y)]$$

## 14.- Gráficos de las funciones hiperbólicas.



## 15.- Derivadas de las funciones hiperbólicas.

$$\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx} \quad \frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx} \quad \frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \operatorname{coth} u \frac{du}{dx} \quad \frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \frac{du}{dx} \quad \frac{d}{dx}(\operatorname{coth} u) = -\operatorname{csch}^2 u \frac{du}{dx}$$

## 16.- Integrales de las funciones hiperbólicas.

$$\int \sinh u \, du = \cosh u + C \quad \int \cosh u \, du = \sinh u + C \quad \int \tanh u \, du = -\ln|\operatorname{sech} u| + C$$

$$\int \operatorname{csch} u \, du = \ln|\tanh(\frac{1}{2}u)| + C \quad \int \operatorname{sech} u \, du = \tan^{-1}(\sinh u) + C \quad \int \operatorname{coth} u \, du = \ln|\sinh u| + C$$

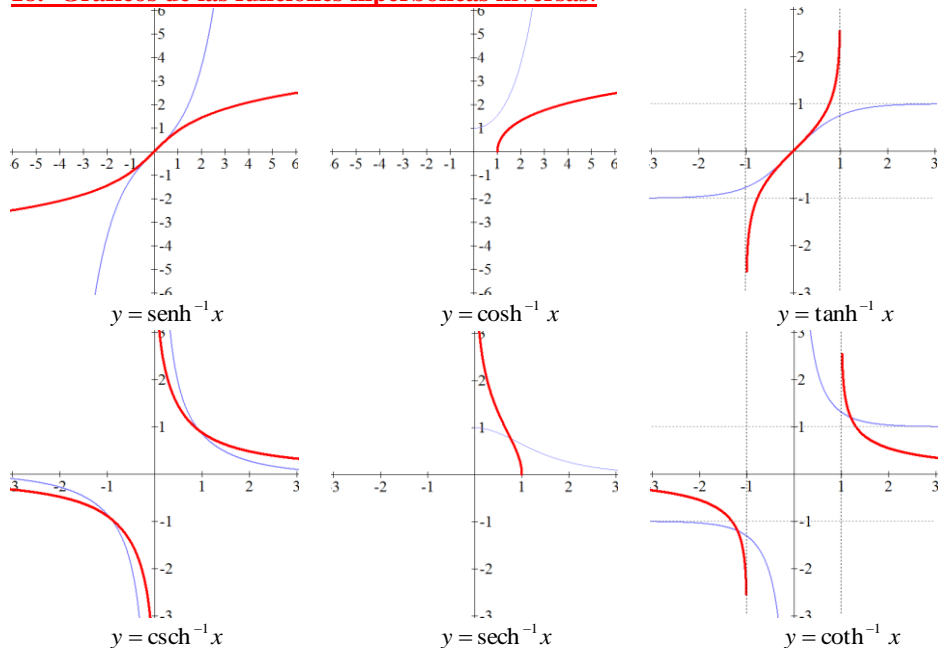
$$\int \operatorname{sech}^2 u \, du = \tanh u + C \quad \int \operatorname{csch}^2 u \, du = -\operatorname{coth} u + C \quad \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$\int \operatorname{csch} u \operatorname{coth} u \, du = -\operatorname{csch} u + C$$

## 17.- Definición de las funciones hiperbólicas inversas.

	Definición	Domino	Rango
Seno hiperbólico inverso	$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$	$R$	$R$
Coseno hiperbólico inverso	$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$	$[1, \infty)$	$[0, \infty)$
Tangente hiperbólica inversa	$\tanh^{-1} x = \ln \sqrt{\frac{1+x}{1-x}}$ $\tanh^{-1} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right)$	$(-1, 1)$	$R$
Cosecante hiperbólica inversa	$\operatorname{csch}^{-1} x = \ln \left( \frac{1 + \sqrt{1+x^2}}{x} \right)$ $\operatorname{csch}^{-1} x = \ln \left( \frac{1}{x} + \sqrt{\frac{1+x^2}{x^2}} \right)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
Secante hiperbólica inversa	$\operatorname{sech}^{-1} x = \ln \left( \frac{1 + \sqrt{1-x^2}}{x} \right)$ $\operatorname{sech}^{-1} x = \ln \left( \frac{1}{x} + \sqrt{\frac{1-x^2}{x^2}} \right)$	$(0, 1]$	$[0, \infty)$
Cotangente hiperbólica inversa	$\operatorname{coth}^{-1} x = \ln \sqrt{\frac{x+1}{x-1}}$ $\operatorname{coth}^{-1} x = \frac{1}{2} \ln \left( \frac{x+1}{x-1} \right)$	$(-\infty, -1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$

## 18.- Gráficos de las funciones hiperbólicas inversas.



## 27.- Identidades hiperbólicas inversas.

	$\sinh^{-1} \theta$	$\cosh^{-1} \theta$	$\tanh^{-1} \theta$	$\operatorname{csch}^{-1} \theta$	$\operatorname{sech}^{-1} \theta$	$\operatorname{coth}^{-1} \theta$
$\sinh^{-1} \theta$	1	$\cosh^{-1} \sqrt{1+\theta^2}$	$\tanh^{-1} \frac{\theta}{\sqrt{1+\theta^2}}$	$\operatorname{csch}^{-1} \frac{1}{\theta}$	$\operatorname{sech}^{-1} \frac{1}{\sqrt{1+\theta^2}}$	$\operatorname{coth}^{-1} \frac{\sqrt{1+\theta^2}}{\theta}$
$\cosh^{-1} \theta$	$\sinh^{-1} \sqrt{\theta^2-1}$	1	$\tanh^{-1} \frac{\sqrt{\theta^2-1}}{\theta}$	$\operatorname{csch}^{-1} \frac{1}{\sqrt{\theta^2-1}}$	$\operatorname{sech}^{-1} \frac{1}{\theta}$	$\operatorname{coth}^{-1} \frac{\theta}{\sqrt{\theta^2-1}}$
$\tanh^{-1} \theta$	$\sinh^{-1} \frac{\theta}{\sqrt{1-\theta^2}}$	$\cosh^{-1} \frac{1}{\sqrt{1-\theta^2}}$	1	$\operatorname{csch}^{-1} \frac{\sqrt{1-\theta^2}}{\theta}$	$\operatorname{sech}^{-1} \sqrt{1-\theta^2}$	$\operatorname{coth}^{-1} \frac{1}{\theta}$
$\operatorname{csch}^{-1} \theta$	$\sinh^{-1} \frac{1}{\theta}$	$\cosh^{-1} \frac{\sqrt{\theta^2+1}}{\theta}$	$\tanh^{-1} \frac{1}{\sqrt{\theta^2+1}}$	1	$\operatorname{sech}^{-1} \frac{\theta}{\sqrt{\theta^2+1}}$	$\operatorname{coth}^{-1} \sqrt{\theta^2+1}$
$\operatorname{sech}^{-1} \theta$	$\sinh^{-1} \frac{\sqrt{1-\theta^2}}{\theta}$	$\cosh^{-1} \frac{1}{\theta}$	$\tanh^{-1} \sqrt{1-\theta^2}$	$\operatorname{csch}^{-1} \frac{\theta}{\sqrt{1-\theta^2}}$	1	$\operatorname{coth}^{-1} \frac{1}{\sqrt{1-\theta^2}}$
$\operatorname{coth}^{-1} \theta$	$\sinh^{-1} \frac{1}{\sqrt{\theta^2-1}}$	$\cosh^{-1} \frac{\theta}{\sqrt{\theta^2-1}}$	$\tanh^{-1} \frac{1}{\theta}$	$\operatorname{csch}^{-1} \sqrt{\theta^2-1}$	$\operatorname{sech}^{-1} \frac{\sqrt{\theta^2-1}}{\theta}$	1

## 19.- Derivadas de las funciones hiperbólicas inversas.

$$\frac{d}{dx}(\sinh^{-1} u) = \frac{1}{\sqrt{u^2+1}} \frac{du}{dx} \quad \frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2-1}} \frac{du}{dx} \quad \frac{d}{dx}(\tanh^{-1} u) = \frac{1}{1-u^2} \frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{csch}^{-1} u) = \frac{-1}{|u|\sqrt{1+u^2}} \frac{du}{dx} \quad \frac{d}{dx}(\operatorname{sech}^{-1} u) = \frac{-1}{|u|\sqrt{1-u^2}} \frac{du}{dx} \quad \frac{d}{dx}(\operatorname{coth}^{-1} u) = \frac{1}{1-u^2} \frac{du}{dx}$$

## 20.- Integrales de las funciones hiperbólicas inversas.

$$\int \sinh^{-1} x \, dx = x \sinh^{-1} x - \sqrt{1+x^2} + c \quad \int \cosh^{-1} x \, dx = x \cosh^{-1} x - \sqrt{x^2-1} + c$$

## 21.- Integrales cuyas primitivas son funciones hiperbólicas inversas.

$$\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u + \sqrt{u^2+a^2}) + C$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln(u + \sqrt{u^2-a^2}) + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln \left( \frac{a+u}{a-u} \right) + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{a} \tanh^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a} \ln \left( \frac{a-u}{a+u} \right) + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{a} \operatorname{coth}^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a} \ln \left( \frac{a + \sqrt{a^2+u^2}}{u} \right) + C$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a} \operatorname{csch}^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a} \ln \left( \frac{a + \sqrt{a^2-u^2}}{u} \right) + C$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a} \operatorname{sech}^{-1} \frac{u}{a} + C$$

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Puerto La Cruz, abril de 2026.